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Optically induced phase transition of excitons in coupled quantum dots*

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The weak classical light excitations in many semiconductor quantum dots have been chosen as important solid-state quantum systems for processing quantum information and implementing quantum computing. For strong classical light we predict theoretically a novel phase transition as a function of magnitude of this classical light from the deformed to the normal phases in resonance case, and the essential features of criticality such as the scaling behaviour, critical exponent and universality are also present in this paper.

Keywords: phase transition, universality, excitation in quantum dot
PACC: 6460

With the variation of a controllable parameter, the ground-state energy spectrum of a many-body quantum system becomes nonanalytical at a certain value of this parameter, which is called phase transition.[1] It has been known that in condense matter systems the phase transition has become a key concept in the study of electrical and magnetic properties.[2−6] Recently, it has been exhibited to have the interesting relations to both quantum entanglement and Berry phase in quantum information and quantum computing.[7−15]

Semiconductor quantum dot,[16] which has the discrete energy-level structure resulting from their three-dimensional confinement, is a promising solid-state material to explore the quantum phenomena in quantum optics such as Rabi oscillations,[17] photon antibunching,[18] the optical Stark effect,[19] etc. The progress in ultrafast optoelectronics and nanostructure fabrication leads to an intensive study of the coherent-carrier control in semiconductor quantum dots.[20] The present ultrafast laser technology allows the carrier (electron and hole) wavefunctions to perform coherent manipulation on a time scale shorter than typical dephasing time.[21] Therefore, the weak driven field optical excitations in semiconductor quantum dots have been chosen as important solid-state quantum systems for processing quantum information and implementing quantum computing.[22−36] It is also very interesting that Bose–Einstein condensate for atom can be realized in excitons in semiconductor quantum dots at a temperature of a few kelvin by using the standard cryogenic techniques.[37−39]

In the present paper we predict theoretically a novel phase transition of excitons in coupled quantum dots in resonance case, which can be controlled by the strong classical light and therefore easily realized by using the current experimental technique, for example, ultrafast optical spectroscopy of semiconductor quantum dot. As the essential features of criticality the scaling behaviour, critical exponent and universality are also given.

We consider a system of N identical semiconductor quantum dots radiated by time-dependent classical light. Ignoring any constant energy terms, the time-dependent Hamiltonian in units of $\hbar$, which describes the formation of single-excitons with the individual semiconductor quantum dots and their interdot hopping, is given by[22−24]

$$\begin{aligned} H(t) = & \frac{\epsilon}{2}\sum_{n=1}^{N}(e_n^+ e_n - h_n h_n^+) \\ & -\frac{V_{n,n'}}{2}\sum_{n=1}^{N}(e_n^+ h_{n'} e_{n'} h_n^+ + h_n e_{n'}^+ h_{n'}^+ e_n) \\ & \times\xi(t)\sum_{n=1}^{N} e_n^+ h_n^+ + \xi^*(t)\sum_{n=1}^{N} h_n e_n, \end{aligned} \tag{1}$$

where $e_n^+(e_n)$ and $h_n^+(h_n)$ are the electron and hole creation (annihilation) operators in the nth semiconductor quantum dot, respectively, and they obey

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the anticommutation relations, i.e. $\{e_n, e_n^+\}_+ = \{h_n, h_n^+\}_+ = 1$; ϵ is the band gap of semiconductor quantum dot; $V_{n,n'}$ represents the Förster interaction which can transfer an exciton from one semiconductor quantum dot to another; for simplicity, the semiconductor quantum dots are assumed to be equidistant from each other, i.e. $N = 2$ dots on a line, $N = 3$ dots at the vertices of an equilateral triangle, $N = 4$ dots at the vertices of a pyramid, etc., hence, $V_{n,n'}$ is not a function of n or n' and satisfies the condition $V \ll \epsilon$; $\xi(t)$ is chosen to be $\xi(t) = A\exp[-\mathrm{i}\omega t]$ with ω being the central frequency and the magnitude A denoting the electron–photon coupling strength. Such a pulse is relatively straightforward to achieve in experiment. For processing quantum information and implementing quantum computing the weak driven field is used, namely, $A \ll V$.[22−24] However, if the strong driven field, namely $A \gg V$, is taken into account, a novel phase transition will occur.

By means of the Schwinger relations defined as $J_+ = \sum_{n=1}^{N} e_n^+ h_n^+$, $J_- = \sum_{n=1}^{N} h_n e_n$ and $J_z = \frac{1}{2}\sum_{n=1}^{N}(e_n^+ e_n - h_n h_n^+)$, which satisfy the well-known SU(2) angular momentum commutation relationships, i.e. $[J_+, J_-] = 2J_z$ and $[J_\pm, J_z] = \mp J_\pm$ with the length of the total angular momentum being $j = N/2$, the time-dependent Hamiltonian (1) can be written, regardless of the constant energy $-VJ^2$, as

$$H(t) = VJ_z^2 + \epsilon J_z + A\mathrm{e}^{-\mathrm{i}\omega t}J_+ + A\mathrm{e}^{\mathrm{i}\omega t}J_-. \quad (2)$$

The well-known unitary transformation operator

$$R(t) = \exp[-\mathrm{i}\omega J_z t] \quad (3)$$

can be used to transform the time-dependent Hamiltonian (2) into a time-independent Hamiltonian. In terms of the unitary transformation $|\Psi_n(t)\rangle = R^+(t)\,|\psi_n(t)\rangle$, the Schrödinger equation $\mathrm{i}\mathrm{d}\,|\psi_n(t)\rangle/\mathrm{d}t = H(t)\,|\psi_n(t)\rangle$ becomes $\mathrm{i}\mathrm{d}\,|\Psi_n(t)\rangle/\mathrm{d}t = H_r\,|\Psi_n(t)\rangle$ with $H_r = R^+(t)H(t)R(t) + \mathrm{i}R(t)\mathrm{d}R^+(t)/\mathrm{d}t$. After a simple algebraic calculation Hamiltonian (2) turns into a time-independent Hamiltonian

$$H_r = VJ_z^2 - \Delta J_z - AJ_x, \quad (4)$$

where $\Delta = \omega - \epsilon$ is the detuning parameter. The well-known resonant condition $\Delta = 0$ can lead to the following Hamiltonian: $H_r = VJ_z^2 - AJ_x$.

In order to analytically show the phase transition induced strong classical light, we here employ the Holstein–Primakoff transformation of the angular momentum operators defined as $J_+ = b^+\sqrt{N - b^+b}$, $J_- = (J_+)^+$, and $J_z = (b^+b - N/2)$,[40] where the introduced boson operators satisfy the commutation relation $[b, b^+] = 1$. It has been known that in order to expand these operators in a standard way we should suppose $b^+b/N \ll 1$, which means that the quantum system is assumed to be in the ground state or fully polarized in the z-direction in the thermodynamical limit. If the collective behaviour for Hamiltonian (4), induced by the long-range corrections, is considered, we can either perform a rotation of the z axis to the semiclassical spin direction,[41] or shift the bosonic operators by setting $d^+ = b^+ - \sqrt{N}\beta$.[42] It should be noticed that the displacement $d^+ = b^+ + \sqrt{N}\beta$ also leads to the same result except for the change of sign. The latter procedure provides a macroscopic expectation value of J_z which is of the order of N and then $b^+b/N \ll 1$ is satisfied. By using the latter approach, a rotation of coordinates in which $x \to z$ and $z \to x$ is implemented and therefore Hamiltonian (4) can be rewritten as $H_r = \frac{\lambda}{N}(J_+^2 + J_-^2 + J_+J_- + J_-J_+) - \Delta J_x - AJ_z$ with $\lambda = NV/4$, which can be expanded in terms of the new boson operators d^+ into a power series of $1/\sqrt{N}$ as expressed as

$$H_r = NH_0 + N^{1/2}H_1 + N^0H_2 + \cdots, \quad (5)$$

where $H_0 = [8\lambda\eta^2 + A(1-\eta^4) - 2\Delta\eta(1+\eta^2)]/(2(1+\eta^2)^2)$, $H_1 = \{[8\lambda\eta(1-\eta^2) - \Delta(1-\eta^4) - 2A\eta(1+\eta^2)]/2(1+\eta^2)^{3/2}\}(d^+ + d)$, $H_2 = \{[-2A(1+\eta^2) + 4\lambda(1-6\eta^2) - \Delta\eta(1+\eta^2)(4+\eta^2)]d^+d + [2\lambda(1-4\eta^2) - \Delta\eta(1+\eta^2/2)(1+\eta^2)][(d^+)^2+d^2] + 2\lambda(1-2\eta^2) - \Delta\eta(1+\eta^2)\}/(2(1+\eta^2))$, with $\beta = \eta/\sqrt{1+\eta^2}$.

It has been known that the first term (NH_0) can give rise to the Hartree–Bogoliubov ground-state energy.[42] By minimizing this ground-state energy we can cancel the term of order $N^{1/2}$, which implies that the free energy should be fixed at a minimum value in the quantum system. A simple study of the minimized Hartree–Bogoliubov ground-state energy shows that the parameter η_0 is an odd function of Δ for all $\Delta \neq 0$. However, for $\Delta = 0$, namely, the resonance case, we have a novel second-order phase transition as will be shown.

According to the condition for the resonance case $\Delta = 0$ in which $4\lambda\eta(1-\eta^2) - A\eta(1+\eta^2) = 0$ resulting from $H_1 = 0$, the parameter η_0 can be immediately obtained to be

$$\eta_0 = \begin{cases} \sqrt{(4\lambda - A)/(4\lambda + A)}, & A < A_\mathrm{c}, \\ 0, & A > A_\mathrm{c}, \end{cases} \quad (6)$$

where the critical electron–photon coupling strength is given by

$$A_{\rm c} = 4\lambda = NV. \tag{7}$$

The corresponding scaled Hartree–Bogoliubov ground-state energy E_0/N becomes

$$\frac{E_0}{N} = \begin{cases} \lambda + A^2/(16\lambda), & A < A_{\rm c}, \\ A/2, & A > A_{\rm c}. \end{cases} \tag{8}$$

Figure 1 shows the ground-state energy and its second-derivative with respect to A as a function of A. It can be seen that the second-derivative of the ground state energy possesses a discontinuity at the transition point $A_{\rm c}$ (its first-derivative with respect to A is continuous at the transition point $A_{\rm c}$), which clearly illustrates the nature of phase transition. If the spin Hamiltonian (4) is mapped into a single-particle problem with an effective potential,[43] it can be shown that in the case $A < A_{\rm c}$, this effective potential is a symmetric double-well potential in which the quantum tunnelling occurs, whereas in the case $A > A_{\rm c}$, this effective potential becomes a single-well potential in which the quantum tunnelling is suppressed. Therefore, the behaviour similar to that in the Lipkin–Meshkov–Glick model of nuclear physics is called the deformed phase for $A < A_{\rm c}$ and the normal phase for $A > A_{\rm c}$.

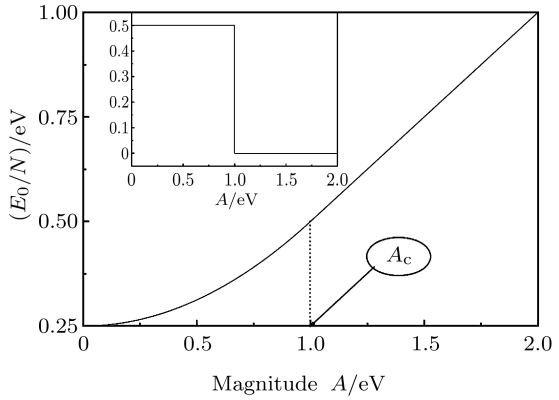


Fig.1. The scaled ground-state energy versus the magnitude of the classical light A with $V = 0.05\,\text{eV}$, $N = 20$ and $A_{\rm c} = 1.0\,\text{eV}$. Insert: its second-derivative with respect to A.

Experimental observation of this phase transition should be possible by using the present ultrafast semiconductor optical techniques.[21] For instance, for a wide-gap ZnSe-based semiconductor quantum dot, we have the band gap $\epsilon = 2.8\,\text{eV}$, which leads to an optical resonance frequency $\omega = 4 \times 10^{15}\,\text{s}^{-1}$, and the Coulomb energy $V = 0.05\,\text{eV}$.[44] Furthermore, the relatively straightforward experimental situation of global excitation pulse, the pulse that acts simultaneously on the entire semiconductor quantum dot system, is taken into account. The critical magnitude of the classical light is therefore obtained to be $A_{\rm c} = 0.5\,\text{eV}$ for $N = 10$ and $1\,\text{eV}$ for $N = 20$, which can be successfully achieved by using the recent experiment technique. It should be pointed out that the rigorous phase transition can be realized only in the thermodynamic limit $N \to \infty$, but some of the basic features of phase transition can still be demonstrated in a finite number of particles, which is of primary interest and belongs to a hot topic in modern physics.

To better understand this phase transition, we should evaluate the critical exponent of the excited state energy which is the essential feature of quantum criticality. By diagonalizing the third term $(N^0 H_2)$ under the resonance condition $\Delta = 0$ through the Bogoliubov transformation the excited state energies can be given by $\varepsilon^2 = \{[-A(1+\eta_0^2)+2\lambda(1-6\eta_0^2)]^2 - 4\lambda^2(1-4\eta_0^2)^2\}/(1+\eta_0^2)^2$, whose numerical results are plotted in Fig.2. When the magnitude of the classical light A approaches the critical value $A_{\rm c}$, this excited state energy vanishes as

$$\varepsilon(A \to A_{\rm c}) \approx \sqrt{4\lambda}\,|A - A_{\rm c}|^{\frac{1}{2}}\,. \tag{9}$$

The corresponding characteristic length scale is $l \approx |A - A_{\rm c}|^{-v}$ with $v = 1/2$. Therefore, the dynamical critical exponent can be derived from $\varepsilon = |A - A_{\rm c}|^{zv}$ by using $z = 1$, which shows the universality principle of phase transition.[1]

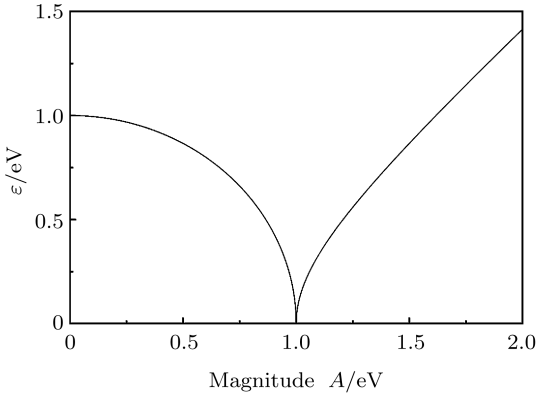


Fig.2. The excited-state energy versus the magnitude of the classical light A with the same parameters as those in Fig.1.

In conclusion, by means of Holstein–Primakoff transformation we have predicted theoretically a novel second-order phase transition of excitons in many

coupled semiconductor quantum dots induced by the time-dependent strong classical light in resonance case. It has been shown that as a function of magnitude of the classical light this phase transition occurs from the deformed to the normal phase. In addition, the essential features of criticality, such as the scaling behaviour, critical exponent and universality, have been also given.

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