Vol 17 No 3, March 2008
1674-1056/2008/17(03)/1008-06
Chinese Physics B
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Energy exchange between (3+1)D colliding spatiotemporal optical solitons in dispersive media with cubic–quintic nonlinearity*

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(Received 28 March 2007; revised manuscript received 13 April 2007)

We investigate the energy exchange between (3+1)D colliding spatiotemporal solitons (STSs) in dispersive media with cubic–quintic (CQ) nonlinearity by numerical simulations. Energy exchange between two (3+1)D head on colliding STSs caused by their phase difference is observed, just as occurring in other optical media. Moreover, energy exchange between two head-on colliding STSs with different speeds is firstly shown in the CQ and saturable media. This phenomenon, we believe, may arouse some interest in the future studies of soliton collision in optical media.

Keywords: spatiotemporal solitons, cubic–quantic nonlinearity, energy exchange
PACC: 4265S, 4265J, 4265T

1. Introduction

The most fascinating features of soliton phenomena may perhaps be seen in interactions between solitons.[1−4] Interactional optical spatial solitons[2,3] and spatiotemporal optical solitons[5] even may have potential applications in optical logic and switching.[6,7] In light of the effects of interference occurring in the processes of soliton collision, interactions between optical solitons can be classified into two categories: coherent and incoherent interaction.[2,3] The first one appears in media for which either the response time is extremely fast or the response time is much shorter than the time that the relative phase of two interactional optical solitons can remain stationary. In these cases, the force between optical solitons is either attractive or repulsive, depending on their phase difference.[2,3] On the other hand, if the relative phase of the two interactional optical solitons varies much faster than the response time of the medium, the medium cannot respond to interference effects but responds only to the time-averaged intensity, which leads to a simple superposition of the intensities in the overlapping region. In this case, the resultant force is always attractive.[2,3]

For coherent interactions, there is energy exchange between two solitons when the phase difference is intermediate between 0 and π. The energy is scattered from one soliton to the other because the phase of the interference pattern in the overlapping region is different from those of both solitons.[2] This phenomenon has been observed experimentally and demonstrated numerically in interactions between diverse solitons, such as Kerr solitons,[3] quadratic solitons[8,9] and solitons in saturable Kerr media or photorefractive media.[10] In these cases, energy exchange appears to be sensitive to the phase difference between interactional solitons.[3,8−10]

Energy exchange also occurs in interactions between spatiotemporal solitons in (3+1) dimensions.[11,12] (3+1)D STSs are nondiffracting and nondispersing wave packets localized in all transverse dimensions of space, as well as in time.[5] They are called light bullets,[13] which will collapse in Kerr media.[13,14] To arrest the catastrophic collapse, various schemes have been proposed.[5] To mention but a few, the saturable nonlinearity[15,16] and the CQ nonlinearity[17,18] were introduced in the media. In saturable media, where the varying index saturates with increasing intensity, energy exchange between colliding (3+1)D STSs has been studied.[11,12] It is shown that as the phase difference between STSs is intermediate between 0 and π, the light bullet leading in phase grows (due to energy gain) at the expense of the other.[11,12]

*Project supported by the Key Project of Hunan Provincial Educational Department of China (Grant No 04A058).
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In this paper, we investigate the energy exchange between (3+1)D colliding STSs in media with CQ nonlinearity numerically. Our extensive simulations revealed that the energy exchange between colliding (3+1)D STSs not only depends on the initial phase difference between the STSs, but also on their initial velocities. Moreover, the same phenomenon occurs in saturable media as well.

2. Theoretical model

The evolution of the electromagnetic field envelope E in the dispersive CQ media is governed by the CQ nonlinear Schrödinger equation:

$$2\mathrm{i}\kappa_0 E_z + \nabla_\perp^2 E + \kappa_0 D E_{\tau\tau} + 2\kappa_0^2 (n_2/n_0)|E|^2 E - 2\kappa_0^2 (n_4/n_0)|E|^4 E = 0. \quad (1)$$

Here z is the propagation coordinate. The Laplacian $\nabla_\perp^2$ (representing the spatial diffraction) acts on the transverse coordinates x and y. The retarded time is $\tau \equiv t - z/v_\mathrm{g}$, where t is the real time and v_g is the group velocity of the carrier wave. κ_0 is the propagation constant; and $D > 0$ is the coefficient of temporal dispersion. Introducing rescaled variables $U = \sqrt{n_4/n_2}E$, $T = n_2\sqrt{2\kappa_0/Dn_0n_4}\tau$, $Z = (\kappa_0 n_2^2/n_0 n_4)z$, and $(X, Y) = \kappa_0 n_2\sqrt{2/n_0 n_4}(x, y)$, we transform Eq.(1) into a normalized form:[19]

$$\mathrm{i}U_Z + (U_{XX} + U_{YY} + U_{TT}) + |U|^2 U - |U|^4 U = 0. \quad (2)$$

Spherically symmetric solitary wave solutions to Eq.(2) in the form of $U(X, Y, Z, T) = V(r)\exp(\mathrm{i}kZ)$, where $r = \sqrt{X^2 + Y^2 + T^2}$ and k is a real propagation parameter, has been obtained.[17] It is assumed that the real amplitude V reaches its maximum value at $r = 0$ and $V_r \to 0$ as $r \to \infty$. With this assumed form, Eq.(2) can be reduced to an ordinary nonlinear differential equation:

$$V'' + (2/r)V' - kV + V^3 - V^5 = 0. \quad (3)$$

The above equation is usually solved by means of shooting method.[17] Figure 1 shows a set of solutions corresponding to different values of k.[17]

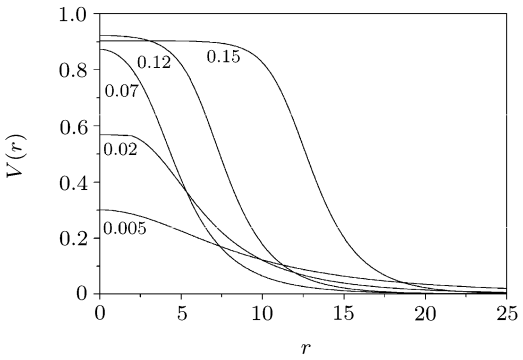


Fig.1. Radial solitary wave profiles $V(r)$. The values of k are indicated near the curves.

The energy of these (3+1)D zero-spin STSs is defined as

$$E(k) = 4\pi \int_0^\infty V^2(r, k) r^2 \mathrm{d}r. \quad (4)$$

Figure 2 displays the energy of (3+1)D STSs versus k.[17] There appears a distinctive U-shaped energy curve, where the minimum energy is $E_\mathrm{min}/(4\pi) \approx 15.08$ at $k_\mathrm{cr} \approx 0.026$.[17] In light of the well-known Vakhitov–Kolokov criterion,[5,19] the curve is divided into two branches. The stable and unstable branches correspond to the ones where $k_\mathrm{cr} > 0.026$ and $k_\mathrm{cr} < 0.026$, respectively. In our numerical simulations, we choose (3+1)D STSs in the stable branch.

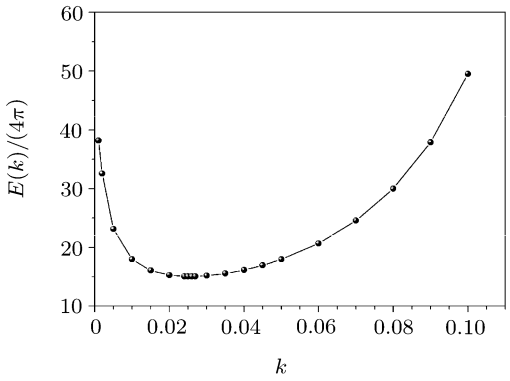


Fig.2. The energy of 3D STSs versus their propagation constant k.

3. Numerical method

With the obtained stable (3+1)D STSs, one can study the interaction processes between STSs by numerical simulations using the split-step Fourier method that is acknowledged to be one of the most efficient ways for solving the NLS equation.[20,21] The

second-order derivative with respect to X, Y and T are solved in Fourier space, so periodic boundary conditions are used in X, Y and T. Most of our numerical simulations are carried out on a $256 \times 256 \times 128$ (or $512 \times 256 \times 256$ when necessary) mesh and the step size is taken to be 0.005. For the Eq.(2), total energy is a conserved quantity.[22] We monitor this quantity in simulations to check the numerical accuracy. Moreover, we also change the step size to minimize the numerical artifacts.

4. Results

Here, the propagation constant is chosen to be $k = 0.07$ in the dispersive CQ media in order that the selected STSs stay in the stable branch. To introduce initial constant phase factor and initial velocity, one can multiply individual initial input profile $U(X, Y, Z = 0, T)$ by a factor $\exp(\mathrm{i}vT + \mathrm{i}\phi_0)$,[12] where v denotes the velocity relative to the group velocity and ϕ_0 represents the initial phase factor (for simplicity, we choose $0 \leq \phi_0 < 2\pi$ in this paper). Because of the spherical symmetry of the solutions, it is informative to draw the contour plots of $|U(X, T, Y = 0, Z = Z_0)|$ in $X - T$ plan, where Z_0 is a constant. However, to clearly show the whole interaction process between the two STSs, plot of $|U(X = 0, T, Y = 0, Z)|$ is necessarily presented, where distribution of light intensity in cross section cannot be shown.

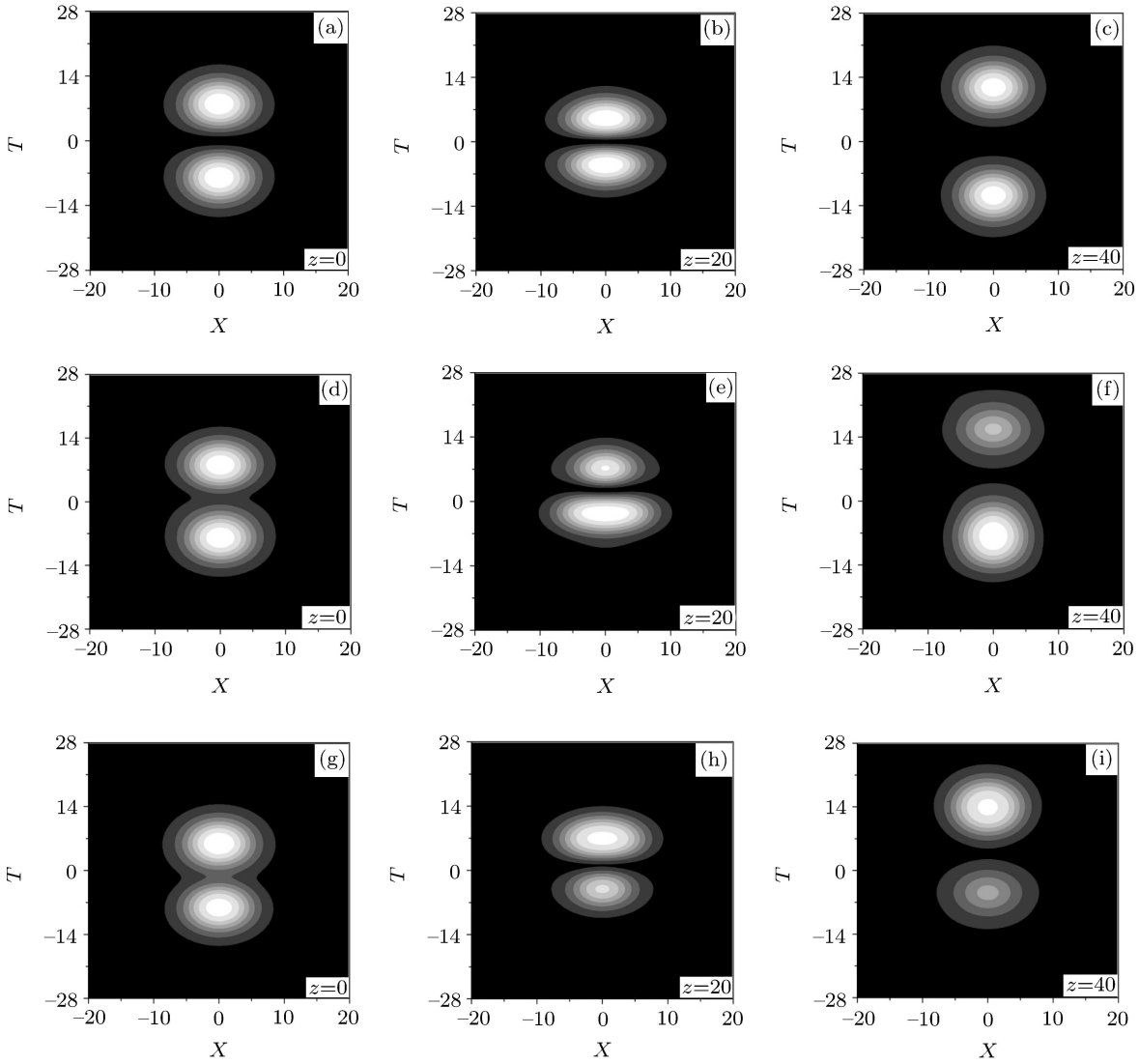


Fig.3. The contour plots of $|U(X, T, Y = 0, Z = Z_0)|$ in the $X - T$ plane displaying three head-on collisions of two STSs in the CQ media. In (a) and (d), The two colliding STSs have equal initial speeds, but the initial phase differences are π and $\pi/2$, respectively. In (g), the two colliding STSs have different initial speeds and the initial phase difference is π. Luminosity is proportional to the intensity of the light.

Figures 3(a), 3(b) and 3(c) are the contour plots of $|U(X,T,Y=0,Z=Z_0)|$ in $X-T$ plane to demonstrate the evolution of two repulsive STSs in the CQ media. The two STSs are initially moving in opposite directions with equal speed and their initial phase difference is π (the phase of the top light bullet in Fig.3(a) is chosen to be 0, the same for other figures). They firstly move head on, a repulsive force arising and increasing between them as they get nearer causes them to slow down and obviously change the light intensity distribution (i.e., change the shape of light bullets). As they rebound away from each other, they accelerate in opposite directions and regain their origin shapes approximately at $Z = 30.0$. It turns out to be almost no energy exchange between the two STSs in this process. We show the whole interaction process in Fig.4(f). When the two head on colliding STSs with equal speeds and 0 phase difference, they attract each other and fuse to one soliton after a finite number of oscillations of decreasing amplitude and period. Meanwhile, a small amount of energy of STSs is lost to radiation. Figure 4(a) shows this interacting process. It is clearly that the two STSs almost merge to one after the third oscillation. A number of cases of other phase differences are shown in Figs.4(b)–4(e) and Figs.4(g)–4(i). Figures 3(d), 3(e), 3(f) and Fig.4(d) show the collision of two STSs with equal and opposite initial velocities and $\pi/2$ initial phase difference.

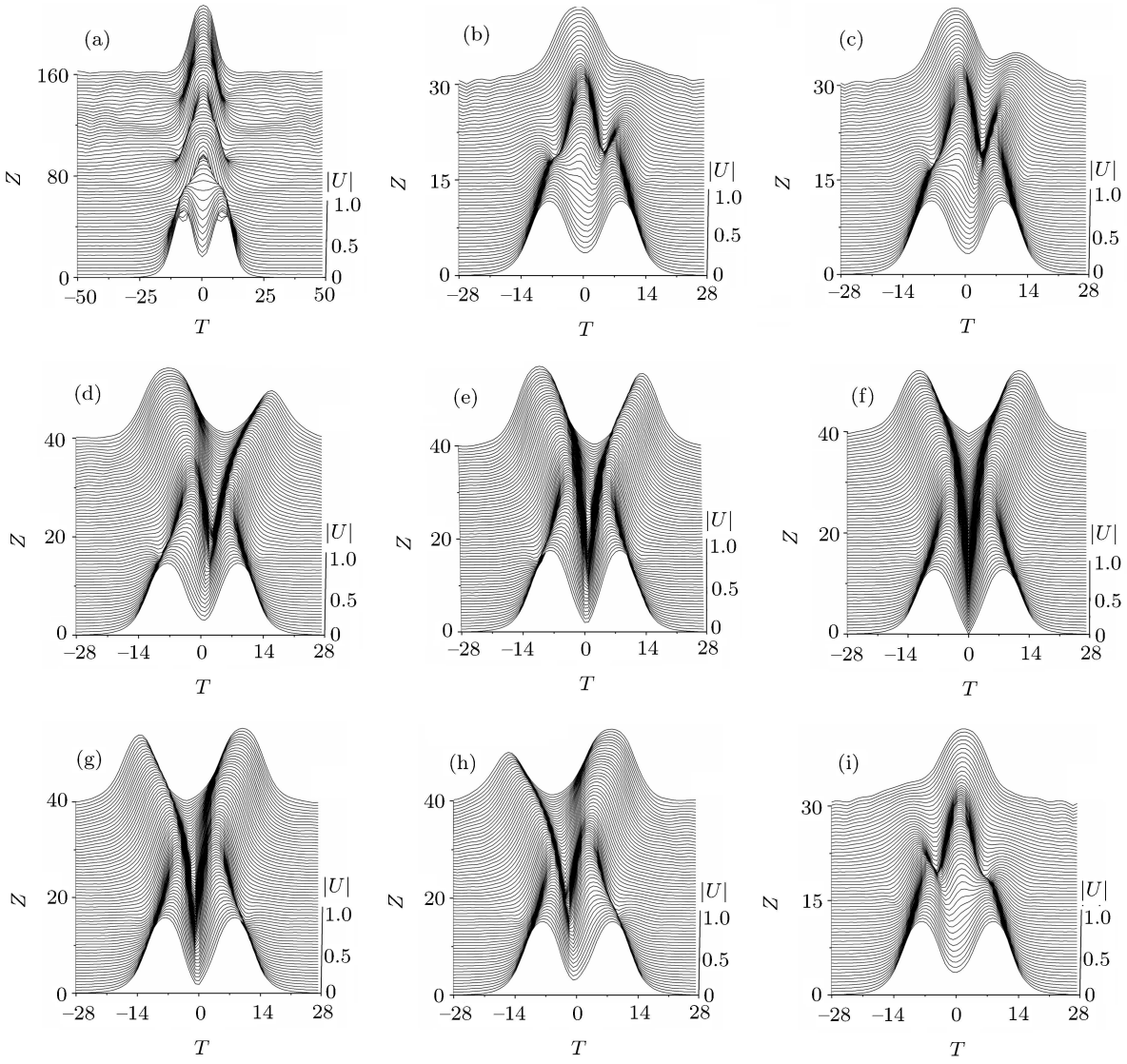


Fig.4. The whole interacting processes of two STSs are shown by plots of $|U(X=0,T,Y=0,Z)|$. (a)–(h) display the evolution of two head on colliding STSs with equal initial speeds, in which the initial phase differences are 0, $\pi/8$, $\pi/4$, $\pi/2$, $3\pi/4$, π, $5\pi/4$, $3\pi/2$, $15\pi/8$, respectively.

The energy is seen to be transferred from the light bullet initially lagging in phase to the one initially leading in phase. When the phase difference between the two STSs is $3\pi/2$ (in Fig.4(h)), it is found that the energy exchange is reversed. Energy is transferred from the light bullet leading in phase to the one lagging in phase. Extensive simulations reveal that when the phase difference between two colliding STSs of equal and opposite velocities varies from 0 to π, the light bullet leading in phase gains energy. While the phase difference varies from π to 2π, the one leading in phase loses energy.

Head on colliding STSs with different initial speeds have received little attention in previous studies. Here, we consider collisions of two STSs with different initial speeds and π-phase difference. Initially, we set the bottom optical bullet in Fig.3(g) at speed of 0.2, the top one 0. Figures 3(g)–3(i) and Fig.5(a) show that the bottom optical bullet moves towards the top one firstly. The top one, which is initially still, accelerates gradually at the expense of the bottom one slowing down. Interestingly, the energy is transferred from the bottom one to the top one. Another case of STSs with different speeds has also been consider, see Fig.5(b). In addition, the amount of transferred energy depends on the difference of initial speeds of the two STSs.

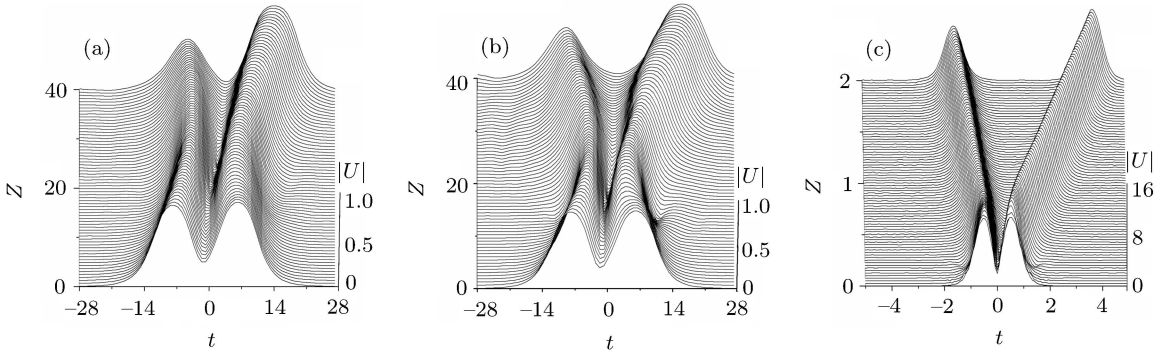


Fig.5. In (a)–(c), the two head on colliding STSs all have the same phase difference π, but the initial speeds of STSs are different. The speeds in the left hand side are 0.2, 0.3 and 0.5, and in the right hand side are 0, 0.1 and 0. The case of (c) is calculated in saturable media.

Moreover, as is shown in Fig.5(c), the same phenomenon occurs in the saturable media, where the propagation constant is chosen to be $k = 6$ to ensure the STSs being stable.

5. Conclusion

To summarize, we investigate energy exchange between two (3+1)D colliding STSs in dispersive media with CQ nonlinearity numerically and provide clear pictures for the processes. Extensive simulations reveal that there exists energy exchange between two head on colliding STSs with the same initial speeds. There appears to be two energy transfer modes, depending on whether the phase difference is intermediate between 0 and π or π and 2π. Importantly, energy exchange caused by the difference of the initial speeds of two head on colliding STSs is firstly shown in CQ media, and then in saturable media, which not only is helpful for a full grasp of properties of colliding STSs, but also may be instructive for future applications of STSs.

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