Vol 17 No 3, March 2008
1674-1056/2008/17(03)/0971-07
Chinese Physics B
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Effects of dipole–dipole interaction on entanglement transfer*

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(Received 29 April 2007; revised manuscript received 12 September 2007)

A system consisting of two different atoms interacting with a two-mode vacuum, where each atom is resonant only with one cavity mode, is considered. The effects of dipole–dipole (dd) interaction between two atoms on the atom–atom entanglement and mode–mode entanglement are investigated. For a weak dd interaction, when the atoms are initially separable, the entanglement between them can be induced by the dd interaction, and the entanglement transfer between the atoms and the modes occurs efficiently; when the atoms are initially entangled, the entanglement transfer is almost not influenced by the dd interaction. However, for a strong dd interaction, it is difficult to transfer the entanglement from the atoms to the modes, but the atom–atom entanglement can be maintained when the atoms are initially entangled.

Keywords: dipole–dipole (dd) interaction, concurrence, entanglement transfer
PACC: 4250, 0367, 0365

1. Introduction

Quantum entanglement is a fundamental resource for quantum information. Since atoms and photons were found to be able to be used as qubits, much attention has been paid to generating atom–atom entanglement,[1−6] atom–photon entanglement[7,8] and photon–photon entanglement.[9−12] In addition, atoms are usually applied to storing information while photons are always employed to carry and process information. Therefore, storing information in atoms[13] and transferring information between atoms and photons[14−16] are also topics of much current interest.

On the other hand, the dipole–dipole (dd) interaction between the atoms has given rise to a myriad of fascinating phenomena.[17−21] Especially, in the field of quantum information, the dd interaction plays an important role in generating the entanglement. According to the two-photon excitation of resonant transitions modified by a dd interaction, Lukin and Hemmer[22] described a method of manipulating entangled metastable states of pairs of neighbouring atoms. By laser-induced dd interactions, Opatrný *et al* [23] realized the entanglement of atom pairs confined to adjacent sites of two mutually shifted optical lattices. In order to achieve a robust entangled state, many researchers have made efforts to stabilize the atomic entanglement via the dd interaction in different engineering scenarios, such as a squeezed vacuum field,[24,25] an electro-magnetic bath[26] and a classical driving field,[27,28] etc. Therefore, it is very necessary to investigate further into the effect of dd interaction on quantum entanglement.

Recently, the entanglement transfer from a two-mode field to a pair of separable atoms, each of which interacts only with one of the modes, has been intensively studied.[29−32] Kraus and Cirac[29] have found that the two atoms can be entangled where the two cavity fields are driven by the common two-mode squeezed light. Then Paternostro *et al* [30] found the complete conditions for the entanglement transfer from a common two-mode driving field to a pair of qubits. Very recently, it is shown in Ref.[33] that the entanglement can be transferred reciprocally between maximally entangled atoms and continuous-variable systems which are initially prepared in a pure separable Gaussian field. In addition, we have found that the entanglement can be fully exchanged between two entangled atoms and two cavity modes initially in vacuum.[34] In these studies, the entanglement can

*Project supported by the National Natural Science Foundation of China (Grant No 60478049), the Natural Science Foundation of Hubei Province, China (Grant No 2006ABB015) and the Natural Science Foundation of Huazhong Normal University, China.
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be transferred between atoms and field modes, but it cannot be generated by the system itself due to the non-interaction between the two atom–mode subsystems. In the present paper, we show that the two separable atoms can be entangled by the dd interaction between them. Consequently, the entanglement transfer between the atoms and the modes is stimulated.

The rest of this paper is organized as follows. In Section 2, we describe our model and present the degrees of atom–atom entanglement and mode–mode entanglement. In Sections 3 and 4, the effects of the dd interaction on the two entanglements are discussed when the two atoms are initially separable and maximally entangled, respectively. Finally, the conclusions are presented in Section 5.

2. Model and entanglement

The model discussed in the present paper is based on a two-mode Jaynes–Cummings (J–C) model,[35] in which there are two different two-level atoms inside an ideal cavity supporting two modes with frequencies ω_1 and ω_2. Here the dd interaction between the two atoms is also considered. In the rotation wave approximation, the Hamiltonian for the system can be written as ($\hbar = 1$)

$$H = H_0 + H_{\rm af} + H_{\rm aa}, \tag{1}$$

$$H_0 = \sum_{i=1}^{2} \omega_i (a_i^\dagger a_i + S_i^z), \tag{2}$$

$$H_{\rm af} = \sum_{i=1}^{2} g_i (a_i^\dagger S_i^- + {\rm H.c.}), \tag{3}$$

$$H_{\rm aa} = \Omega (S_1^+ S_2^- + {\rm H.c.}), \tag{4}$$

where, $a_i^\dagger$ and a_i are the creation and annihilation operators for the ith mode with frequency ω_i, while ω_i is also the transition frequency of the ith atom; S_i^z and $S_i^\pm$ denote the atomic spin-like operations with $S_i^z = (1/2)(|e_i\rangle\langle e_i| - |g_i\rangle\langle g_i|)$ and $S_i^+ = (S_i^-)^\dagger = |e_i\rangle\langle g_i|$, g_i is the coupling constant between the ith atom and the ith mode; Ω refers to the strength of dd interaction between the atoms; H_0 describes the free Hamiltonian of the atoms and the modes; $H_{\rm af}$ denotes the interactions between the atoms and the respective modes; $H_{\rm aa}$ represents the dd interaction between the atoms. For simplicity, we suppose that the atom–mode couplings are equal, i.e. $g_1 = g_2 = g$.

The total state of the system at time $t = 0$ is

$$\begin{aligned} |\psi(0)\rangle_{\rm af} =& |\psi(0)\rangle_{\rm a} \otimes |\psi(0)\rangle_{\rm f} \\ =& \cos\theta |e_1 g_2, 0_1 0_2\rangle \\ &+ {\rm e}^{{\rm i}\phi} \sin\theta |g_1 e_2, 0_1 0_2\rangle \end{aligned} \tag{5}$$

with $0 \le \theta \le \pi/2$, $-\pi \le \phi < \pi$.

In the Schrödinger picture, the total state of the system at any time t can be expressed as

$$\begin{aligned} |\psi(t)\rangle_{\rm af} =& c_1(t)|e_1 g_2, 0_1 0_2\rangle + c_2(t)|g_1 e_2, 0_1 0_2\rangle \\ &+ c_3(t)|g_1 g_2, 1_1 0_2\rangle + c_4(t)|g_1 g_2, 0_1 1_2\rangle, \end{aligned} \tag{6}$$

where

$$\begin{aligned} c_1(t) =& r_1\alpha(t) - {\rm i}(r_1\Theta + r_2\Omega)\beta(t) + r_1(g^2 + \Theta^2)\gamma(t) \\ &+ {\rm i}\Theta[r_1(g^2 - \Theta^2) - r_2\Theta\Omega]\eta(t), \\ c_2(t) =& r_2\alpha(t) + {\rm i}(r_2\Theta - r_1\Omega)\beta(t) + r_2(g^2 + \Theta^2)\gamma(t) \\ &- {\rm i}\Theta[r_2(g^2 - \Theta^2) + r_1\Theta\Omega]\eta(t), \\ c_3(t) =& g\{-{\rm i}r_1\beta(t) - (2r_1\Theta + r_2\Omega)\gamma(t) \\ &- {\rm i}[r_1(g^2 - \Theta^2) - r_2\Theta\Omega]\eta(t)\}, \\ c_4(t) =& g\{-{\rm i}r_2\beta(t) + (2r_2\Theta - r_1\Omega)\gamma(t) \\ &- {\rm i}[r_2(g^2 - \Theta^2) + r_1\Theta\Omega]\eta(t)\} \end{aligned} \tag{7}$$

with

$$\begin{aligned} \alpha(t) =& \frac{a^2\cos(at) - b^2\cos(bt)}{a^2 - b^2}, \\ \beta(t) =& \frac{a\sin(at) - b\sin(bt)}{a^2 - b^2}, \\ \gamma(t) =& \frac{\cos(at) - \cos(bt)}{-a^2 + b^2}, \\ \eta(t) =& \begin{cases} \dfrac{b\sin(at) - a\sin(bt)}{-a^3 b + a b^3}, & \text{for } b \ne 0, \\ \dfrac{at - \sin(at)}{a^3}, & \text{for } b = 0, \end{cases} \end{aligned} \tag{8}$$

and

$$\begin{aligned} a =& \sqrt{\frac{p + \sqrt{q}}{2}}, \\ b =& \sqrt{\frac{p - \sqrt{q}}{2}}, \\ p =& \Omega^2 + 2g^2 + 2\Theta^2, \\ q =& \Omega^4 + 4g^2\Omega^2 + 16g^2\Theta^2. \end{aligned} \tag{9}$$

Here $r_1 = \cos\theta$, $r_2 = {\rm e}^{{\rm i}\phi}\sin\theta$, and $\Theta = \Delta/2$ with $\Delta = \omega_1 - \omega_2$, the difference between the atomic transition frequencies.

It is convenient to quantify the degrees of atom–atom entanglement and mode–mode entanglement by concurrence defined as[36,37]

$$C = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \tag{10}$$

where $\lambda_i (i = 1, 2, 3, 4)$ are the eigenvalues, in decreasing order, of matrix $R = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$; σ_y is the Pauli spin matrix and the asterisk denotes complex conjugation; ρ is the density matrix of an arbitrary two-qubit state. The two qubits are maximally entangled when $C = 1$, while they are separable when $C = 0$.

Following our previous work,[34] it is easy to calculate the atomic concurrence $C_{\mathrm{a_1a_2}}(t)$ and the field concurrence $C_{\mathrm{f_1f_2}}(t)$ as follows:

$$\begin{aligned} C_{\mathrm{a_1a_2}}(t) &= 2|c_1(t)||c_2(t)|, \\ C_{\mathrm{f_1f_2}}(t) &= 2|c_3(t)||c_4(t)|, \end{aligned} \tag{11}$$

where the symbol $|\cdot|$ denotes the modulus.

3. Two atoms separable initially

Now we consider the case of $\theta = 0$, that is to say, the two atoms are initially in a separable state $|e_1 g_2\rangle$, which is sensitive to the effect of the dd interaction. In the following, we will discuss the effects of weak and strong dd interactions respectively.

3.1. The effect of weak dd interaction

When the difference between the atomic transition frequencies, Δ, the strength of dd interaction Ω and the atom–mode coupling constant g satisfy the relation of $\Delta \ll \Omega \ll g$, we can make an approximation for a and b, respectively, in expression (9) as

$$a \approx g + \frac{\Omega}{2}, \quad \text{and} \quad b \approx g - \frac{\Omega}{2}. \tag{12}$$

Then expressions (7) and (11) are easily reduced to

$$\begin{aligned} c_1(t) &\approx \cos(gt)\cos\left(\frac{1}{2}\Omega t\right), \\ c_2(t) &\approx -\,\mathrm{i}\cos(gt)\sin\left(\frac{1}{2}\Omega t\right), \\ c_3(t) &\approx -\,\mathrm{i}\sin(gt)\cos\left(\frac{1}{2}\Omega t\right), \\ c_4(t) &\approx -\sin(gt)\sin\left(\frac{1}{2}\Omega t\right) \end{aligned} \tag{13}$$

and

$$\begin{aligned} C_{\mathrm{a_1a_2}}(t) &\approx \cos^2(gt)|\sin(\Omega t)|, \\ C_{\mathrm{f_1f_2}}(t) &\approx \sin^2(gt)|\sin(\Omega t)|. \end{aligned} \tag{14}$$

From expression (14) we see that the two initially separable atoms can be entangled due to the weak dd interaction between them. We also see that the two entanglements both have two periods, one is the oscillation period with a value of π/g determined by atom–mode coupling, the other is the envelope oscillation period, which is π/Ω arising from the dd interaction. Obviously, at $gt = n\pi$ and $\Omega t = (m + 1/2)\pi$ $(n, m = 0, 1, 2, \cdots)$, the atoms are maximally entangled and the modes are separated, while at $gt = (n+1/2)\pi$ and $\Omega t = (m+1/2)\pi$, the modes are maximally entangled and the atoms are completely disentangled into their initially separable states. That is to say, entanglement can be completely transferred from the atoms to the modes because of the atom–mode couplings. In addition, it is possible for the entanglement to be restored to the atoms. Thus the entanglement transfer between the two subsystems is almost periodical and perfect.

In order to study the dependence of the maximum entanglement obtained during the time evolution on the difference between the atomic transition frequencies, we extract the corresponding maximum value on the time scale $(0, 100/g)$, which is large enough for our model. As shown in Fig.1, $\mathrm{MC_{a_1a_2}}$ and $\mathrm{MC_{f_1f_2}}$ represent the corresponding maximum concurrences of atom–atom and mode–mode subsystems derived from the time evolution respectively.

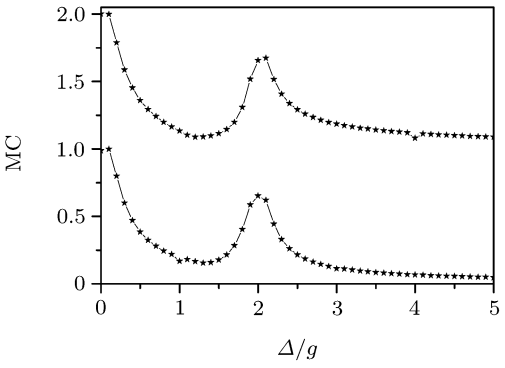


Fig.1. The maximum concurrences ($\mathrm{MC_{a_1a_2}} + 1.0$) (upper) and $\mathrm{MC_{f_1f_2}}$ (lower) versus Δ/g with $\theta = 0$ and $\Omega = 0.1g$.

When $\Delta \ll \Omega$, the maximum values of the two entanglements both are about 1.0. As the value of Δ increases as shown in Fig.1, the maximum values of the two entanglements decrease first, then increase and go through a peak value at about $\Delta = 2g$, finally drop close to zero.

Since the dd interaction here is weak, i.e. $\Omega \ll g$, we can explain these results via dressed states. The dressed states, which are the eigenstates of the Hamil-

tonian $\mathcal{H} = H_0 + H_{\rm af}$, are

$$|\mathcal{E}^1_\pm\rangle = \frac{1}{\sqrt{2}}(|g_1g_2, 1_10_2\rangle \pm |e_1g_2, 0_10_2\rangle),$$
$$|\mathcal{E}^2_\pm\rangle = \frac{1}{\sqrt{2}}(|g_1g_2, 0_11_2\rangle \pm |g_1e_2, 0_10_2\rangle), \quad (15)$$

the corresponding eigenvalues are

$$\mathcal{E}^1_\pm = \frac{\Delta}{2} \pm g,$$
$$\mathcal{E}^2_\pm = -\frac{\Delta}{2} \pm g. \quad (16)$$

Figure 2 shows the transitions induced by the dd interaction between two atoms. The transitions of $|\mathcal{E}^2_-\rangle - |\mathcal{E}^1_-\rangle$ and $|\mathcal{E}^2_+\rangle - |\mathcal{E}^1_+\rangle$ when $\Delta \simeq 0$ and $|\mathcal{E}^2_+\rangle - |\mathcal{E}^1_-\rangle$ when $\Delta \simeq 2g$ occur easily because the transitions need no additional energy. As a result, when $\Delta \simeq 0$ and $\Delta \simeq 2g$, the two atoms are easily and deeply entangled, and the maximum entanglement of two atoms when $\Delta \simeq 0$ is larger than when $\Delta \simeq 2g$. However, in other cases, it is difficult for the transitions to occur as shown in Fig.2, which need additional energies, thus the maximum entanglement between two atoms is small. In addition, due to the strong atom–mode couplings, the entanglement transfer between the atoms and the modes happens easily and almost completely, thus the maximum entanglement between the modes is always similar to that between the atoms in behaviour. Therefore, the maximum entanglements between two atoms and between two modes strongly depend on the difference between the atomic transition frequencies.

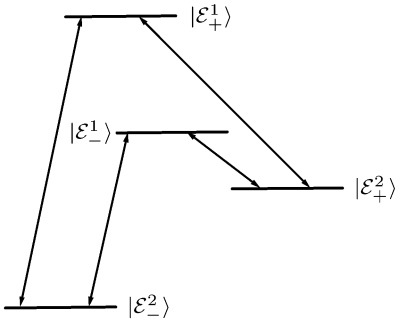


Fig.2. The eigenstates of the Hamiltonian $\mathcal{H}$ and the transitions induced by the dd interaction between two atoms.

3.2. The effect of strong dd interaction

Under the condition of $\Delta \ll \dfrac{g^2}{\Omega} \ll \Omega$, the approximations for a and b in expression (9) are

$$a \approx \Omega, \quad b \approx \frac{g^2}{\Omega}. \quad (17)$$

Substituting expression (17) into expressions (8) and (7), the population amplitudes of the four basic states are

$$c_1(t) \approx \cos(\Omega t),$$
$$c_2(t) \approx -\,\mathrm{i}\sin(\Omega t),$$
$$c_3(t) \approx -\,\mathrm{i}\frac{g}{\Omega}\left[\sin(\Omega t) + \sin\left(\frac{g^2}{\Omega}t\right)\right],$$
$$c_4(t) \approx \frac{g}{\Omega}\left[\cos(\Omega t) - \cos\left(\frac{g^2}{\Omega}t\right)\right]. \quad (18)$$

Then combining expression (18) with expression (11), the two concurrences become

$$C_{\rm a_1a_2}(t) \approx |\sin(2\Omega t)|,$$
$$C_{\rm f_1f_2}(t) \approx 0. \quad (19)$$

From expression (18) we see that when Ω becomes very large, the system state can be approximately written as

$$|\psi(t)\rangle_{\rm af} \approx [\cos(\Omega t)|e_1g_2\rangle - \mathrm{i}\sin(\Omega t)|g_1e_2\rangle] \otimes |0_10_2\rangle. \quad (20)$$

This implies that due to the strong dd interaction, the energy exchange in the whole system mainly occurs between the two atoms, which is determined by the Hamiltonian $H_{\rm aa}$ described by expression (4). Consequently, the degree of atom–atom entanglement ranges from 0 to 1 with a frequency of Ω/π; the mode–mode entanglement reaches a value zero, this is because in this situation the atoms and the modes are approximately decoupled, thus it is difficult to transfer the entanglement from the two atoms to the two modes.

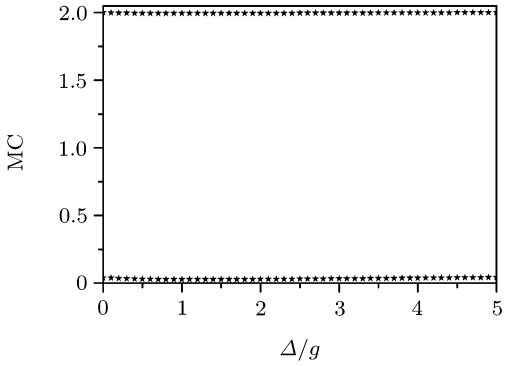


Fig.3. The maximum concurrences ($\mathrm{MC_{a_1a_2}} + 1.0$) (upper) and $\mathrm{MC_{f_1f_2}}$ (lower) versus Δ/g with $\theta = 0$ and $\Omega = 10g$.

Figure 3 shows the maximum values of entanglements for different values of Δ. As contrasted with the results in Fig.1, here the maximum values of the

entanglements of atom–atom and mode–mode are almost unchanged as the value of Δ increases. This implies that the dependence of the maximum values of the two entanglements on the difference between the atomic transition frequencies is obviously suppressed by the strong dd interaction.

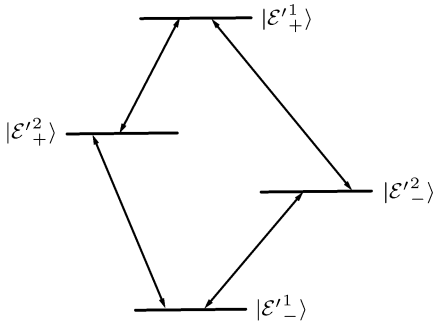


Fig.4. The eigenstates of the Hamiltonian $\mathcal{H}'$ and the transitions induced by the atom–mode couplings.

Because of the relation of $g \ll \Omega$, the Hamiltonian $H_{\rm af}$ denoting the atom–mode coupling is taken as a perturbation. The eigenvalues and the corresponding eigenstates of the unperturbed Hamiltonian $\mathcal{H}' = H_0 + H_{\rm aa}$ are easily derived respectively as

$$\begin{aligned} \mathcal{E}'^1_{\pm} =& \pm\frac{\sqrt{\Delta^2+4\Omega^2}}{2}, \\ \mathcal{E}'^2_{\pm} =& \pm\frac{\Delta}{2}, \end{aligned} \tag{21}$$

and

$$\begin{aligned} |\mathcal{E}'^1_{\pm}\rangle =& \frac{\lambda_{\pm}}{\Lambda_{\pm}}|e_1g_2,0_1 0_2\rangle + \frac{1}{\Lambda_{\pm}}|g_1e_2,0_1 0_2\rangle, \\ |\mathcal{E}'^2_{+}\rangle =& |g_1g_2,1_1 0_2\rangle, \\ |\mathcal{E}'^2_{-}\rangle =& |g_1g_2,0_1 1_2\rangle, \end{aligned} \tag{22}$$

with $\lambda_{\pm} = \dfrac{\Delta \pm \sqrt{\Delta^2+4\Omega^2}}{2\Omega}$ and $\Lambda_{\pm} = \sqrt{\lambda_{\pm}^2+1}$. As a large additional energy is needed in all the transitions induced by the atom–mode interaction shown in Fig.4, no matter what the difference between the atomic transition frequencies is, it is really hard for the atoms to transfer their entanglement to the modes, and thus the two modes are almost always disentangled.

4. The two atoms maximally entangled initially

In this section, we will discuss the case of $\theta = \pi/4$, i.e. the two atoms are initially in a Bell state $(|e_1g_2\rangle + |g_1e_2\rangle)/\sqrt{2}$, which is a maximally entangled state with $C = 1$. In the same way, the effects of weak and strong dd interactions will be studied respectively.

4.1. The effect of weak dd interaction

When Δ, Ω and g satisfy the condition of $\Delta \ll \Omega \ll g$, the approximate values for a and b are given in expression (13). Accordingly, expressions (7) and (11), respectively, become

$$\begin{aligned} c_1(t) \approx& c_2(t) \approx \frac{1}{\sqrt{2}}\cos(gt)\mathrm{e}^{-\frac{\mathrm{i}}{2}\Omega t}, \\ c_3(t) \approx& c_4(t) \approx -\frac{\mathrm{i}}{\sqrt{2}}\sin(gt)\mathrm{e}^{-\frac{\mathrm{i}}{2}\Omega t}, \end{aligned} \tag{23}$$

and

$$\begin{aligned} C_{\mathrm{a_1a_2}}(t) \approx& \cos^2(gt), \\ C_{\mathrm{f_1f_2}}(t) \approx& \sin^2(gt). \end{aligned} \tag{24}$$

In this case, the atom–atom entanglement and the mode–mode entanglement both range from 0 to 1 with the same oscillation period π/g, and the latter retards by half the oscillation period compared with the former. Hence, the entanglement can almost be fully transferred between the atoms and the modes, which is similar to the case of $\Omega = 0$.[34] Moreover, this result is nearly not affected by the weak dd interaction, which is because the dd interaction is so weak that it just induces a phase factor in the total state of the system at any time t.

Figure 5 displays the dependence of maximum values of the two entanglements on the value of Δ. With the value of Δ increasing, the maximum values of the mode–mode entanglement are always 1, which indicates that, no matter what value Δ has, the entanglement between two atoms can always be fully transferred to the two modes and be restored.

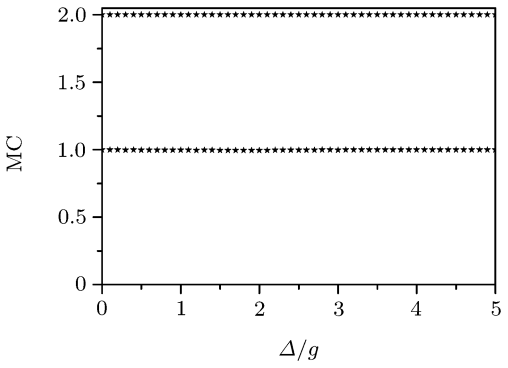


Fig.5. The maximum concurrences ($\mathrm{MC_{a_1a_2}} + 1.0$) (upper) and $\mathrm{MC_{f_1f_2}}$ (lower) versus Δ/g with $\theta = \pi/4$ and $\Omega = 0.1g$.

Figures 6–8 plot the time evolutions of the entanglements for the two subsystems for different values of Δ. It is obvious that when $\Delta \approx 0$ (see Fig.6) and $\Delta = 2g$ (see Fig.7), the time evolution of each entanglement appears to be modulated by the difference between the atomic transition frequencies. When the value of Δ is far from these two points (see Fig.8), the modulation behaviour disappears almost completely. Comparing these results with those in the case of $\Omega = 0$,[34] it is found that because of the dd interaction between two atoms, the modulation behaviour of each entanglement is strongly dependent on the difference between the atomic transition frequencies. However, no matter what the value of Δ is, the entanglement transfer between the atoms and the modes are always nearly complete. This indicates that the possibility of transferring entanglement between the atoms and the modes does not depend on the difference between the atomic transition frequencies even when it is affected by the weak dd interaction.

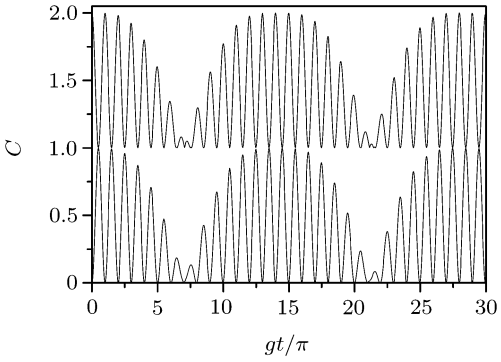


Fig.6. The concurrences $(C_{\mathrm{a_1a_2}}(t) + 1.0)$ (upper) and $C_{\mathrm{f_1f_2}}(t)$ (lower) versus gt with $\theta = \pi/4$, $\Omega = 0.1g$ and $\Delta = 0.1g$.

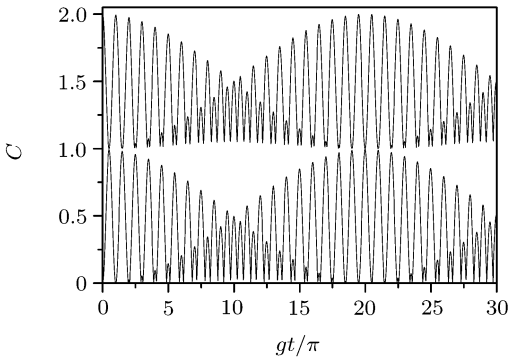


Fig.7. The concurrences $(C_{\mathrm{a_1a_2}}(t) + 1.0)$ (upper) and $C_{\mathrm{f_1f_2}}(t)$ (lower) versus gt when $\theta = \pi/4$, $\Omega = 0.1g$ and $\Delta = 2g$.

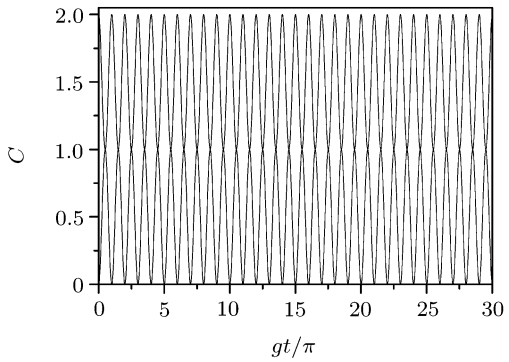


Fig.8. The concurrences $(C_{\mathrm{a_1a_2}}(t) + 1.0)$ (upper) and $C_{\mathrm{f_1f_2}}(t)$ (lower) versus gt with $\theta = \pi/4$, $\Omega = 0.1g$ and $\Delta = 4g$.

4.2. The effect of strong dd interaction

Next, we consider the case of $\Omega \gg g$. Under the condition of $\Delta \ll \dfrac{g^2}{\Omega}$, a and b are given in expression (17) approximately. Accordingly, the population amplitudes and the two concurrences can be, respectively, expressed as

$$\begin{aligned} c_1(t) \approx & c_2(t) \approx \frac{1}{\sqrt{2}} \mathrm{e}^{-\mathrm{i}\Omega t}, \\ c_3(t) \approx & c_4(t) \approx \frac{1}{\sqrt{2}} \frac{g}{\Omega} (\mathrm{e}^{-\mathrm{i}\Omega t} - \mathrm{e}^{\mathrm{i}\frac{g^2}{\Omega} t}), \end{aligned} \tag{25}$$

and

$$\begin{aligned} C_{\mathrm{a_1a_2}}(t) \approx & 1, \\ C_{\mathrm{f_1f_2}}(t) \approx & 0. \end{aligned} \tag{26}$$

In this case, we find that the populations are not only concentrated in the states of $|e_1g_2, 0_1 0_2\rangle$ and $|g_1e_2, 0_1 0_2\rangle$, but also almost equal. Thus the system state can be approximately expressed as

$$|\psi(t)\rangle_{\mathrm{af}} \approx \mathrm{e}^{-\mathrm{i}\Omega t} \frac{1}{\sqrt{2}} (|e_1g_2\rangle + |g_1e_2\rangle) \otimes |0_1 0_2\rangle. \tag{27}$$

Compared with the state when $t = 0$, the state when $t > 0$ has an additional phase factor, which is induced by the dd interaction. Consequently, the two atoms can always maintain their initial entangled state due to the strong dd interaction, which is different from the case in which the two atoms are initially separable. This indicates that the strong dd interaction can prevent the two atoms maximally entangled initially from disentangling. In addition, because of $g \ll \Omega$, it is difficult for the two atoms to transfer their entanglement to the two modes, thus the mode–mode entanglement always approaches to a value near zero.

For different differences between atomic transition frequencies, we calculate the time evolutions of the two entanglements numerically for $0 \leq \Delta \leq 5g$. It is found

that, as the value of Δ increases, the obtained results are almost similar to those in expression (26). That is to say, the maintaining of the atomic initial entanglement is also independent of the difference between atomic transition frequencies, for which the reason is the same as that shown in Section 3.2.

5. Conclusions

We have investigated the effects of dd interaction on entanglement transfer of a system composed of two different atoms each interacting resonantly only with one of two cavity modes.

For small values of Ω, it is shown that the entanglement of the two initially separable atoms can be generated by the weak dd interaction between them, so that the entanglement can be transferred to the two cavity modes and restored to the atoms through the atom–mode couplings. Interestingly, whether the atoms are initially separable or entangled, the entanglement transfer between the atoms and the modes is nearly not influenced by the weak dd interaction for any given value of Δ.

For large values of Ω, the dd interaction is destructive to the entanglement transfer between the atoms and the modes. However, it makes the two atoms maximally entangled periodically when they are initially separable and maintain the atomic initial state when they are maximally entangled initially, which is also independent of the value of Δ.

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