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Theoretical investigation of an electron beam propagating through a wide gap cavity

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This paper presents a self-consistent nonlinear theory of the current and energy modulations when an electron beam propagates through an inductively-loaded wide gap cavity. The integro-differential equations are obtained to describe the modulation of the beam current and kinetic energy. A relativistic klystron amplifier (RKA) model is introduced, which uses an inductively-loaded wide gap cavity as an input cavity. And a numerical code is developed for the extended model based on the equations, from which some relations about the modulated current and modulated energy are numerically given.

Keywords: RKA, wide gap cavity, intense electron beam, modulation
PACC: 4180, 5240M, 2925B

1. Introduction

The relativistic klystron amplifier (RKA) is a coherent transition radiation device which uses the bunched electron beam to produce high power microwave (HPM). It has been proved as one of the most promising high power microwave and radio frequency (RF) sources in the frequency range of 0.1–10GHz and gigawatt power range.[1,2] After its initial development at the Naval Research Laboratory (NRL) there are now several areas of investigation into long pulse, repeated, L band, S band, J band, and X band RKAs. Compared to other sources of HPM,[1,3,4] such as relativistic magnetrons, vicators, or backward wave oscillators (BWOs), the RKA is characterized by an inherent phase and amplitude stability and electrostatic insulation.

In the most common klystronlike devices, the electron beams have been modulated by a unidirectional RF voltage applied to a narrow gap whose length is less than a quarter wavelength. To decrease the RF electric field and reduce the nonlinear beam loading, a novel gap design which incorporates an inductively-loaded return-current structure has been developed at the NRL.[5,6] In this geometry the gap length was about a half wavelength. Using the wide-gap cavity as the modulation cavity, we find that the current modulation will be different from the previous one because of the long transit time. Based on the theory about the narrow-gap RKA, A self-consistent nonlinear theory of the current and energy modulation of an electron beam crossing the wide gap cavity is developed in this paper.

2. Theoretical model and basic assumptions

In this section we present a nonlinear theoretical model of the wide-gap cavity based on Uhm's theory[7] which is under the assumption of a narrow gap cavity.

To simplify the following analysis, we give some assumptions as follows: (1) the electron beam is spatially uniform and the energy spread is zero at the entrance of the gap; (2) under the strong magnetic field, the transverse motion of the electron is neglected; (3) the thickness of the beam is neglected; (4) the DC field between the gap of the cavity is neglected; (5) the long wavelength approximation, i.e., $k_z R_\mathrm{b} < 1$, where k_z is axial wave number and R_b is beam radius.

2.1. RF self-electric field

When a segment of electron beam pulse begins to pass through the cavity, it gains or loses energy. The velocity nonuniformity will transform into den-

sity nonuniformity in the electron distribution. The axial RF electric field component $E_{\rm sc}(r,z,t)$ will be generated with the charge and current deviations according to the Maxwell equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}E_{\rm sc}\right)+\frac{\partial^2}{\partial z^2}E_{sc}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}E_{\rm sc} =\mu\frac{\partial}{\partial t}(j-j_0)+\frac{1}{\varepsilon_0}\frac{\partial}{\partial z}(\rho-\rho_0), \quad (1)$$

where j, ρ are the total beam current and charge density, and, j_0, ρ_0 are the DC current and charge density, respectively.

The propagation distance z is related to time t and initial time t_0 by $z=\int_{t_0}^{t}c\beta(t')\,{\rm d}t'$, where βc is the instantaneous velocity of the electron at the time t, $\theta_0=\omega t_0$ $\theta=\omega t$ are the instantaneous phase at the time t_0 and t, respectively.

The charge and current densities for the beam segment θ_0 are approximately related to one another by

$$\rho\simeq\frac{j}{\beta c},\ \frac{\partial\rho}{\partial z}=\frac{1}{\beta c}\frac{\partial\rho}{\partial t}. \quad (2)$$

Under the long wavelength approximation, which is good for the fundamental mode, substituting Eq.(2) into Eq.(1) we obtain

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}E_{\rm sc}\right)=-\mu_0\frac{1}{\gamma^2\beta^2}\frac{\partial j}{\partial t}. \quad (3)$$

It is very useful to define the normalized current $F(\zeta,\theta)$:

$$F(\zeta,\theta_0)=I(\zeta,\theta_0)/I_0, \quad (4)$$

where $\zeta=\omega z/(\beta_{\rm b}c)$ is the normalized propagation distance, I_0 is the initial current, and $I(\zeta,\theta_0)$ is the instantaneous beam current

$$I(\zeta,\theta_0)=2\pi\int_0^{R_w}rj(r,\zeta,\theta_0)\,{\rm d}r.$$

Integrating Eq.(2), we can obtain the average axial electric field:

$$E_{\rm sc}(\zeta,\theta_0)=\frac{\mu_0\ln(R_{\rm w}/R_{\rm b})}{2\pi}\frac{\omega I_0}{\gamma^2-1}\frac{\partial F}{\partial\theta}, \quad (5)$$

where $R_{\rm w}$ is the radius of the drift tube. From Eq.(5) we can see that the self-electric field of the intense electron beam consists of three factors: the first factor is related to the parameters of the configuration, the second one is related to the initial condition of the electron beam, and the last one is related to the current modulation.

2.2. The current and energy modulation

We introduce the following phase relation,

$$\theta=\theta_0+\int_0^{\zeta}\frac{\beta_0}{\beta}{\rm d}\zeta'. \quad (6)$$

The beam current at z=0 is a constant value of I_0. When this segment arrives at z at time t, it is stretched by a factor of $\partial t/\partial t_0$. According to the conservation of charge, the ratio F of the current $I(\zeta,\theta)$ to the initial current I_0 is expressed as:

$$F(\zeta,\theta_0)=\frac{I(\zeta,\theta_0)}{I_0}=\frac{1}{\partial\theta/\partial\theta_0}, \quad (7)$$

$$\frac{\partial\theta}{\partial\theta_0}=1-\beta_0\int_0^{\zeta}\frac{1}{(\gamma^2-1)^{3/2}}\left(\frac{\partial\gamma}{\partial\theta_0}\right){\rm d}\zeta'. \quad (8)$$

Combining Eqs.(7) and (8), we find the normalized current:

$$\frac{\partial F}{\partial\zeta}=\frac{\beta_0F^2}{(\gamma^2-1)^{3/2}}\frac{\partial\gamma}{\partial\theta_0}. \quad (9)$$

With the above mentioned assumptions, the equation of motion of electron is given by

$$mc^2\frac{{\rm d}\gamma}{{\rm d}z}=-eE, \quad (10)$$

where $\gamma=1/\left(1-\beta^2\right)^{1/2}$, $E(z,t)$ is the axial component of the total electric field at the location of the electron.

The axial electric field $E(z,t)$ is comprised of: (1) the RF electric field associated with space-charge bunching $E_{\rm sc}$; (2) an externally applied, modulating electric field $E_{\rm mod}$, depending on the cavity configuration and the power in the input cavity. Thus

$$E=E_{\rm sc}+E_{\rm mod}. \quad (11)$$

In the wide gap cavity, the modulated electric field is given by

$$E_{\rm mod}(\zeta,\theta)=E_0\alpha(\zeta)f(\theta). \quad (12)$$

So, the energy factor of this electron is

$$\gamma=\gamma_0-\kappa f(\theta)\int_0^{\zeta}\alpha(\zeta')\,{\rm d}\zeta' -\frac{\eta}{(\gamma^2-1)}\int_0^{\zeta}{\rm d}\zeta'\left(\frac{\partial F^2}{\partial\theta_0}\right), \quad (13)$$

where we define two factors: $\kappa=\dfrac{eE_0\beta_0}{\omega mc}$, relevant to the external electric field; $\eta=\dfrac{I_0\beta_0}{2I_{\rm s}}$, relevant to the electron beam and the configuration parameters, where $I_{\rm s}$ is the current parameters

$$I_{\rm s}=\frac{2\pi\varepsilon_0mc^3}{e\ln(R_{\rm w}/R_{\rm b})}=\frac{8.5{\rm kA}}{\ln(R_{\rm w}/R_{\rm b})}.$$

Using Eqs.(9) and (13), we find

$$\frac{\partial F}{\partial \zeta}=\frac{-\dfrac{\kappa\beta_0 F f'(\theta)}{(\gamma^2-1)^{3/2}}\displaystyle\int_0^{\zeta}\alpha(\zeta')\,\mathrm{d}\zeta'-\dfrac{\eta\beta_0 F^2}{(\gamma^2-1)^{5/2}}\int_0^{\zeta}\mathrm{d}\zeta'\left(\dfrac{\partial F^2}{\partial^2\theta_0}\right)}{1-\dfrac{2\eta\gamma}{(\gamma^2-1)^2}\displaystyle\int_0^{\zeta}\mathrm{d}\zeta'\left(\dfrac{\partial F^2}{\partial\theta_0}\right)}. \tag{14}$$

This is the expression of the normalized current, with initial condition $F(0,\theta)=1$. Eqs.(6), (13) and (14) are the expressions of the electron phase, energy factor and normalized current when an electron beam crosses a wide gap cavity. The current modulation and the energy modulation are coupled. They can be solved simultaneously by numerical method.

3. Numerical results

To solve Eqs.(6), (13) and (14), we develop a numerical code for the extended model. A RKA model is introduced which uses an inductively- loaded wide gap cavity as an input cavity,[5,6] as illustrated in Fig.1(a). The increase of potential energy in the wide gap is prevented by stacking the gap with thin metallic washers which are connected inductively to ground. The length of the wide gap cavity is 6.0cm, about a half wavelength. To keep the electrons in synchronism with the RF field while crossing such a long gap, a bidirectional electric field is imposed, i.e., a field which at any given time changes its sign at the midpoint of the gap. The axial electric field distribution is shown in Fig.1(b).

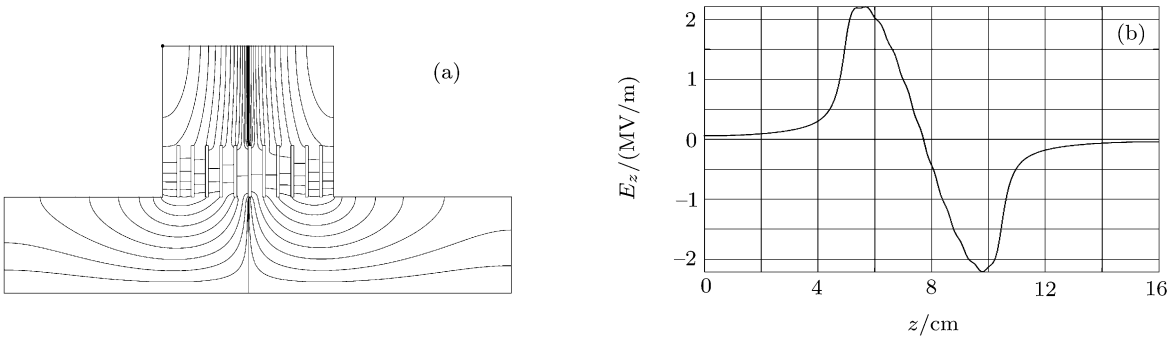


Fig.1. (a) Schematic figure of an inductively- loaded wide gap cavity; (b) The axial electric field distribution in the wide gap cavity.

Electron-beam parameters are: (1) radius, $r_0=2.4$cm; (2) current, $I_0=6$kA; (3) voltage, $V_0=500$kV.

The modulating electric field in the gap is given by

$$E_{\mathrm{mod}}(z,\theta)=E_0\alpha(z)f(\theta),$$

where $\alpha(z)$, $f(\theta)$ are the spatial and temporal distributions,

$$\alpha(z)=\cos\left(\frac{2\pi z}{z_{\mathrm{gap}}}\right),\ f(\theta)=\sin(\theta).$$

The modulation frequency, $f_{\mathrm{mod}}=2.86$GHz, is identical to the TM011 mode eigenfrequency.

The initial normalized current is unity for arbitrary values of θ_0, i.e. $F(0,\theta_0)=1$, and the initial energy factor is $\gamma(0,\theta_0)=\gamma_0$ for arbitrary θ_0. With a reasonably small integration step of $\delta\zeta$, we repeat numerical calculations of the $\theta(\delta\zeta)$, $\gamma(\delta\zeta)$ and $F(\delta\zeta)$ until arrive at the propagation distance ζ.

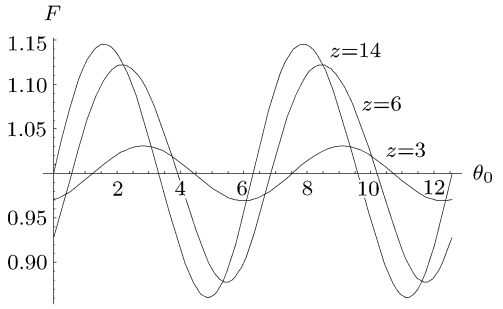


Fig.2. Plots of the normalized current profile versus initial phase.

Plots of the normalized current profile $F(\zeta, \theta)$ are shown in Fig.2, in which different curves indicate different propagation distance. Along with the increasing of the drift distance, the current modulation depth is enhanced.

Figure 3(a) is a plot of normalized kinetic energy versus axial distance z for each electron with the different initial phase. The associated electric field of the space-charge wave on a drift distance decelerates fast electrons and accelerates slow electrons, thereby reducing the kinetic energy spread. At a distance of 8cm beyond the gap, energy spread is reduced to a minimum value, meanwhile the current modulation coefficient becomes maximum. So it is the best place where we put the second cavity in RKA. The maximum modulation coefficient of the current is near 20%.

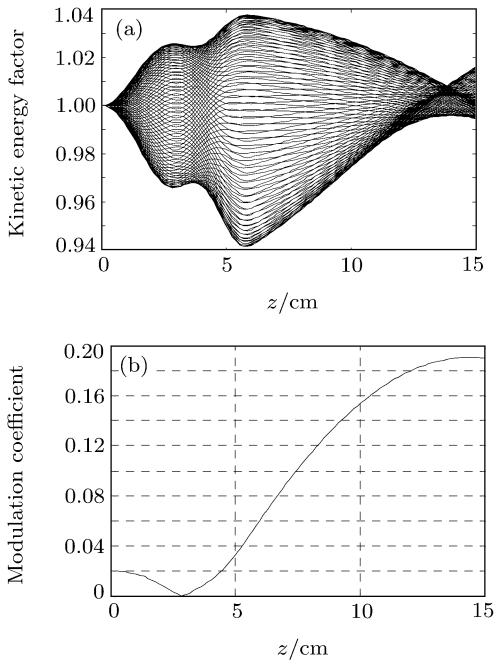


Fig.3. (a) Plot of electron normalized kinetic energy versus axial distance z; (b) plot of modulation coefficient versus axial distance z

The amount of current modulation coefficient depends sensitively on the magnitude of the modulated electric field and on beam parameters. We change the beam density and have obtained the curve of the modulation coefficient with different beam density, as shown in Fig.4. When the beam current increases, the modulated coefficient decreases and shorter drift distance is required to achieve maximum modulation.

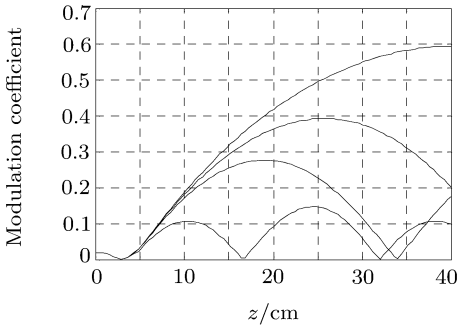


Fig.4. Plots of the modulation coefficient with different beam density versus axial distance z

The integro-differential equations are so complex that require a long computer time to calculate. More work is needed to simplify the equation and extract simple physical meanings.

4. Summary and discussion

Thus, a self-consistent nonlinear theory in an electron beam propagating through a wide gap cavity is investigated. The integro-differential equations are obtained to describe the modulation of the beam current and energy. An inductively-loaded wide gap cavity model is introduced and solved by numerical method. Under the modulation of the external electric field and the space-charge electric field, a 500kV, 6kA electron beam propagates through the wide gap and the modulation coefficient of the current can reach 20%. Some relations about the modulated current, modulated energy and modulation coefficient are given by solving the equations numerically.

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