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Parametric resonance and cooling on an atom chip*

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This paper observes the parametric excitation on atom chip by measuring the trap loss when applying a parametric modulation. By modulating the current in chip wires, it modulates not only the trap frequency but also the trap position. It shows that the strongest resonance occurs when the modulation frequency equals to the trap frequency. The resonance amplitude increases exponentially with modulation depth. Because the Z-trap is an anharmonic trap, there exists energy selective excitation which would cause parametric cooling. We confirm this effect by observing the temperature of atom cloud dropping.

Keywords: atom chip, parametric excitation, parametric cooling
PACC: 3280P, 4250

1. Introduction

The atom chip is a configuration using current carrying wires to create a magnetic trap.[1,2] It has become an important method in cold atom physics.[3−6] Since the trap is very close to the wires, it has large magnetic field gradients and curvatures which are very useful for evaporative cooling, leading to fast production of Bose–Einstein condensation (BEC).[1,7] It is easy to form various traps by carefully organizing the current wires.[8,9] The commonest kinds are U-trap and Z-trap with the wire bent U-shaped and Z-shaped respectively.[10] In addition, there is another advantage of the atom chip: we can easily modulate the wire current due to small inductance of the wire. In traditional BEC experiments, people use large coils to form a magnetic trap, so the frequency of modulation current can only reach the KHz range. But for chip wires, it is easy to achieve MHz range and even higher. It provides a chance to extend the DC trap to AC trap by using suitable AC current. In the KHz range, it is useful to create a time-orbiting potential (TOP) trap,[11] and by introducing a KHz range current modulation, it can be used to suppress the roughness of a wire-based, magnetic atom guide.[12] When we apply an AC current in the MHz range, there will be a radio frequency field which can be used in evaporative cooling, or used to realize atom splitter.[13]

Parametric excitation is an effective method to study atoms in a trap. By modulating the trap, observing the behaviour of atoms, we can get much information about the trap and the atoms. Such a method has been applied in the optical trap,[14] magnetic trap,[15,16] and so on. Usually, the trap is harmonic, and the modulation parameter is confined to the trap frequency. This leads to the strongest resonance which occurs at $\omega_{\rm m} = 2\omega$, where $\omega_{\rm m}$, ω are frequencies of modulation and the trap respectively. In this paper, we report some experiments about parametric excitation on atom chip. By modulating the wire current, we modulate not only the frequency but also the position of the trap, and find that the strongest resonance occurs at $\omega_{\rm m} = \omega$. We also study the roles of frequency modulation and position modulation. Because the Z-trap is anharmonic as shown in Fig.1, the oscillation frequency of an atom in the trap is relative to its energy. So there exists energy selective excitation which would leads to parametric cooling, and we have observed this cooling effect.

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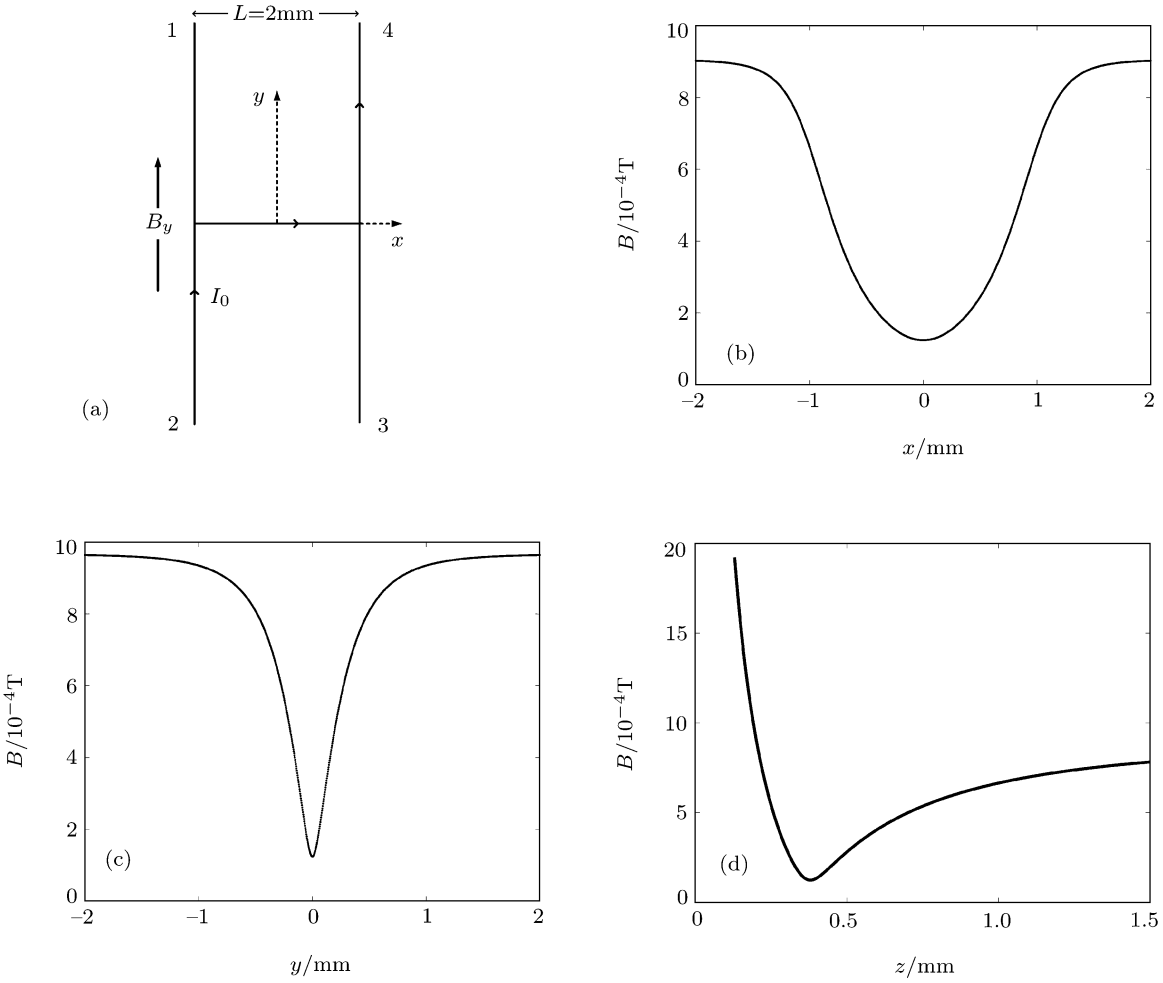


Fig.1. (a) The H wire in our chip. If we connect two ends of 2 and 4, it is a Z-wire. The bias field is also shown. (b) (c) (d) are the depiction of magnetic field of x, y, z directions created by Z wire with 1.85A current and bias field B_y 0.9mT. They are anharmonic especially in the z direction as shown in (d).

2. Parametric exciting and parametric resonance

A classical picture of parametric excitation is described in Ref.[17]. But it is easy to understand when we use the quantum picture. Let us assume that an atom moves in a trap, the Hamiltonian is

$$H_0 = \frac{P^2}{2M} + V_{\rm trap}(\boldsymbol{r}). \tag{1}$$

Then we apply a modulation to the trap, and the Hamiltonian will become

$$H = \frac{P^2}{2M} + V_{\rm trap}(\boldsymbol{r}, \epsilon(t)). \tag{2}$$

Here $\epsilon(t)$ represents a small modulation. Using Taylor series, and keeping the first order, we obtain

$$H = \frac{P^2}{2M} + V_{\rm trap}(\boldsymbol{r}) + \frac{\partial V_{\rm trap}}{\partial \epsilon(t)}\epsilon(t). \tag{3}$$

The first and second term is unmodulated Hamiltonian H_0, and the last term can be taken as a perturbation. We label the eigenstates of H_0 as $|n\rangle$, and eigenenergies as E_n. The perturbation will cause transition between different eigenstates, and the transition rate is dominated by the transition matrix element

$$T_{m,n} = \langle m|\frac{\partial V_{\rm trap}}{\partial \epsilon(t)}\epsilon(t)|n\rangle. \tag{4}$$

If the modulation is periodic, such that

$$\epsilon(t+T) = \epsilon(t), \tag{5}$$

it results in a resonance. As shown in Fig.2, both sine and square wave modulation resulted in some resonances. Where the resonance occurs depends on trap potential and transition matrix element.

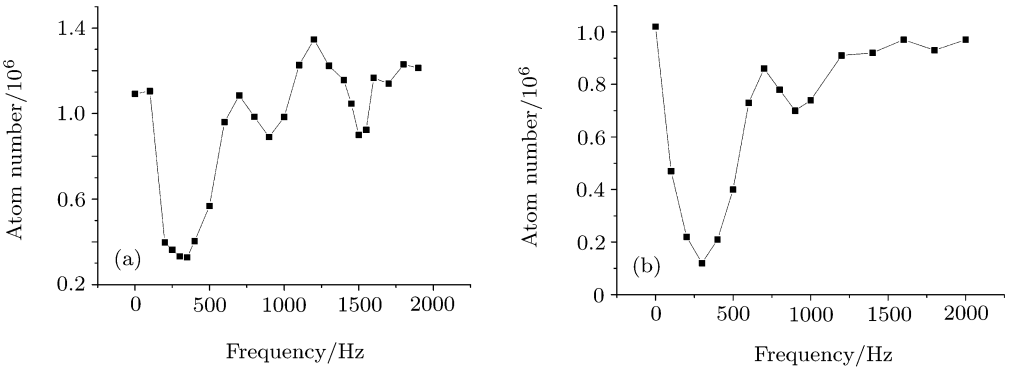


Fig.2. The residual atom number versus the modulation frequency when we apply a 5% modulation to wire current. The wire current is 1.85A, the bias field is 0.9mT. (a) The modulation is a sine wave. (b) The modulation is square wave. In both conditions the trap losses have resonances near $\omega_{\rm m} = \omega = 300$Hz.

Let us consider a simple case: the trap is harmonic, and only the trap frequency is modulated with $\sin(\omega_{\rm m}t)$ form. The Hamiltonian is

$$H = \frac{P^2}{2M} + \frac{1}{2}M\omega^2 x^2[1 + \epsilon\sin(\omega_{\rm m}t)]^2, \qquad (6)$$

so

$$\frac{\partial V_{\rm trap}}{\partial\epsilon(t)} = M\omega^2 x^2. \qquad (7)$$

But x^2 will lead to transition only from $|n\rangle$ to $|n\rangle$ and $|n\rangle$ to $|n\pm 2\rangle$, so there will be a resonance at $\omega_{\rm m} = 2\omega$. Using k-fold perturbation theory, we find that there will be resonances at $\omega_{\rm m} = 2\omega/k$, k is an integer, as pointed out by Ref.[17].

Another case is trap position modulation: the Hamiltonian is

$$H = \frac{P^2}{2M} + \frac{1}{2}M\omega^2[x - \epsilon_x\sin(\omega_{\rm m}t)]^2, \qquad (8)$$

thus

$$\frac{\partial V_{\rm trap}}{\partial\epsilon(t)} = -M\omega^2 x. \qquad (9)$$

Since x leads to transition between only $|n\rangle$ to $|n\pm 1\rangle$. So the strongest resonance occurs when $\omega_{\rm m} = \omega$. Also using k-fold perturbation theory, we find that there will be resonances at $\omega_{\rm m} = \omega/k$, where k is an integer.

In our experiment, we modulate the trap by modulating the chip wire current $I_{\rm m} = I_0[1 + \epsilon\sin(\omega_{\rm m}t)]$. Since the trap centre is $x_0 = 0$, $y_0 = 0$, and z_0 is determined by

$$\frac{\mu_0 I}{4\pi}\frac{L}{z_0\sqrt{(L/2)^2 + z_0^2}} = B_y, \qquad (10)$$

where B_y is the bias field, I is the current in the wire, L is the length of the middle wire, as shown in Fig.1(a), and z_0 is the position of trap centre in the z direction. There does not exist a simple formula for the position modulation, but we can numerically calculate the position modulation by using our experimental data listed in Section 3. First, we fit the relation between z_0 and I near the experimental data. We obtain $z_0 = A + B\cdot I$. When the current modulation depth is ϵ, the position modulation depth is $BI_0/z_0\cdot\epsilon$. In our case, the trap centre is modulated by

$$z = z_0[1 + 0.89\epsilon\sin(\omega_{\rm m}t)]. \qquad (11)$$

Similarly, for frequency, $\omega_0 = D + E\cdot I$, current modulation with depth ϵ will result in frequency modulation with depth $EI_0/\omega_0\cdot\epsilon$. Using the same experimental data, we find that the frequency modulation is

$$\omega = \omega_0[1 - 1.6\epsilon\sin(\omega_{\rm m}t)]. \qquad (12)$$

In order to study the role of position modulation, we modulate both the Z-wire current I and the bias field B_y with the same depth and phase, the trap centre will not change as indicated in equation (3) only the frequency is modulated,

$$\omega = \omega_0[1 + 0.5\epsilon\sin(\omega_{\rm m}t)]. \qquad (13)$$

By carefully organizing the modulation depth of the Z-wire current and the bias field, we can make the modulation a pure position modulation. First, we can get the relation near a certain group of parameters by using linear approximation:

$$z_0 = A + B\cdot I + C\cdot B_y, \qquad (14)$$

$$w_0 = D + E\cdot I + F\cdot B_y. \qquad (15)$$

If the modulation depth ratio between Z-wire current and bias field is $-F/E$, the frequency modulation

will be eliminated. We numerically calculate and find if

$$I_{\rm m} = I_0[1 + \epsilon \sin(\omega_{\rm m} t)], \tag{16}$$

and

$$B_{ym} = B_{y0}[1 + 0.756\epsilon \sin(\omega_{\rm m} t)], \tag{17}$$

the position modulation depth is 0.244ϵ and the frequency modulation depth is less than 1% ϵ, we can consider that there exists only position modulation.

3. Experiment

The wire in our chip is a H-shape wire, as depicted in Fig.1. We connected two ends of 2 and 4, thus obtained a Z-wire. The magnetic field created by the Z-wire with a bias field B_y is shown in Fig.1. Additional details about our configuration is described in Ref.[18]. The experimental procedure is as follows. First, we use the mirror-magneto-optic trap (MOT) to capture about 4×10^6 atoms, then transfer them to the chip U-MOT, compress the MOT and molasses. After that, we load atoms to the Z-trap.

The Z-trap is created by a Z-wire with 1.85A current and a bias field about 0.9mT. It creates a trap with potential minimum about 400μm below the chip. The frequency of the Z-trap is $2\pi \times 25$ Hz for axial direction and $2\pi \times 301$ Hz for radial direction respectively. After the Z-trap loading, we hold the atoms for 0.3s, in order to thermalize them. After that, there are about 1×10^6 atoms in the Z-trap and the temperature is about 70μK.

Then we apply a 200ms modulation of the Z-wire current $I_{\rm m} = I_0[1 + \epsilon \sin(\omega_{\rm m} t)]$, modulation depth is $\epsilon = 5\%$, the modulation frequency changes from 100Hz to 2000Hz. According to Eqs.(4) and (5), there will be both frequency and position modulation.

After modulation, we measure the number of atoms remained in the trap. The trap loss is a direct consequence of parametric excitation. As shown in Fig.3, we can see that there is a strong resonance when $\omega_{\rm m} = \omega = 300$Hz for both sine and square wave modulation, unlike usually frequency modulation where the strongest resonance is at $\omega_{\rm m} = 2\omega$. It is due to the anharmonicity of the trap and the modulation of trap position. And the resonance width is very wide, because the Z-trap is an anharmonic trap. We will discuss the anharmonicity of the trap potential in the next section.

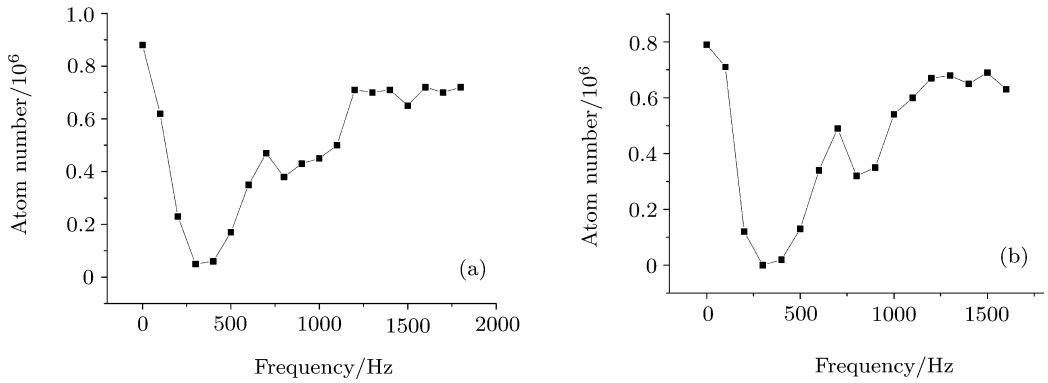


Fig.3. The residual atom number versus modulation frequency for pure frequency modulation and position modulation. (a) When we apply a 10% modulation to Z-wire current and 10% modulation to bias field B_y with the same phase, there is only frequency modulation; the trap position does not change. (b) When we apply a 10% modulation to wire current and 7.56% modulation to bias field B_y with the same phase, there is only position modulation; the trap frequency modulation can be ignored. Both these two conditions have the strongest resonances at $\omega_{\rm m} = \omega = 300$Hz.

In order to investigate the roles of frequency modulation and position modulation, firstly we applied a modulation with 10% depth for both current and bias field. It will result in only frequency modulation, and the modulation depth is determined by formula (13). Secondly, we applied a current modulation with depth 10% and a bias field modulation with depth 7.56%. It will result in almost pure position modulation. In both cases, we found a strong resonance near $\omega_{\rm m} = \omega$. For pure frequency modulation, it is different from the simple harmonic model, where the strongest resonance occurs at $\omega_{\rm m} = 2\omega$. We ascribed this phenomenon to

anharmonicity of the Z-trap potential. And we can see that both frequency modulation and position modulation play important roles in parametric excitation.

We also measure the trap loss versus modulation depth and modulation time when we modulate the Z-wire current with $\omega_{\rm m} = \omega = 300$Hz. As shown in Fig.4, the resonance amplitude exponentially increases with modulation depth. From these data, we can fit the relation between residual atom number and modulation depth,

$$N = N_0 \mathrm{e}^{\frac{1}{2}\epsilon\omega t}. \tag{18}$$

So the modulation depth is important for parametric excitation. If it is too small, we cannot observe this phenomenon obviously, and if it is too large, the excitation will become too strong and we cannot find any details.

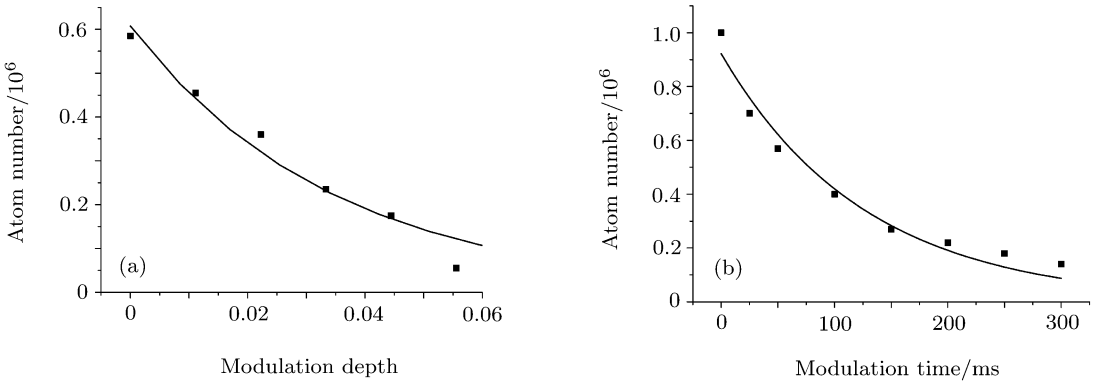


Fig.4. (a) The response of trap loss versus modulation depth when we fix the modulation frequency at 300Hz, and the modulation time is 0.2s. (b) The behaviour of trap loss for different time when the modulation is 300Hz, depth is 5%. We fit these two plots with exponential curves, and find $N = N_0 \mathrm{e}^{\frac{1}{2}\epsilon\omega t}$.

4. Anharmonic trap and parametric cooling

When atoms are trapped in a trap, parametric excitation will heat the atoms.[19,20] But if the trap is anharmonic, there will exist parametric cooling effect.[14,15] Because the Z-trap is an anharmonic trap, atoms in the trap with different energies will have different oscillation periods. The energy selective excitation can lead to a cooling effect for atoms.

An atom with energy E in a trap will have a period of oscillation[17]

$$T(E) = \sqrt{2M} \int_{x_1}^{x_2} \frac{\mathrm{d}x}{\sqrt{E - V_{\rm trap}}}. \tag{19}$$

When the trap is harmonic, $V_{\rm trap} = \dfrac{1}{2} M\omega^2 x^2$, the period is independent of energy. But if the trap is anharmonic, it depends. In our experiment, the trap potential can be approximately described by

$$V_{\rm trap} = \left\{ \left[\frac{\mu_0 I}{4\pi} \frac{L}{z\sqrt{(L/2)^2 + z^2}} - By \right]^2 + \left[\frac{\mu_0 I}{4\pi} \frac{2z}{(L/2)^2 + z^2} \right]^2 \right\}^{1/2}. \tag{20}$$

We calculate numerically the frequency versus energy in Fig.5, it shows that high energy atoms have low oscillation frequencies. If we apply a parametric modulation with the frequency which is a bit less than the resonance frequency, it will excite more high energy atoms. If the excited atoms can escape from the trap, the residual atoms will have lower temperature.

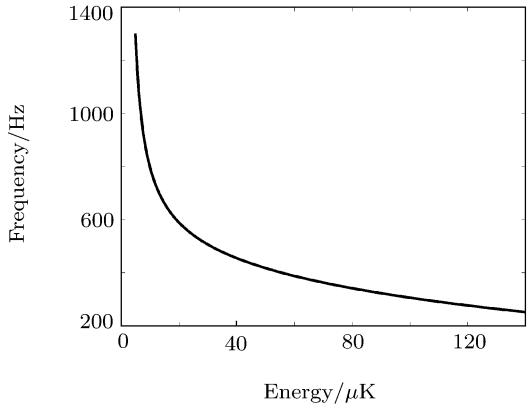


Fig.5. The frequency versus energy about our Z-trap. The Z-wire current is 1.85A, bias field B_y is 0.9mT. We can see that an atom with high energy has low frequency.

First, we parametrically excited the trap by us-

ing the parameters listed in Section 3, and found the resonance of trap loss. But the temperature of atoms remaining in the trap has not dropped, because the trap depth was about 600μK, almost 10 times higher than that of the trapped atoms, it is difficult for atoms to escape from the trap. In order to make it easy for excited atoms to escape the trap, we apply a radio frequency field to lower the trap depth. First, we applied a 5MHz radio frequency cutoff magnetic field, but still, it is hard to say that the temperature drops. We further lowered down the cutoff frequency to 3.5MHz, the corresponding trap depth is about 170μK. At this time, we found that the temperature drops at the modulation frequency a bit lower than resonance frequency. When no parametric excitation is applied, the temperature is 62μK and atom number is 5×10^5. When 250Hz parametric excitation is applied, the temperature became 52μK and atom number is 2×10^5. In both conditions the atom cloud was held for 200ms with a 3.5MHz radio frequency field present.

5. Conclusion and acknowledge

We have modulated both the frequency and the position of Z-trap, and found that a strong resonance occurs at $\omega_{\rm m} = \omega$. Because the inductance of chip wire is almost zero, it is easy to modulate the current of chip wire. This will be widely used in the future by taking the DC current as a special case on atom chip. Due to the anharmonicity of the Z-trap, we found a parametric cooling effect, but the temperature drop is unimpressive. We believe that we will obtain better cooling effect by optimizing the potential and frequency.

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