Vol 17 No 3, March 2008
1674-1056/2008/17(03)/0954-06
Chinese Physics B
© 2008 Chin. Phys. Soc.
and IOP Publishing Ltd

Aberration-free two-thin-lens systems based on negative-index materials*

Lin Zhi-Li(林志立)†, Ding Jie-Chen(丁婕琛), and Zhang Pu(张 朴)

Centre for Optical and Electromagnetic Research, State Key Laboratory for Modern Optical Instrumentation, Zhejiang University, Hangzhou 310058, *China*

(Received 6 May 2007; revised manuscript received 27 June 2007)

Since the complete correction of all five monochromatic Seidel aberrations for a singlet lens with random shape or a two-thin-lens system is unprocurable merely by using the conventional positive-index materials both in theory and practice, this paper proposes that when one or both of the two lenses is/are made from negative-index materials, an imaging system composed of a pair of spherical thin lenses is possible to form a real image, in air, free from all five monochromatic Seidel aberrations. The calculated numerical solutions to the structural parameters of such lens systems possessing superior performance are provided and examples of them are illustrated for the given combinations of the two lenses' refractive indices, including an ultimately-remote imaging system.

Keywords: negative refraction, negative-index materials, Seidel aberrations, thin lens
PACC: 4215, 4215E, 4230, 4230F

1. Introduction

Over the past several years, artificial negative-index materials (NIMs) have received considerable attention by the international academy of electromagnetics and optics for their unique properties and potential applications,[1,2] especially when after the experimental demonstration and preparation of them with the various methods such as the appropriate combinations of metallic wires and split-ring resonators deposited on a substrate, or the LC-loaded transmission line networks at the microwave frequencies.[3−6] Several recent reports even experimentally verified the existence of a negative index of refraction in the near-visible and infrared band for the novel fabricated structures with many periodical arrays of nanoscale wires and magnetic resonators, or paired metallic rods.[7−10] Moreover, the photonic-crystal structures were shown to display an effective negative-refraction and have already been used to fabricate novel focusing lenses with performance superior to their counterparts made from the conventional positive-index materials (PIMs).[11−14] By the ray-tracing methods and confirmatory experimental results,[15,16] it also has been proved that the explicit form of the expressions for basic formulas found in the optics literature are kept unchanged for NIMs so long as negative values of refractive indices were substituted, providing a wider selectable range of materials available for lens designs.

We know that the monochromatic imaging quality of a centred lens system depends on the five Seidel aberrations, in this case only a very small part of the object plane close to the optical axis is to be imaged by rays making small slope angles with the axis. To totally correct these primary aberrations, a sufficiently large number of refracting surfaces or lenses is required. Among all the traditional lens systems merely made of the conventional PIMs with refractive indices above one positive unit, some possibilities of the total correction of monochromatic Seidel aberrations are known as the famous Cooke triplet[17] (three lenses with six spherical surfaces), the Dyson unit magnification system (a spherical mirror and a plano-convex lens with the object and image plane coinciding with each other in the planar surface of the lens)[18] and the thick doublet lenses (with two aspherical surfaces and one spherical centred interfaces) as proposed by Schulz.[19] However, based on the use of NIMs, we find that the required number of optical elements can be reduced and the shape of lens can be simplified, even to a singlet flat-slab lens with refractive index equal to negative unit.[2] The NIM multilayer flat lenses are also shown to have great advantages in both low and high order aberration corrections.[20] Unfortunately,

*Project partially supported by the National Basic Research Program of China (Grant No 2004CB719802) and an additional support from the Science and Technology Department of Zhejiang Province, China.
†Corresponding author. E-mail: zllin@coer.zju.edu.cn

due to the inherent short working-distances of the flat lenses, about in the order of their thicknesses, they can operate to form real images only when the objects are close enough. So in order to focus far-field radiations, NIM lenses with curved surfaces are generally demanded. But it has been verified that a singlet NIM lens with arbitrary surface shapes is still not sufficient to totally correct all five monochromatic Seidel aberrations, although possible regions of refractive index with small values of aberrations are proposed.[16,21] As the impossibility of total correction of monochromatic Seidel aberrations by a two-thin-lens system using conventional materials is well known, which is also verified in our work, we think it is necessary to further investigate the feasibility of designing a two-thin-lens system without all five monochromatic Seidel aberrations when one or both of the two spherical lenses are made from NIMs.

2. Equations of Seidel aberration sums to be solved

According to wave-front aberration theory,[17] the aberration polynomial of third-order written in terms of Seidel aberration sums has the form,

$$\begin{aligned} & W(y,\rho,\cos\theta) \\ = & -\frac{1}{8}S_{\rm I}\rho^4+\frac{1}{2}S_{\rm II}y\rho^3\cos\theta-\frac{1}{2}S_{\rm III}y^2\rho^2\cos^2\theta \\ & -\frac{1}{2}(S_{\rm III}+S_{\rm IV})y^2\rho^2+\frac{1}{2}S_{\rm V}y^3\rho\cos\theta, \end{aligned} \tag{1}$$

where y is the normalized object height, ρ and θ are the normalized polar coordinates of exit-pupil plane. For an imaging system composed of two spherical thin lenses, the five Seidel (primary) aberration sums, $S_{\rm I}$, $S_{\rm II}$, $S_{\rm III}$, $S_{\rm IV}$, and $S_{\rm V}$ can be formulated as[22,23]

$$S_{\rm I}=h_1^4U_1+h_2^4U_2, \tag{2a}$$

$$S_{\rm II}=\sum_{i=1}^{2}h_i^4k_iU_i+h_i^2V_i, \tag{2b}$$

$$S_{\rm III}=\sum_{i=1}^{2}h_i^4k_i^2U_i+2h_i^2k_iV+\varphi_i, \tag{2c}$$

$$S_{\rm IV}=\varphi_1/n_1+\varphi_2/n_2, \tag{2d}$$

$$S_{\rm V}=\sum_{i=1}^{2}h_i^4k_i^3U_i+3h_i^2k_i^2V_i+k_i(3+1/n_i)\varphi_i, \tag{2e}$$

where $(i=1,2)$

$$\begin{aligned} U_i=\varphi_i\Bigg[&\frac{n_i+2}{4n_i(n_i-1)}X_i+\frac{n_i+1}{n_i(n_i-1)}X_iY_i \\ &+\frac{3n_i+2}{4n_i}Y_i+\frac{n_i^2}{4(n_i-1)^2}\Bigg], \end{aligned} \tag{3a}$$

$$V_i=\varphi_i[\frac{n_i+1}{2n_i(n_i-1)}X_i+\frac{2n_i+1}{2n_i}Y_i], \tag{3b}$$

$$\varphi_i=(n_i-1)\left(\frac{1}{r_i}-\frac{1}{r_i'}\right)=\frac{1}{s_i'}-\frac{1}{s_i}, \tag{3c}$$

$$X_i=\frac{r_i'+r_i}{r_i'-r_i}, \tag{3d}$$

$$Y_i=\frac{s_i'+s_i}{s_i'-s_i}=-1-\frac{2}{s_i\varphi_i}=1-\frac{2}{s_i'\varphi_i}, \tag{3e}$$

$$h_1=\frac{s_1}{t_1-s_1}, \tag{3f}$$

$$h_2=\frac{h_1\varphi_1(Y_1-1)}{\varphi_2(Y_2+1)}, \tag{3g}$$

$$k_1=t_1\left(\frac{t_1-s_1}{s_1}\right), \tag{3h}$$

$$k_2=k_1+\frac{d}{h_1h_2}. \tag{3i}$$

Moreover, we define the above notation as follows: h_i is the incidence height of a paraxial aperture ray at the ith lens; s_i and s_i' are the object and image distances from the ith lens; X_i and Y_i are the shape and position factors of the ith lens; $\varphi_i, n_i, r_i, r_i'$ are the focal power, refractive index, and two radii of curvature of the ith lens; d is the distance between the two lenses and t_1 is the distance from the front lens to the aperture stop.

Indeed, in a monochromatic Seidel aberration-free lens system, the aperture stop's position is of no importance, because in the end all the Seidel sums become zero and thus they are unaffected by shifting the stop.[24] Usually, the aperture stop is assumed to be in the principal plane of the first lens to facilitate the intricate calculations, that is, $t_1=0$ and thence we have

$$h_1=-1 \quad \text{and} \quad k_1=0 \tag{4}$$

according to Eqs.(3f) and (3h). Then in order to correct the monochromatic Seidel aberrations, all the five aberration sums should be set equal to zero, i.e. the following five equations need to be fulfilled:

$$0=S_{\rm I}=U_1+h_2^4U_2, \tag{5a}$$

$$0=S_{\rm II}=V_1+h_2^4k_2U_2+h_2^2V_2, \tag{5b}$$

$$0=S_{\rm III}=\varphi_1+\varphi_2+h_2^4k_2^2U_2+2h_2^2k_2V_2, \tag{5c}$$

$$0=S_{\rm IV}=\varphi_1/n_1+\varphi_2/n_2, \tag{5d}$$

$$0=S_{\rm V}=h_2^4k_2^3U_2+3h_2^2k_2^2V_2+k_2(3+1/n_2)\varphi_2. \tag{5e}$$

Viewing the forms of the explicit expressions of Eqs.(5a–5e) by the substitution of Eqs.(3a–3i), we find that it is more suitable to choose the six parameters, X_i, Y_i, and φ_i $(i = 1, 2)$, as the unknowns to solve these quartic equations and find possible system solutions for the given values of refractive indices, n_1 and n_2. Subsequently, with the obtained numerical solutions to the six intermediate variables, we can further calculate the actual structural parameters, r_i, r'_i, s_i, s'_i, and d with the following relationships:

$$r_i = 2(n_i - 1)/(\varphi_i(X_i + 1)), \tag{6a}$$

$$r'_i = 2(n_i - 1)/(\varphi_i(X_i - 1)), \tag{6b}$$

$$s_i = -2/(\varphi_i(Y_i + 1)), \tag{6c}$$

$$s'_i = -2/(\varphi_i(Y_i - 1)), \tag{6d}$$

and

$$d = -2/(\varphi_1(Y_1 - 1)) + 2/(\varphi_2(Y_2 + 1)), \tag{6e}$$

which are derived from Eqs.(3c–3i) and the transfer formula $s_2 = s'_1 - d$. For the convenience of comparisons of the imaging properties between two designed lens systems, the size of lens system should be normalized. In this paper, the distance between the two thin lenses is assumed to be the unit of length such that $d = 1$. Therefore, besides Eqs.(5a–5e), now we have one more constraint condition on the six unknowns,

$$1 = -2/\varphi_1/(Y_1 - 1) + 2/\varphi_2/(Y_2 + 1). \tag{7}$$

Of course, the to-be-obtained numerical solutions to the given system parameters represent actual imaging systems with both real object and image spaces only when certain conditions are fulfilled, such as $s_1 < 0$ and $s'_2 > 0$. All lens systems free from total monochromatic Seidel aberrations discussed in the next section should and do implicitly satisfy these constraint conditions.

3. Solutions and examples

Based on the discussions in Section 2 , we now have the two arbitrary parameters,

$$n_1, n_2,$$

and the six intermediate variables,

$$X_1, X_2, Y_1, Y_2, \varphi_1, \varphi_2,$$

to fulfil the six equations, Eqs.(5a–5e) and (7). Unsurprisingly, on solving these equations using numerical computation method, we note that no solutions possibly can be found when n_1 and n_2 are assigned with arbitrary positive values. This verifies the fact that a pair of spherical thin lenses both made of the conventional PIMs is impossible to form an imaging system without all the five monochromatic Seidel aberrations, which is also the basic reason for the Cooke triplet. Thus in the following discussions, without loss of generality, the front lens is assumed to be made of NIMs and, to facilitate the following illustration, the ratio of n_2 to n_1 is also defined here that

$$\eta = n_2/n_1,$$

where for $\eta < 0$, the back lens is supposed to be made of PIM, while for $\eta > 0$, NIM.

In Fig.1, we show the possible distributions of n_1 for the NIM–PIM combinations of the two lenses with several negative values of η in the given region of n_1, $-4 \leq n$, and for the NIM–NIM cases with several positive values of η in the given region of n_1, $-6 \leq n$, where by these given values of the two lenses' refractive indices, valid numerical solutions to the system parameters can be found. The number of horizontal lines in each row stands for how many sets of solutions being found for the corresponding value of η. The small

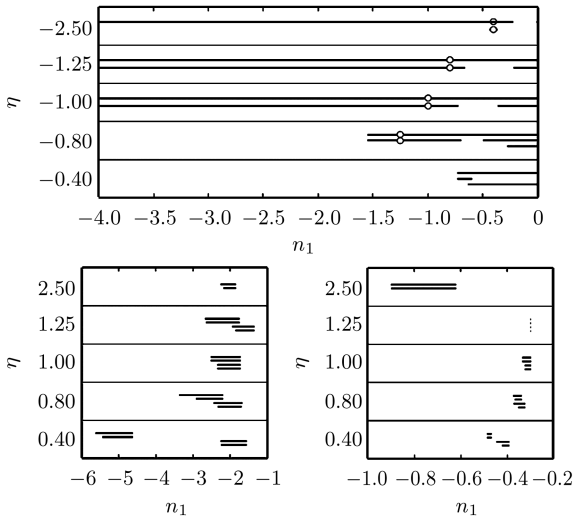


Fig.1. Possible distributions of n_1 for the several values of η, where valid solutions to imaging systems composed of a pair of spherical thin lenses are found free from all five monochromatic Seidel aberrations based on the introduction of NIMs.

circles in the upper part of Fig.1 represent those points which should be excluded, where the back lens is actually absent in those cases with $n_2 = 1$. It is worth

noting that the number of the solution sets, as well as the possible distributions of n_1, varies with the given values of η. In the NIM–PIM combinations with $\eta = -1$, for example, there exist two sets of solutions in the regions of n_1, $-4.0 \le n_1 \le -0.7179$ and $-0.3658 \le n_1 < 0$, excluding the special point with $n_1 = -1$, while only one set of solutions can be found for $-0.7179 < n_1 < -0.3658$. On the other hand, for the NIM–NIM combinations with $\eta = 1$, where the two lenses have a same negative index, as many as four sets of valid solutions are possible in the corresponding regions of n_1, $-2.339 \le n_1 \le -1.707$ and $-0.3205 \le n_1 \le -0.2929$. As an inevitable result of the mutual self-compensation effects for the NIM–PIM combinations, it is apparent that for cases with $\eta < 0$, the corresponding distributions of n_1 are much wider than the NIM–NIM cases with $\eta > 0$. In Fig.2, two sets of numerical solutions to the system parameters are plotted as functions of n_1 for the case that $\eta = -1$, which are found in the regions of n_1 as given in Fig.1. To the first set of solutions, the values of the four radii of curvature are all positive and satisfy the four inequalities, $r_1 > r_1' > 0$ and $r_2' > r_2 > 0$. So the front lens is supposed to be a negative meniscus, while the back lens being a positive meniscus. As for the second set of solutions, the front lens is a negative meniscus of another type with $r_1' < r_1 < 0$ and the back lens retains a positive meniscus with $r_2' > r_2 > 0$. It should be noted that the diameters of the back lenses are relatively small, limiting the extent of field of view for such lens systems. Indeed, as only rays making small angles with the axis are traced for the calculations of the Seidel aberrations, such lens systems are suitable for the imaging with not-too-wide fields of view in the object space. In Fig.3, we provide the schematic drawings of the two example of such lens systems, represented by the two dotted lines in Fig.2 where $n_1 = -1.6$ and $n_2 = 1.6$. The calculated numerical solutions to other system parameters of the two imaging systems are as follows:

for (a): $s_1 = -1.764$, $s_1' = 1.178$, $s_2 = 0.1779$, $s_2' = 0.1421$, $r_1 = 2.525$, $r_1' = 1.063$, $r_2 = 0.2091$, $r_2' = 0.4128$

for (b): $s_1 = -0.6750$, $s_1' = 1.159$, $s_2 = 0.1592$, $s_2' = 0.1159$, $r_1 = -0.6906$, $r_1' = -0.1830$, $r_2 = 0.1490$, $r_2' = 0.3564$

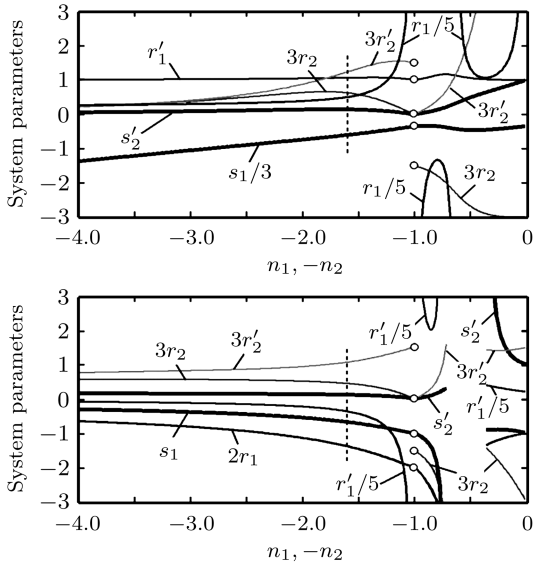


Fig.2. Two sets of numerical solutions of two-thin-lens systems free from all five mono-chromatic Seidel aberrations are shown as the functions of the two lenses' refractive indices in the special case that $\eta = -1$.

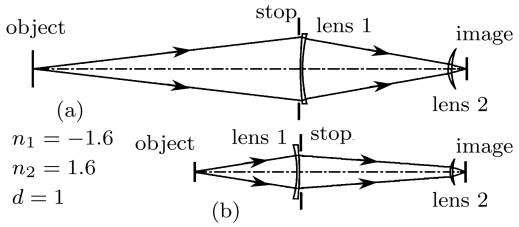
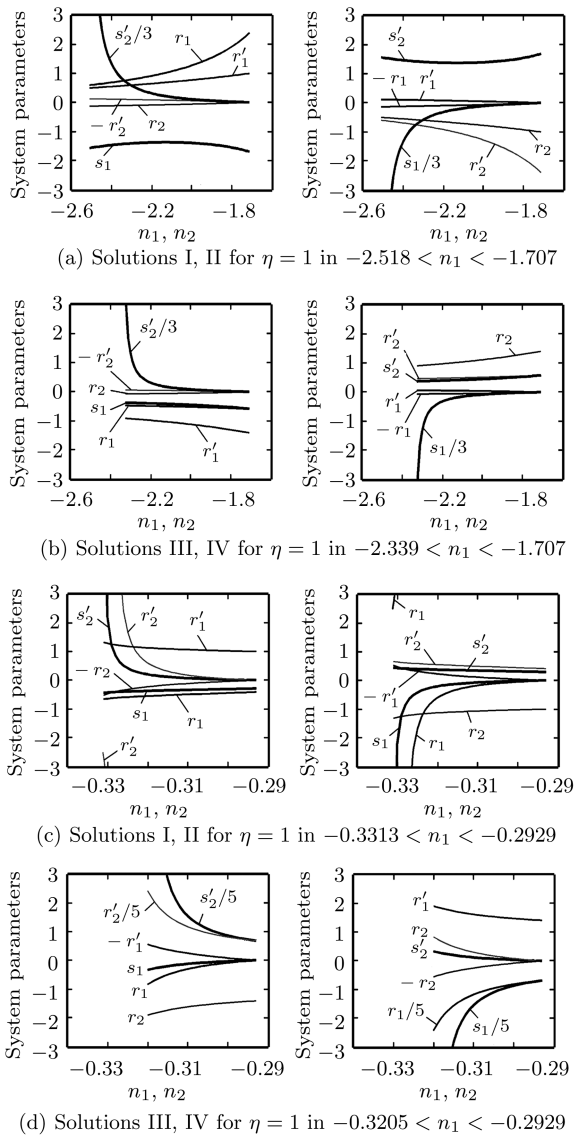


Fig.3. Schematic drawings for the two examples of monochromatic Seidel aberration-free imaging systems composed of a NIM spherical thin lens with $n_1 = -1.6$ and a PIM one with $n_2 = 1.6$.

Although the remote-imaging systems are of interest to us, the obtained values of the object distances for the NIM–PIM cases such as those given in Fig.2, however, are fairly comparable in length to the interval between the two thin lenses and then unpractical to focus the radiation from a faraway object. But is it possible for a couple of spherical thin lenses both made from NIMs to attain the aim of remote imaging? In Fig.4, we give the four sets of numerical solutions to the imaging system parameters for the NIM–NIM combinations with $\eta = 1$, distributing in the two narrow regions of n_1, approximately centred at the two points, -0.023 and -0.3067, respectively. Due to the two thin lenses having a same index of refraction, the first and second sets of numerical solutions are conjugated and thence are plotted together in the same row. The third and fourth sets of solutions are with the same operation. From Fig.4, we find that the calculated solutions to the object distances for each set of conjugates do distribute from the negative infinity to zero in the variation of n_1. Therefore, regardless how

far the object locates, we are always able to design a lens system composed of two NIM thin-lenses to focus



(a) Solutions I, II for $\eta = 1$ in $-2.518 < n_1 < -1.707$

(b) Solutions III, IV for $\eta = 1$ in $-2.339 < n_1 < -1.707$

(c) Solutions I, II for $\eta = 1$ in $-0.3313 < n_1 < -0.2929$

(d) Solutions III, IV for $\eta = 1$ in $-0.3205 < n_1 < -0.2929$

Fig.4. Four sets of numerical solutions to the imaging system composed of a pair of spherical thin lenses with a same negative index and free from all five monochromatic Seidel aberrations are shown as functions of the refractive index.

the radiation emitting from the object into a real image free from all five monochromatic Seidel aberrations. Note that the numerical values of the solutions to the radii of the first lens are much larger in the region near $n_1 = -0.32$ than those in the region near $n_1 = -2.3$. So it is recommendable to design such type of thin-lens systems for far-field focusing according to the given solutions shown in Fig.4(d). In light of potential application, an ultimately-remote imaging system is illustrated in Fig.5 with $s_1 = -\infty$ and $n_1 = n_2 = -0.3205$. The explicit values of the structural parameters of such a special system are given as follows $s_1 = -\infty$, $s_1' = 1.280$, $s_2 = 0.2796$, $s_2' = 0.3579$, $r_1 = -12.89$, $r_1' = 1.942$, $r_2 = 0.5814$, $r_2' = 0.8869$.

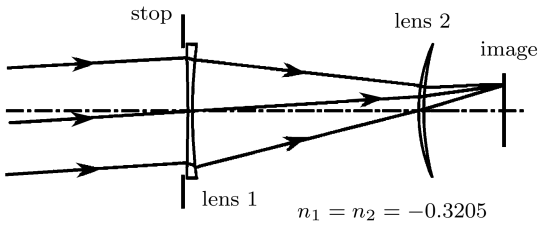


Fig.5. Ray-tracing of an ultimately-remote imaging system without monochromatic Seidel aberrations and composed of two spherical thin lenses with a same negative index of refraction.

4. Conclusions

Imaging systems free from all the five monochromatic Seidel aberrations are always holding interest for many lens designers and engineers. Based on solving the quartic equations of the five Seidel aberration sums provided, we show that an imaging system composed of a pair of spherical thin lenses is possible to form a real image in air and free from all the five Seidel aberrations when one or both of them are made of NIMs,[25] as which is impossible to achieve when only the conventional PIMs are utilized. The calculated numerical solutions found to structural parameters of such lens system with common spherical surfaces are presented for the several combinations of refractive indices and examples of them are given, respectively. Unlike the NIM flat-slab-lenses only with short working distances,[20] such kinds of lens systems proposed in this work seem useful for high-quality remote-imaging where reduced number of lenses and simpler lens surfaces are required, as compared to their conventional counterparts. They mostly can be applied in the astronomical or radar lens systems with high remote-focusing performance.

References

[1] Veselago V G 1967 *Sov. Phys. Solid State* **8** 2854
[2] Pendry J B 2000 *Phys. Rev. Lett.* **85** 3966
[3] Smith D R, Padilla W J, Vier D C, Nemat-Nasser S C and Schultz S 2000 *Phys. Rev. Lett.* **84** 4184
[4] Shelby R A, Smith D R and Schultz S 2001 *Science* **292** 77
[5] Iyer A K, Kremer P C and Eleftheriades G V 2003 *Opt. Express* **11** 696
[6] Zheng Q, Zhao X P, Fu Q H, Zhao K L and Li M M 2005 *Acta Phys. Sin.* **54** 5683 (in Chinese)
[7] Podolskiy V A, Sarychev A K and Shalaev V M 2003 *Opt. Express* **11** 735
[8] Zhang S, Fan W, Panoiu N C, Malloy K J, Osgood R M and Brueck S R J 2005 *Phys. Rev. Lett.* **95** 137404
[9] Shalaev V M, Cai W, Chettiar U, Yuan H, Sarychev A K, Drachev V P and Kildishev A V 2005 *Opt. Lett.* **30** 3356
[10] Zhang S, Fan W, Malloy K J, J. Brueck S R, Panoiu N C and Osgood R M 2005 *Opt. Express* **13** 4922
[11] Gralak B, Enoch S and Tayeb G 2000 *J. Opt. Soc. Am.* A **17** 1012
[12] Luo C, Johnson S G, Joannopoulos J D and Pendry J B 2003 *Phys. Rev.* B **68** 045115
[13] Parimi P V, Lu W T, Vodo P, Sokoloff J, Derov J S and Sridhar S 2004 *Phys. Rev. Lett.* **92** 127401
[14] Jiang T, Chen Y and Feng Y J 2006 *Chin. Phys.* **15** 1154
[15] Vodo P, Parimi P V, Lu W T and Sridhar S 2005 *Appl. Phys. Lett.* **86** 201108
[16] Schurig D and Smith D R 2004 *Phys. Rev.* E **70** 065601
[17] Welford W T 1974 *Aberrations of the Symmetrical Optical System* (New York: Academic)
[18] Dyson J 1959 *J. Opt. Soc. Am.* **49** 713
[19] Schulz G 1980 *J. Opt. Soc. Am.* **70** 1149
[20] Lin Z L and Zou Y Z 2006 *Appl. Opt.* **45** 6925
[21] Chen J, Radu C and Puri A 2006 *Appl. Phys. Lett.* **88** 071119
[22] Born M and Wolf E 1975 *Principles of Optics* (New York: Pergamon)
[23] Mikš A 2002 *J. Opt. Soc. Am.* A **19** 1867
[24] Hopkins H H 1950 *Wave Theory of Aberrations* (Oxford: Clarendon)
[25] Dong Z G, Zhu S N and Liu H 2006 *Chin. Phys.* **15** 1772