Vol 17 No 3, March 2008
1009-1963/2008/17(03)/0988-07
Chinese Physics B
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Surface solitons supported by one-dimensional composite Bessel optical lattices*

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(Received 29 October 2007; revised manuscript received 7 December 2007)

We address the existence of surface solitons at an interface in a defocusing cubic medium with an imprinted one-dimensional (1D) composite Bessel optical lattice. This setting is composed of two Bessel lattices with different orders and different modulation depths, separated beside both sides of an interface. Stability analysis and numerical propagation simulations prove that solitons supported by the model are dynamically stable in the entire domain of their existence. The order of lattice determines the shape of soliton, and the amplitude of soliton depends on the lattice modulation depth. The experimental realization of the scheme is also proposed. Our results may provide another effective way of controlling the shapes of surface solitons and thus their evolutions by introducing a new freedom degree.

Keywords: surface soliton, one-dimensional (1D), Bessel lattice
PACC: 4265, 4225B, 4265S, 9210H

1. Introduction

A special class of transversally localized waves, known as surface solitons or surface modes, can be supported by an interface separating different physical media or different refractive index modulations. The presence of interface substantially modifies the dynamics of solitons propagating along it. Surface solitons have attracted special attention these days due to their novel features in physics, i.e. for exploration of intrinsic and extrinsic surface characteristics, etc, as well as potential applications in optical sensing and switching. Optical surface waves were first observed at a nonlinear interface of a photorefractive crystal.[1] An interface between a waveguide array and a uniform material allows the formation of scalar surface solitons.[2,3] Interfaces of periodic media support gap solitons that were predicted theoretically in Ref.[4] and observed experimentally in Ref.[5]. Besides scalar solitons, stable vector discrete surface solitons[6] and multi-gap (mixed-gap) solitons[7,8] have also been reported. More recently, Siviloglou *et al*[9] have observed discrete surface solitons supported by quadratic nonlinearity. Multipole-mode surface solitons were shown to exist at the interface between two distinct periodic lattices imprinted in Kerr-type nonlinear media.[10] Polychromatic surface solitons in nonlinear photonic lattices were also predicted.[11] Moreover, vortex solitons can be captured stably by an interface between two optical lattices with different modulation depths.[12]

Bessel lattices have received intensive attention these days due to their ability to capture some kinds of solitons that cannot be supported by other schemes. Such lattices support stable vortex solitons,[13] spiraling solitons,[14] etc. Ring-shaped, single-site, discrete, and rotary soliton in modulated Bessel lattices were observed.[15,16] Furthermore, vortex solitons with a winding number larger than unity remained robust in Bessel lattices under appropriate conditions[17] and spatiotemporal solitons were also predicted to exist stably in Bessel lattices.[18] The radial symmetric Bessel lattices can be induced by non-diffracting Bessel beams. Different order Bessel beams can be created experimentally by illuminating a narrow annular split placed in the focal plane of a lens or axicon[19] or by using holographic techniques.[20]

However, all the aforementioned studies on Bessel lattice solitons focus on the two-dimensional cases because Bessel beams were created naturally in two dimensions. Moreover, researches on surface solitons involved mostly the self-focusing medium. So far, surface solitons have been observed only in waveguide arrays.[3,5,9] Observation of surface solitons in bulk

*Project supported by the National Natural Science Foundation of China (Grant No 10704067) and the Scientific Research Foundation of Education Bureau of Zhejiang Province of China (Grant No 20060493).
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media modulated by optical lattices is still an open and challenge problem. Very recently, we[21] have put forward a simple but new model, in which an interface separates 1D Bessel optical lattices with different lattice depths beside both sides of the interface. Such scheme supports stable surface solitons in the entire domain of their existence. Nevertheless, this work was restricted on the discussion about the effects of the lattice depth on the profile and the stability of solitons. Closely related problems arise: 1. whether this model supports the stationary solutions when the 1D Bessel lattices beside both sides of the interface are of different orders; 2. how the change of lattice order influences the profile of solitons; 3. whether the solitons are still stable.

Experimentally, the composite 1D Bessel lattices with variable orders and lattice depths beside both sides of an interface may be created by illuminating them with two Bessel beams with different frequencies and orders and their polarizations orthogonal to those beams guiding into a slab waveguide. Different orders of Bessel beams can be chosen by placing filters with different frequency responses (corresponding to the wavelength of Bessel beams) in front of the waveguide. The depth of refractive-index modulation beside the centre of waveguide can be adjusted by introducing an attenuating plate in front of the input Bessel beams.

In the present paper, we will investigate in detail the dynamics of solitons supported by varying the orders and depths of Bessel potential beside the side of an interface in a defocusing cubic medium. We should note that the difference between the present work and Ref.[21] is evident in that it introduces a new freedom degree for controlling the profile of surface solitons and thus their evolutions.

2. The surface model

We assume that a beam propagates along an interface separating two 1D Bessel lattices imprinted in a slab waveguide with Kerr-type defocusing nonlinearity. The generic equation describing the evolution of optical wave packets in this setting is the nonlinear Schrödinger equation for dimensionless complex amplitude of light field $A(x,z)$

$$\mathrm{i}\frac{\partial A}{\partial z}+\frac{1}{2}\frac{\partial^2 A}{\partial x^2}-|A|^2A+R(x)A=0, \qquad (1)$$

where $x=X/X_0$, and $z=Z/L_\mathrm{d}$ are the normalized transverse and longitudinal coordinate respectively, with X_0 being an arbitrary scale, $L_\mathrm{d}=k_0X_0^2$ the diffraction length, and k_0 the wave number. We suppose that optical lattices are created by $n_\mathrm{l}(x\le 0)$, $n_r(x>0)$-order Bessel beams, whose field is governed by the functions $p_\mathrm{r}J_{n_\mathrm{r}}[\sqrt{2\beta}x]\exp(-\mathrm{i}b_\mathrm{lin}z)$ at $x>0$ and $p_\mathrm{l}J_{n_\mathrm{l}}[\sqrt{2\beta}(-x)\exp(-\mathrm{i}b_\mathrm{lin}z)$ at $x\le 0$, where p_l and n_l $(p_\mathrm{r},n_\mathrm{r})$ describe the depth and the order of the refractive-index modulation on the left (right) hand of the interface located at $x=0$, β determines the transverse scale of the Bessel lattice. Accordingly the lattice profile is determined by the beam intensity in the form of $R(x)=pJ_n^2\left[\sqrt{2\beta}|x|\right]$. Since the lattice is now composed of two Bessel lattices with different parameters, we term it "composite Bessel lattice". The surface scheme will degenerate into a non-surface one if $p_\mathrm{l}=p_\mathrm{r}=p$ and $n_\mathrm{l}=n_\mathrm{r}=n$. Equation (1) conserves several quantities, including the power P and the Hamiltonian H, expressed as:

$$P=\int_{-\infty}^{\infty}|A|^2\mathrm{d}x$$

and

$$H=\frac{1}{2}\int_{-\infty}^{\infty}\left[|\partial A/\partial x|^2-2R(x)|A|^2+|A|^4\right]\mathrm{d}x. \qquad (2)$$

We search for stationary solutions in the form of $A(x,z)=q(x)\exp(\mathrm{i}bz)$, where b is the real propagation constant, $q(x)$ is the real function which must vanish at $|x|\to\infty$ and converge to the same value at the interface. Substitute the expression of A into Eq.(1), then we will obtain the following equation:

$$\frac{\mathrm{d}^2q}{\mathrm{d}x^2}-2bq-2q^3+2Rq=0. \qquad (3)$$

The soliton profiles are found numerically by using a standard relaxation method. Mathematically, the soliton families are defined by the propagation constant b and lattice parameters, i.e. $n_{\mathrm{l(r)}}, p_{\mathrm{l(r)}}$, and b_lin. Since one can always use the scaling transformation $q(x,z,p)\to\chi q(\chi x,\chi^2 z,\chi^2 p)$ to obtain various families of solitons from a given one,[13] we fix the transverse scale by setting $\beta\equiv 2$, and vary the lattice parameters, i.e. $n_{\mathrm{l(r)}}$, and $p_{\mathrm{l(r)}}$ and the propagation constant b without loss of generality.

The stability of the solitons can be analysed by considering the perturbed stationary solution to be in the form of $A(x,z)=q(x)\exp(\mathrm{i}bz)+u(x)\exp[\mathrm{i}(b+\lambda)z]+v(x)\exp[\mathrm{i}(b-\lambda)z]$, where the perturbation components u and v could grow with a

complex rate λ during propagation. The soliton is stable if all imaginary parts of eigenvalues (λ) equal zero. The eigen-vector corresponding to the real part of λ is the so-called 'internal mode' which may lead to the rotating, stretching, or oscillating effects on the evolution of solitons if they are stable.[22,23] Substitute the perturbed solution into Eq.(1) and linearize the perturbation functions $u(x)$, and $v(x)$ around the stationary solution $q(x)$, then we will obtain the linear eigen-equations as follows:

$$\lambda u = \frac{1}{2}\frac{\mathrm{d}^2 u}{\mathrm{d}x^2} - q^2(2u+v) + Ru - bu,$$

and

$$\lambda v = -\frac{1}{2}\frac{\mathrm{d}^2 v}{\mathrm{d}x^2} + q^2(u+2v) - Rv + bv, \qquad (4)$$

which can also be solved numerically by the relaxation algorithm.

3. Stationary solutions and propagation of surface solitons

First we discuss the properties of surface solitons in a composite Bessel lattice with fundamental order on the left side of the interface and higher order on the other side for fixed p (Fig.1). The power of such solitons decreases monotonically with the increase of propagation constant (Fig.1(a)) and vanishes at the upper cutoff (b_{cutoff}) which determines the region where solitons exist. One can immediately find that the power of solitons with $n_{\mathrm{r}} = 0$ is larger than with $n_{\mathrm{r}} = 2, 4$. The reason is that the potential induced by 0th-order Bessel lattice is higher than by higher-order ones for the same parameters b and p, thus more energy can be captured. Also shown in Fig.1(a) is that the power curve of $n_{\mathrm{r}} = 2$ coincides with that of $n_{\mathrm{r}} = 4$. This implies that the energy captured is dominated by the lowest order lattices.

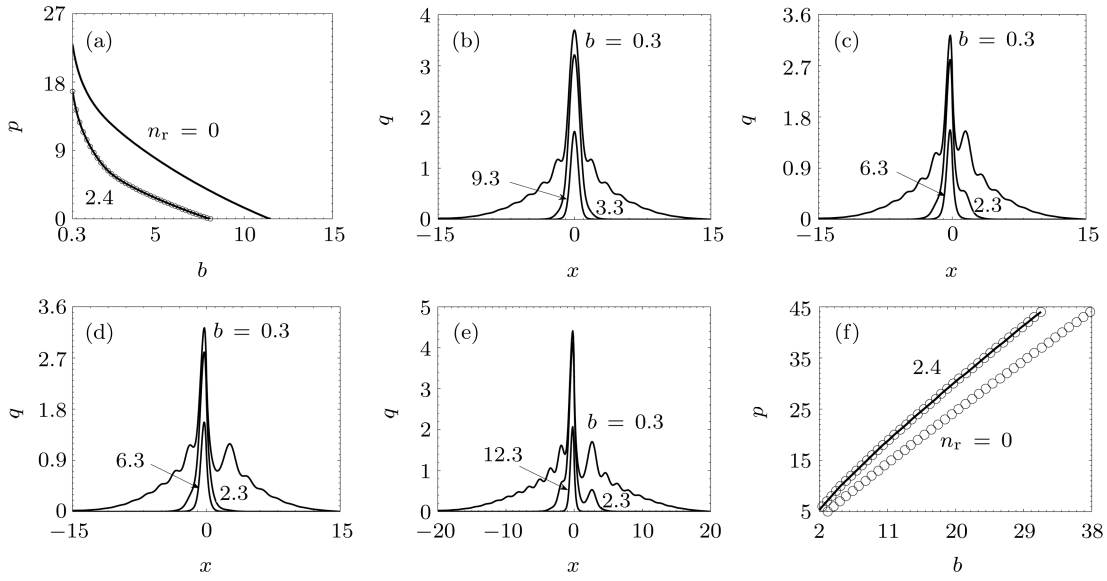


Fig.1. Properties of surface soliton in 1D composite Bessel lattice with different orders beside the side of the interface. (a) Power of surface soliton versus lattice depth with $n_{\mathrm{l}} = 0, p_{\mathrm{l(r)}} = 15$ for different values of n_{r}. (b)-(d) Profiles of surface soliton with $p_{\mathrm{l(r)}} = 15, n_{\mathrm{l}} = 0$ and (b) $n_{\mathrm{r}} = 0$. (c) $n_{\mathrm{r}} = 2$. (d) $n_{\mathrm{r}} = 4$. (e) $n_{\mathrm{l}} = 0, n_{\mathrm{r}} = 4$, and $p_{\mathrm{l(r)}} = 25$. (f) Typical b_{cutoff} curve versus lattice depth for $n_{\mathrm{l}} = 0$ and different values of n_{r}.

The soliton with a symmetrical profile corresponds to the ground-state soliton if $n_{\mathrm{l}} = n_{\mathrm{r}}$ (Fig.1(b)). The solitons reside mainly in the centre of nth-order Bessel lattice if the value of b is large. They will extend and cover several nodes of Bessel function with the decrease of propagation constant. On the other hand, the soliton profiles are asymmetric when $n_{\mathrm{l}} \neq n_{\mathrm{r}}$ (Figs.1(c)–1(e)). The asymmetry becomes apparent when the value of $|n_{\mathrm{l}} - n_{\mathrm{r}}|$ is larger. The peaks of soliton profiles are located at the peaks of composite Bessel lattice. This indicates that the shapes of solitons are determined by the shapes of composite lattices, i.e. the orders of the Bessel functions. Minor dissimilarity cannot be seen clearly from Figs.1(c)–1(e) in that the presence of lattice with larger-n_{r} slightly modulates the left part of soliton profiles. We should

emphasis that it is the interface and the lattice together that support such modes in the present scheme, that is, either component of surface soliton beside the interface cannot exist alone if the other component varnishes or is deformed seriously. We show in Fig.1(f) that those stationary solutions exist only on the upper side of b_{cutoff} curves. The region where the surface solitons exist slightly shrinks when the parameter $n_{\text{r}} \neq 0$. Again, the b_{cutoff} curve of $n_{\text{r}} = 2$ completely coincides with that of $n_{\text{r}} = 4$. This result is in agreement with the power curves shown in Fig.1(a). The difference in b_{cutoff} between the case $n_{\text{r}} = 0$ and the case $n_{\text{r}} \neq 0$ slowly increases with the growth of p.

Linear stability analysis (Eq.(4)) is implemented on such stationary solutions and the results show that surface solitons are stable in the entire domain of their existence (Fig.1(f)), i.e. all maximal imaginary parts of eigenvalues (λ) equal zero. To confirm the results of instability analysis, we perform extensive numerical simulations by inputting the perturbed initial solutions $A(x, z = 0) = q(x)[1+\rho(x)]$ into Eq.(1), where $\rho(x)$ is the white noise with a variance σ^2_{noise}. Some examples of evolution of the perturbed solitons are shown in Fig.2. Although the stationary solutions are subjected to a strong perturbation, they can propagate stably.

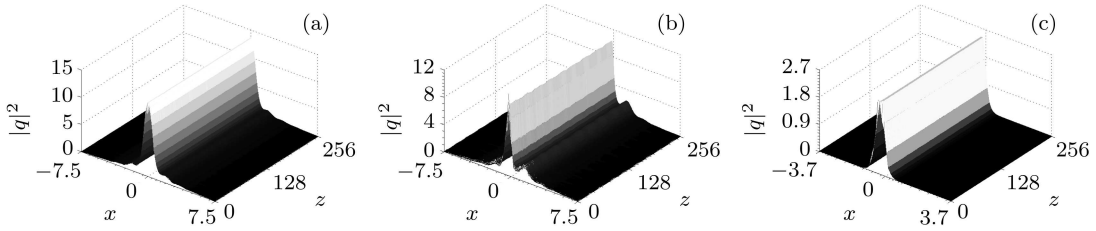


Fig.2. Stable propagation of the surface solitons with $p_{\text{l(r)}} = 15$, and $n_{\text{l}} = 0$. (a) $b = 0.3$, and $n_{\text{r}} = 0$. (b) $b = 0.3$, and $n_{\text{r}} = 2$. (c) $b = 6.3$, and $n_{\text{r}} = 4$. The white noise with a variance (σ^2_{noise}) of 0.01 was added into the initial input.

Next, we consider the properties of surface solitons supported by the composite lattice with n_{l}, $n_{\text{r}} > 0$. The power of surface solitons also decreases monotonically with the growth of propagation constant (Fig.3(a)). The domain where solitons exist is smaller than that with $n_{\text{l}} = 0$ if other lattice parameters remain the same. Some typical soliton profiles are shown in Fig.3. One can see clearly the influence of the different lattices beside the interface on soliton profiles. Similar to case $n_{\text{l}} = 0$, the profile of soliton is symmetrical with n_{l} and n_{r} being equal but asymmetrical with n_{l} and n_{r} being different. Since the Bessel potential now has a local minimum at $x = 0$, the profile of soliton has a dip at the interface for relatively small

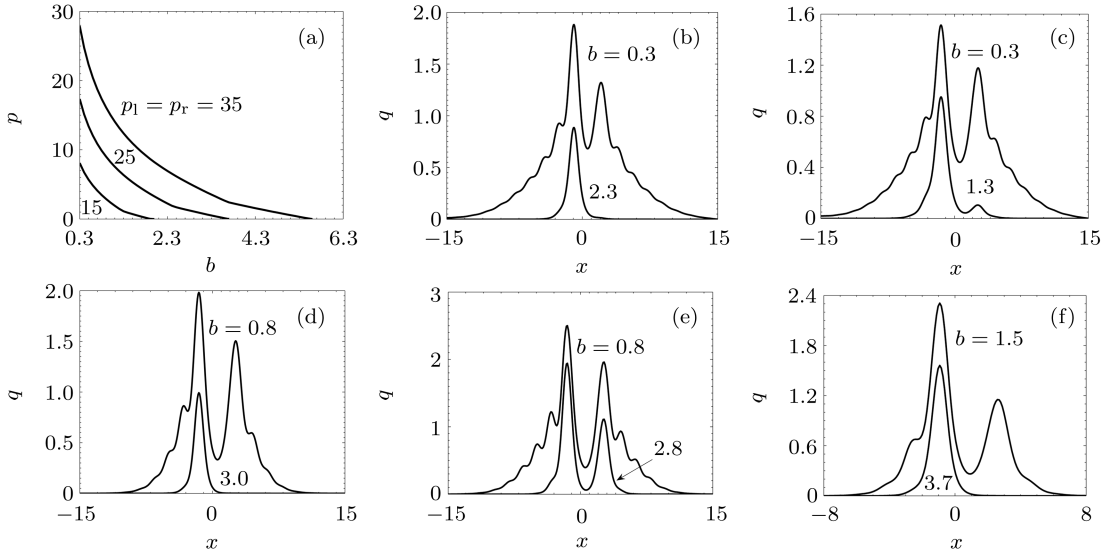


Fig.3. (a) Power of surface soliton versus lattice depth with $n_{\text{l}} = 2$, and $n_{\text{r}} = 4$. (b)–(f) Profile of surface solitons. (b) $n_{\text{l}} = 1, n_{\text{r}} = 3$, $p_{\text{l(r)}} = 15$; (c) $p_{\text{l(r)}} = 15$; (d) $p_{\text{l(r)}} = 25$; (e) $p_{\text{l(r)}} = 35$. $n_{\text{l}} = 2$, and $n_{\text{r}} = 4$ in (c)–(e); (f) $n_{\text{l}} = 1$, $n_{\text{r}} = 4$, and $p_{\text{l(r)}} = 25$.

values of b. Energy of soliton resides mainly at the $\min(n_{\rm l}, n_{\rm r})$ side and the dip is almost invisible when the value of b is larger. Notice that the profiles for $b = 0.8$ in Figs.3(d) and 3(e) have the same shapes but with different amplitudes because the values of $n_{\rm l(r)}$ are the same but the values of p are different. Stable evolution of some examples are displayed in Fig.(4).

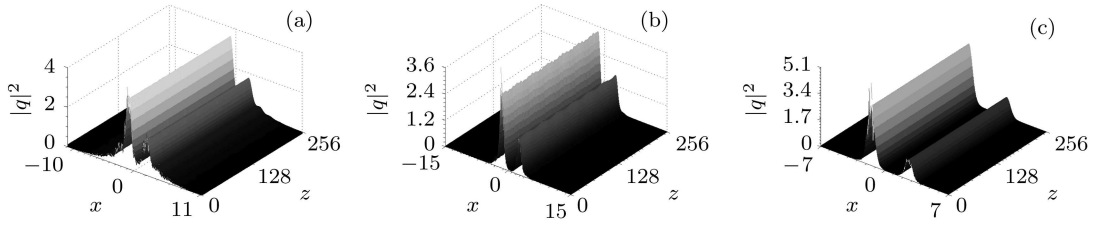


Fig.4. Stable propagation of the surface solitons with different lattice depths and different values of $n_{\rm l}$ and n_r. (a) $p_{\rm l(r)} = 15,\ b = 0.3,\ n_{\rm l} = 1$, and $n_{\rm r} = 3$. (b) $p_{\rm l(r)} = 25,\ b = 1.3,\ n_{\rm l} = 2$, and $n_{\rm r} = 4$. (c) $p_{\rm l(r)} = 35,\ b = 2.8,\ n_{\rm l} = 2$, and $n_{\rm r} = 4$. The white noise with a variance ($\sigma^2_{\rm noise}$) of 0.01 was added into the initial input.

Now, we focus on the cases in which $p_{\rm l} \neq p_{\rm r}$ and $n_{\rm l} \neq n_{\rm r}$ simultaneously. Some soliton solutions are shown in Fig.5. Although the profiles in Fig.5(a) look symmetrical, they are really surface solitons as long as we notice that the centres of profiles shift away from the interface. Compare Fig.5(a) with Fig.5(c), then we will see that the amplitudes of solitons with opposite $p_{\rm l}$, $p_{\rm r}$ differ obviously from each other while the shapes of solitons remain the same if other parameters are fixed (e.g. $b = 0.3$). Thus, the amplitude of soliton is mainly determined by the lattice depth, and the shape of soliton is governed by the order of lattice beside the interface. The energy distribution of soliton beside the interface is proportional to the ratio between $p_{\rm l}\kappa_{\rm l}$ and $p_{\rm r}\kappa_{\rm r}$, where $\kappa_{\rm l}$ and $\kappa_{\rm r}$ denote the maximal peak values of the left and the right Bessel lattices respectively. This can be explained in physics as the fact that the deeper the refractive-index is modulated, the more energy is trapped. The profile extends with the increase of $p_{\rm r}$ when other parameters remain the same. Figure 5(e) displays the region where surface solitons exist with $p_{\rm l}$ and $p_{\rm r}$ varying. The domain shrinks with the decrease of $p_{\rm l}$. Another important result is that the $b_{\rm cutoff}$ is determined by the value of $\max(p_{\rm l}\kappa_{\rm l},\ p_{\rm r}\kappa_{\rm r})$. The $b_{\rm cutoff}$ remains the same when $p_{\rm r}\kappa_{\rm r} < p_{\rm l}\kappa_{\rm l}$ but increases almost linearly with b when $p_{\rm r}\kappa_{\rm r} > p_{\rm l}\kappa_{\rm l}$. All $b_{\rm cutoff}$ curves converge to a collectively asymptotic line and increase linearly with the increase of b for a sufficiently large value of $p_{\rm r}$.

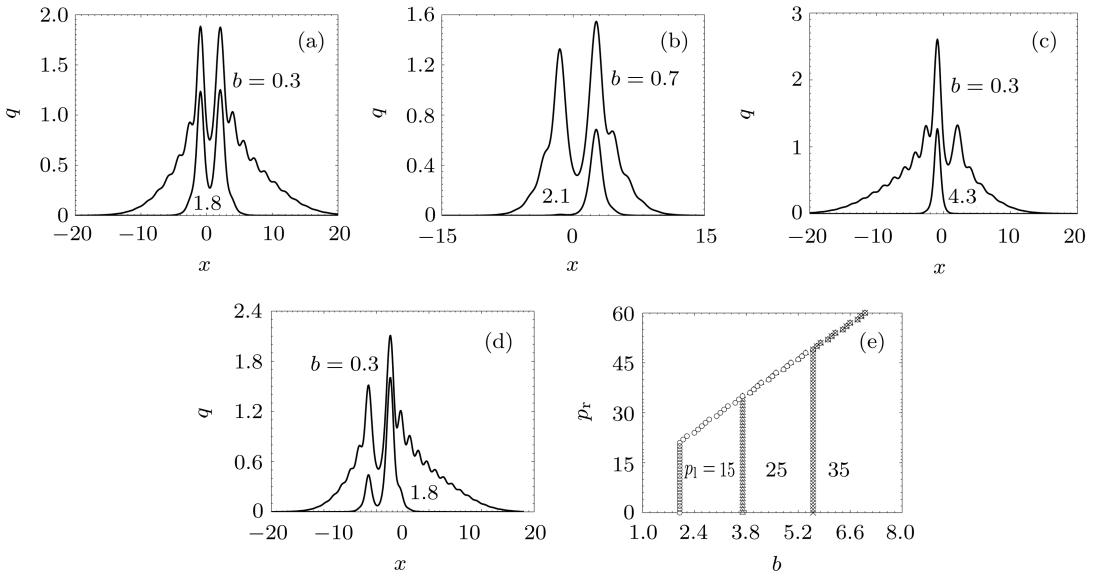


Fig.5. (a)–(d) Profiles of surface solitons with different lattice depths beside the interface. (a) $p_{\rm l} = 15,\ p_{\rm r} = 25,\ n_{\rm l} = 1$, and $n_{\rm r} = 3$. (b) $p_{\rm l} = 15,\ p_{\rm r} = 25,\ n_{\rm l} = 2$, and $n_{\rm r} = 4$. (c) $p_{\rm l} = 25,\ p_{\rm r} = 15,\ n_{\rm l} = 1$, and $n_{\rm r} = 3$. (d) $p_{\rm l} = 15,\ p_{\rm r} = 35,\ n_{\rm l} = 2$, and $n_{\rm r} = 4$. (e) Domain where surface solitons exist with the value of $p_{\rm l}$ fixed and the value of p_r varying, $n_{\rm l} = 2,\ n_{\rm r} = 4$.

We perform again the instability analysis on the stationary solutions with lattice parameters varying and find the results to be similar to those in the former cases, that is, the solitons are stable in the domain of their existence. Numerical simulations also verify the theoretical analysis results but they are not shown here.

Furthermore, to explore the effect of Bessel potential on the stabilization of solitons, we examine the surface solitons supported by a higher depth of Bessel lattice with $n_{\rm l} = 2$, and $n_{\rm r} = 5$, and we find that it also supports the stable surface solitons. The properties of these solitons are similar to those discussed above. The only difference is that the domain of their existence decreases when the n value increases for the same parameters $p_{\rm l}$ and $p_{\rm r}$. Some examples of stable solitons and their propagation simulations are shown in Fig.6. Surface solitons still propagate stably. Note that the soliton profiles with $p_{\rm l} = 50$ and $p_{\rm r} = 100$ are very different from those with $p_{\rm l} = 100$ and $p_{\rm r} = 50$ because the profile of soliton is determined by both $p_{\rm l(r)}$ and $b_{\rm l(r)}$. We perform instability analysis and propagation simulations for even larger values of $n_{\rm l(r)}$ and $p_{\rm l(r)}$ and prove that the stationary solution is stable in the domain of its existence.

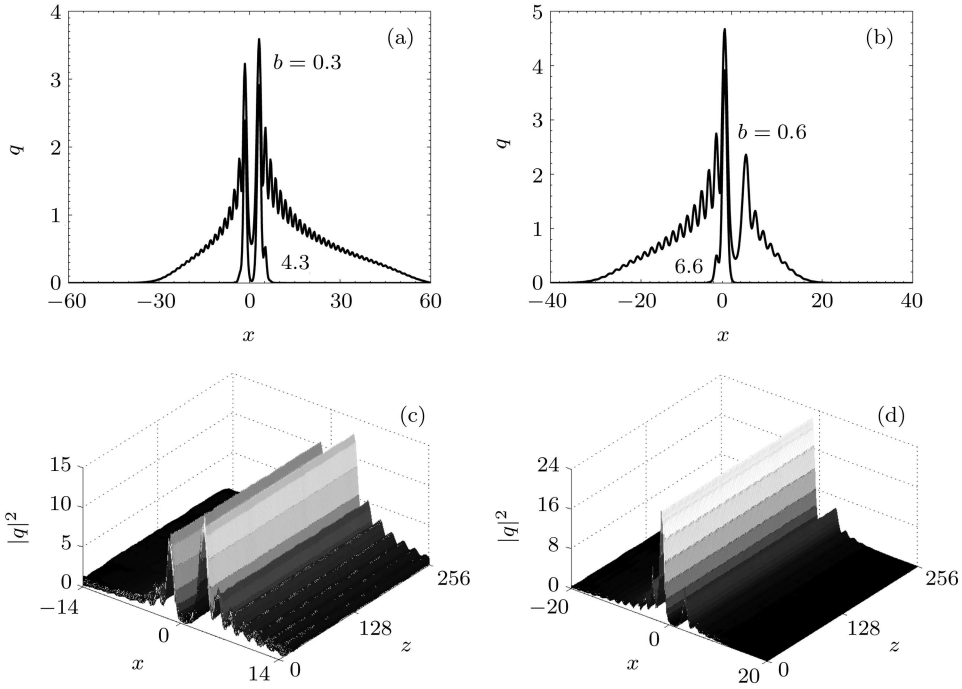


Fig.6. Profile and evolution of the solitons with strong lattice modulation for $n_{\rm l} = 2$, and $n_{\rm r} = 5$. (a) and (c) $p_{\rm l} = 50$, and $p_{\rm r} = 100$. (b) and (d) $p_{\rm l} = 100$, $p_{\rm r} = 50$ and $b = 0.3$ in (c) and 0.6 in (d). The white noise with $\sigma^2_{\rm noise} = 0.01$ was added into the initial input in (c) and (d).

4. Conclusions

We investigate the existence and dynamics of solitons supported by a 1D composite Bessel optical lattice in a defocusing Kerr medium. This composite lattice is composed of two 1D Bessel lattices with different orders and different modulation depths, separated beside both sides of an interface. Power of solitons decreases monotonically with the increase of propagation constant, and solitons no longer exist when the propagation constant exceeds a threshold value. The varying of lattice order leads to the varying of soliton shape and thus the soliton energy distribution. The lattice depth determines the amplitude of soliton when other parameters are fixed. All stationary solutions we found are dynamically stable in the entire domain of their existence. We propose a method of experimentally realizing the presented scheme. Our theoretical results may enrich the concept of surface soliton by providing another way to control the shapes of soliton. They can also be easily generalized to the case of Bose–Einstein condensates trapped in a 1D Bessel potential with repulsive interatomic interactions.

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