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Intense Cherenkov-type terahertz electromagnetic radiation from ultrafast laser–plasma interaction*

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A Cherenkov-type terahertz electromagnetic radiation is revealed, which results efficiently from the collective effects in the time-domain of ultrafast pulsed electron current produced by ultrafast intense laser–plasma interaction. The emitted pulse waveform and spectrum, and the dependence of laser pulse parameters on the structure of the radiation field are investigated numerically. The condition of THz radiation generation in this regime and Cherenkov geometry of the radiation field are studied analytically.

Keywords: terahertz radiation, Cherenkov, ultrashort pulse laser, plasma
PACC: 5240M, 5225P

1. Introduction

Plasmas created by high-intensity laser pulses with ultrashort durations have received considerable attention as rich sources of electromagnetic radiations including coherent and incoherent radiations. Particularly, because the plasmas have no limit to the damage threshold, as a coherent radiation medium, it has the potential of generating high power terahertz (THz) radiations as compared with the conventional THz radiation generation mechanism such as electro-optic effect based on the laser-triggered solid-state.[1] The recent studies demonstrate that the generation of coherent radiation in the THz band from the laser–plasma interaction is achievable by two components, i.e. sub-ps electron bunch produced by a compact laser–plasma accelerator and the coherent transition radiation resulting from the accelerated electron bunch at the boundary of plasma and vacuum.[2−4]

In the process of interaction of an intense ultrashort pulse laser with a plasma, electrons can be efficiently accelerated to a high energy[5] and form a pulsed energetic electron bunch. In this paper, we demonstrate that the collective effect of this ultrafast pulsed electron bunch or current results in the electromagnetic emission of Cherenkov-type radiation in a THz range, which is another powerful mechanism to generate the pulsed THz radiation by the ultrafast laser–plasma interaction. The emitted pulse waveform in time-domain and radiation spectrum, and the dependence of laser pulse parameters on the structure of the radiation field are investigated numerically. The condition of THz radiation generation in this regime and Cherenkov geometry of the radiation field are revealed analytically.

2. Basic equations

When an ultrashort intense laser pulse propagating through a plasma the density of electron current, due to the action of the laser radiation field, is given by[6,7]

$$\boldsymbol{J} = -e(n_0 + n)\boldsymbol{v}, \tag{1}$$

where e is the electronic charge, n_0 and n are the ambient and disturbed electron densities, respectively, and $\boldsymbol{v}$ is the electron velocity. The normalized electron density within the laser pulse can be derived as[8,9]

$$\frac{n}{n_0} = \frac{c}{\omega_p^2}\nabla\cdot\left(\frac{\partial\hat{\boldsymbol{p}}}{\partial t} + c\nabla\gamma\right), \tag{2}$$

where $\omega_{\rm p} = \sqrt{4\pi e^2 n_0/m_{\rm e}}$ is the plasma frequency, $\hat{\boldsymbol{p}} = \dfrac{\gamma\boldsymbol{v}}{c}$ the dimensionless electron momentum, and $\gamma = (1+\hat{p}^2)^{1/2}$ the Lorentz factor. Substituting Eq.(2) into Eq.(1) and expanding the momentum $\hat{\boldsymbol{p}}$ in power into $\hat{p} = \hat{p}_1 + \hat{p}_2 + \cdots$, and retaining terms

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up to the second order of the momentum in Eq.(1), one can obtain

$$J \approx -en_0c(\hat{p}_1+\hat{p}_2)+en_0c\hat{p}_1\frac{c}{\omega_p^2}\nabla\cdot\frac{\partial\hat{p}_1}{\partial t}. \quad (3)$$

The first-order momentum $\hat{\boldsymbol{p}}_1$ is dependent on the form of laser pulse in the linear approximation and usually is expressed as

$$\hat{p}_1=\frac{1}{2}[a(r,t)\exp(-\mathrm{i}\omega_0t+\mathrm{i}k_0\cdot r)+\mathrm{c.c.}], \quad (4)$$

where $\boldsymbol{a}(\boldsymbol{r},t)$ is the complex amplitude that is supposed to be a slowly varying function both in time in the laser radiation period $(2\pi/\omega_0)$ and in space in its wavelength $(2\pi/k_0)$, ω_0 and k_0 are the frequency and the wave vector of the laser pulse, respectively. Using Eq.(4) and the hydrodynamic equations, the second order momentum $\hat{\boldsymbol{p}}_2$ can be presented as[8,9]

$$\hat{\boldsymbol{p}}_2=-\frac{c}{2\omega_\mathrm{p}}\nabla\int_{-\infty}^{t}\mathrm{d}t'\sin[\omega_\mathrm{p}(t-t')]\frac{\partial}{\partial t}\hat{p}_1^2(r,t'). \quad (5)$$

Introduce a dimensionless function

$$\phi=\omega_\mathrm{p}\int_{-\infty}^{t}\mathrm{d}t'\sin[\omega_\mathrm{p}(t-t')]\hat{p}_1^2(r,t'), \quad (6)$$

which is the solution of the equation

$$\frac{\partial^2\phi}{\partial t^2}+\omega_\mathrm{p}^2\phi=\omega_\mathrm{p}^2\hat{p}_1^2, \quad (7)$$

then, in accordance with Eq.(5), we will have

$$\hat{\boldsymbol{p}}_2=-\frac{c}{2\omega_p^2}\nabla\frac{\partial\phi}{\partial t}. \quad (8)$$

Therefore, the quasistatic current for one pulse duration can be derived from Eq.(3) as

$$\bar{\boldsymbol{J}}=en_0\frac{c^2}{2\omega_p^2}\nabla\frac{\partial\bar{\phi}}{\partial t}, \quad (9)$$

where

$$\bar{\phi}=\frac{\omega_\mathrm{p}}{2}\int_{-\infty}^{t}\mathrm{d}t'\sin[\omega_\mathrm{p}(t-t')]|a(\boldsymbol{r},t')|^2. \quad (10)$$

3. Emission of Cherenkov radiation

We now restrict our consideration to a steady axisymmetric laser pulse in a cylindrical coordinate system and assume that the incident laser pulse has the form $|a(\boldsymbol{r},t')|^2=a_0^2f_1(z,t)f_2(\rho)$, where a_0 denotes the peak value of the normalized laser intensity and functions $f_1(z,t)$ and $f_2(\rho)$ determine the shapes of the pulse in the longitudinal and the transverse directions, respectively.

To find the emitted radiation analytically, as an example, the longitudinal envelope of the pulse is taken as the form

$$f_1(z,t)=\sin^2[k_\mathrm{L}(z-v_\mathrm{g}t)], \quad (11)$$

and the transverse shape of the pulse is assumed to be in the Gaussian form

$$f_2(\rho)=\exp[-\rho^2/\rho_0^2], \quad (12)$$

where v_g is the group speed of the laser pulse in the plasma; $k_\mathrm{L}=2\pi/L$; L and ρ_0 are the space length and effective radius of the beam spot of the laser pulse, respectively. From Eq.(10), we have

$$\begin{aligned}\bar{\phi}=&-\frac{a_0^2f_1(\rho)}{4(k_\mathrm{L}^2-k_\mathrm{p}^2)}\\&\times[k_\mathrm{L}^2-k_\mathrm{p}^2+k_\mathrm{p}^2\cos(k_\mathrm{L}(z-v_\mathrm{g}t))\\&-k_\mathrm{L}^2\cos(k_\mathrm{p}(z-v_\mathrm{g}t))]. \quad (13)\end{aligned}$$

Substituting Eq.(13) into Eq.(9), the quasistatic current components for one pulse duration can be presented as follows:

$$\begin{aligned}\bar{J}_\rho=&en_0\frac{c^2}{2\omega_\mathrm{p}^2}\frac{a_0^2\rho f_1(\rho)}{2(k_\mathrm{L}^2-k_\mathrm{p}^2)\rho_0^2}\\&\times[k_\mathrm{p}^2k_\mathrm{L}v_\mathrm{g}\sin(k_\mathrm{L}(z-v_\mathrm{g}t))\\&-k_\mathrm{L}^2k_\mathrm{p}v_\mathrm{g}\sin(k_\mathrm{p}(z-v_\mathrm{g}t))], \quad (14)\\ \bar{J}_z=&-en_0\frac{c^2}{2\omega_\mathrm{p}^2}\frac{a_0^2f_1(\rho)}{4(k_\mathrm{L}^2-k_\mathrm{p}^2)}\\&\times[k_\mathrm{p}^2k_\mathrm{L}^2v_\mathrm{g}\cos(k_\mathrm{L}(z-v_\mathrm{g}t))\\&-k_\mathrm{L}^2k_\mathrm{p}^2v_\mathrm{g}\cos(k_\mathrm{p}(z-v_\mathrm{g}t))]. \quad (15)\end{aligned}$$

Equations (14) and (15) show that the generation of the quasistatic electron current in a pulse duration is due to the fact that a group of electrons inside the laser pulse induces an ultrafast collective effect.

From the d'Alembert equation, the vector potential of the emitted radiation due to the pulsed current, as a function of time at the viewpoint $\boldsymbol{x}$, reads[10,11]

$$\boldsymbol{A}(\boldsymbol{x},t)=\int\frac{\boldsymbol{J}(\boldsymbol{x}',t-R/v_\varphi)}{cR}\mathrm{d}V', \quad (16)$$

where R is the distance from the current centre to the viewpoint, and v_φ is the phase speed of the radiation wave. Assuming $\rho_0 << R$ and considering the far field radiation, we have $R\approx\sqrt{\rho^2+(z-z')^2}$. Substituting Eqs.(14) and (15) into Eq.(16) yields

$$A_\rho = en_0\frac{c^2}{2\omega_{\rm p}^2}\frac{a_0^2 v_{\rm g}k_{\rm p}k_{\rm L}}{2(k_{\rm L}^2-k_{\rm p}^2)\rho_0^2} \times\int\frac{\rho'^2\exp[-\rho'^2/\rho_0^2][k_{\rm p}\sin(k_{\rm L}(z'-v_{\rm g}(t-R/v_\varphi)))-k_{\rm L}\sin(k_{\rm p}(z'-v_{\rm g}(t-R/v_\varphi)))]}{cR}{\rm d}\rho'{\rm d}z' \quad (17)$$

and

$$A_z = -en_0\frac{c^2}{2\omega_{\rm p}^2}\frac{a_0^2 v_{\rm g}k_{\rm p}^2k_{\rm L}^2}{4(k_{\rm L}^2-k_{\rm p}^2)} \times\int\frac{\rho'\exp[-\rho'^2/\rho_0^2][\cos(k_{\rm L}(z'-v_{\rm g}(t-R/v_\varphi)))-\cos(k_{\rm p}(z'-v_{\rm g}(t-R/v_\varphi)))]}{cR}{\rm d}\rho'{\rm d}z'. \quad (18)$$

Therefore, the magnetic field of the radiation, according to the relation $\boldsymbol{B}=\nabla\times\boldsymbol{A}$, can be written as

$$B_\varphi=\frac{\partial A_\rho}{\partial z}-\frac{\partial A_z}{\partial\rho},\quad B_z=0,\quad B_\rho=0. \quad (19)$$

Substituting Eqs.(17) and (18) into Eq.(19), we can carry out the numerical calculation of the radiating magnetic field, and the results are shown in Fig.1.

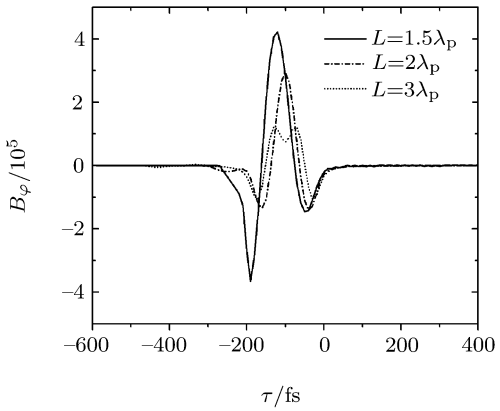


Fig.1. The radiating magnetic field in time-domain for different values of laser pulse and plasma parameters, a_0=0.5, ρ_0=1μm, $\lambda_{\rm L}$= 1.06μm, $\tau_{\rm L}$=200fs, and $L=h\lambda_{\rm p}$ (h=1.5, 2, 3).

In Fig.1, $\tau_{\rm L}$ is the laser pulse duration and $\lambda_{\rm p}$ is the wavelength of the plasma wave. When $h=2$, the corresponding plasma density is about $10^{18}{\rm cm}^{-3}$. Figure 1 shows that the electromagnetic radiation is emitted when the ultrashort intense laser pulse propagates in plasma and the waveform is dependent on plasma density. It implies that, theoretically, the waveform can be shaped by changing the plasma density. When h is an integer, the radiated magnetic field is symmetrical with respect to the centre of radiation pulse as shown in Fig.1 with dot dash and dotted curves, respectively. This is the resonant modulation of the plasma wave to the radiation field. From Fig.1 one can also see that the duration of radiation pulse is close to that of laser pulse, which means that the response of free electron to the laser pulse is fast compared with the response of the laser-triggered solid-state to the laser pulse.

The spectrum of radiation field is obtained by performing the Fourier transform

$$\boldsymbol{B}(\omega)=\int_{-\infty}^{\infty}\boldsymbol{B}(t)\exp({\rm i}\omega t){\rm d}t. \quad (20)$$

Using Eqs.(17)–(20), we perform the numerical calculation and the results are shown in Fig.2.

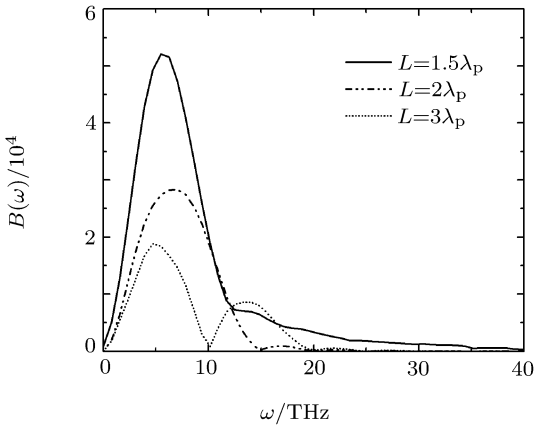


Fig.2. The spectra of the radiation corresponding to those in Fig.1, in which a_0=0.5, ρ_0=1μm, $\lambda_{\rm L}$= 1.06μm, $\tau_{\rm L}$=200fs, and $L=h\lambda_{\rm p}$ (h=1.5, 2, 3).

It is shown in Fig.2 that the frequencies of the emitted radiation field lie in a THz range, which implies another approach to generating a THz source by the ultrashort pulse laser–plasma. One can see from Fig.2 that when $\tau_{\rm L}$=200fs the first central frequency of the radiation spectrum has a shift of 5–7THz for different plasma densities. But it is worthy to be emphasized that the second central frequency appears

and the central frequencies display a diploid relation when $L = 3\lambda_{\rm p}$, which may be the radiation of plasma harmonic field.

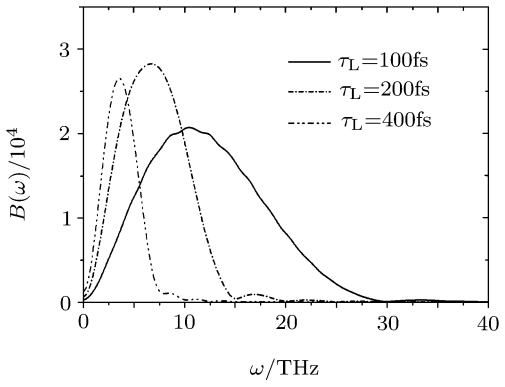


Fig.3. The spectra of the radiation for different laser pulse durations, in which a_0=0.5, ρ_0=1μm, $\lambda_{\rm L}$= 1.06μm, and $L = 2\lambda_{\rm p}$.

Figure 3 displays the radiation spectra for different laser pulse durations. And it shows that the first central frequency appears around $\tau_{\rm L}^{-1}$. For example, as shown in Fig.3, the first central frequency $\omega \approx 11$THz for $\tau_{\rm L}$=100fs; $\omega \approx 6$THz for $\tau_{\rm L}$=200fs; $\omega \approx 3$THz for $\tau_{\rm L}$=400fs.

Figure 4 shows the time dependences of the radiated magnetic field for different laser intensities. Generally, the radiation intensity can be expressed as $I_{\rm R} = (c/8\pi)|\boldsymbol{B}|^2$. For $L = 2\lambda_{\rm p}$, as shown in Fig.4, the magnetic field of the radiation is $B = 11.7 \times 10^5 a_0^2$Gs. Therefore, the radiation intensity is $I_{\rm R} = 1.4 \times 10^{14} a_0^4$W·cm^{-2}. It demonstrates the possibility of generating a high-power THz source by ultrashort intense laser–plasma interaction.

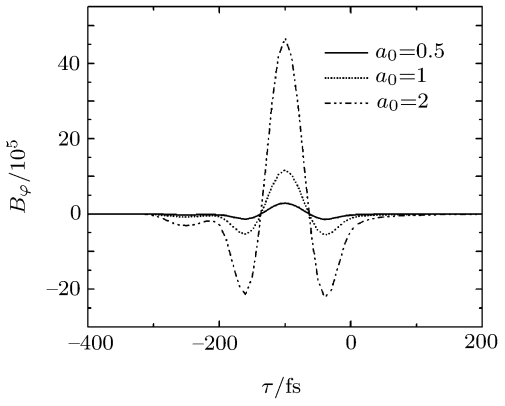


Fig.4. Radiation pulse in time-domain for different laser intensities, in which ρ_0=1μm, $\lambda_{\rm L}$= 1.06μm, $\tau_{\rm L}$=200fs, and $L = 2\lambda_{\rm p}$.

4. Discussion and conclusion

To understand the generation of the radiation field further, we perform the Fourier transform of the current density $\boldsymbol{J}(\boldsymbol{x}', t)$ as follows:

$$\boldsymbol{J}_\omega(\boldsymbol{x}') = \frac{1}{2\pi}\int_{-\infty}^{\infty} \boldsymbol{J}(\boldsymbol{x}', t)\exp(\mathrm{i}\omega t)\mathrm{d}t. \quad (21)$$

Substituting Eq.(21) into Eq.(16), we have

$$\boldsymbol{A}(\boldsymbol{x}, t) = \frac{1}{2\pi c}\int_{-\infty}^{\infty}\exp(-\mathrm{i}\omega t)\mathrm{d}\omega \times \int \frac{\boldsymbol{J}_\omega(\boldsymbol{x}')}{R}\exp(\mathrm{i}\omega R/v_\varphi)\mathrm{d}V'. \quad (22)$$

Therefore

$$\boldsymbol{A}_\omega(\boldsymbol{x}) = \frac{1}{2\pi c}\int_{-\infty}^{\infty}\exp[\mathrm{i}\omega(t' + R/v_\varphi)]\mathrm{d}t' \times \int \frac{\boldsymbol{J}(\boldsymbol{x}', t')}{R}\mathrm{d}V'. \quad (23)$$

Considering the far field radiation, one can have $R = |\boldsymbol{R} - \boldsymbol{R}'| \approx R - \boldsymbol{n}\cdot\boldsymbol{x}(t')$, then

$$\boldsymbol{A}_\omega \approx \frac{1}{Rc}\exp(\mathrm{i}\omega R/v_\varphi) \times \int \exp[\mathrm{i}\omega(1 - \boldsymbol{n}\cdot\boldsymbol{v}_{\rm g}/v_\varphi)t']\,\boldsymbol{\Psi}\mathrm{d}t', \quad (24)$$

where $\Psi = \int J(x,t)\rho\mathrm{d}\rho\mathrm{d}z$. Using Eqs.(14) and (15), we have

$$\Psi_r = \chi\frac{m_{\rm e}c^2}{e}\frac{a_0^2 v_{\rm g} k_{\rm L}^2}{4(k_{\rm L}^2 - k_{\rm p}^2)}\frac{\rho_0\sqrt{\pi}}{4}\left(1 - \cos\frac{2\pi k_{\rm p}}{k_{\rm L}}\right), \quad (25)$$

$$\Psi_z = -\chi\frac{m_{\rm e}c^2}{2e}\frac{a_0^2 v_{\rm g} k_{\rm p} k_{\rm L}^2}{4(k_{\rm L}^2 - k_{\rm p}^2)}\frac{\rho_0^2}{2}\sin\left(\frac{2\pi k_{\rm p}}{k_{\rm L}}\right). \quad (26)$$

Therefore,

$$\boldsymbol{A}_\omega \approx \frac{\boldsymbol{\Psi}}{Rc}\exp(\mathrm{i}\omega R/v_\varphi)\delta[\omega(1 - \boldsymbol{n}\cdot\boldsymbol{v}_{\rm g}/v_\varphi)]. \quad (27)$$

Equation (27) implies that for emitting the radiation (i.e. $\boldsymbol{A}_\omega \neq 0$), the condition

$$\boldsymbol{n}\cdot\boldsymbol{v}_{\rm g}/v_\varphi = 1 \quad (28)$$

must be satisfied, which means that the radiation is of Cherenkov type. The emitting angle θ of Cherenkov photons relative to the propagating direction of the laser pulse is

$$\theta = \arccos(v_\varphi/v_{\rm g}). \quad (29)$$

This Cherenkov condition emerges when the group speed of the propagating ultrashort laser pulse is larger than the phase speed of the generated radiation wave, i.e. $v_{\rm g} > v_\varphi$, which is consistent with the early result reported by Kleinman and Auston.[12]

In conclusion, Cherenkov radiation at a terahertz frequency range can be generated by ultrafast laser–plasma interaction. In physics, this Cherenkov-type THz radiation results from the ultrafast change of the free electron current excited by an ultrashort laser pulse, which resembles the THz emitter by the laser-triggered photoconductive switch.[13] By using the numerical solutions, we present the waveform in time-domain of the radiation field, and then we demonstrate that the relationship between the radiation intensity at a THz frequency range and the pulse laser field satisfies $I_{\rm R} = 1.4 \times 10^{14} a_0^4 {\rm W \cdot cm^{-2}}$. And finally we find out the Cherenkov angle of the electromagnetic radiation, $\theta = \arccos(v_\varphi / v_{\rm g})$, which is consistent with the early theoretical prediction.

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