Vol 17 No 3, March 2008
1674-1056/2008/17(03)/0901-07

Chinese Physics B

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Slice emittance and projected emittance

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(Received 11 January 2007; revised manuscript received 25 September 2007)

This paper presents an analytical description for the growth of the projected emittance (here just referred to the transverse emittance) based on the concept of the slice emittance in an RF photoinjector. In the RF photoinjector, the slice emittance undergoes small changes, but the projected emittance changes significantly even in the drift space after the injector. Carefully adjusting the parameters of the RF photoinjector, which usually means emittance compensation, the projected emittance can be minimized to the value of the slice emittance. The relation between slice emittance and projected emittance is explained in this paper. An emittance function, which shows such a relation, is also introduced. A model about the emittance growth in the RF photoinjector is established, which accords with the particle simulation results by using the code ASTRA. The condition to minimize the emittance is also given by using the emittance function, which means the emittance compensation in the RF photoinjector.

Keywords: photoinjector, emittance, slice emittance, emittance compensation
PACC: 2900, 2915D, 2925B

1. Introduction

For the future x-ray free electron laser (FEL) projects such as TeV-Energy Superconducting Linear Accelerator (TESLA[1]) project and Linac Coherent Light Source(LCLS[2,3]) project, good beam quality is essential for achieving optimal performance. The resulting requirements for beam emittance and energy spread at high peak currents become most stringent for the short-wavelength FELs.[4] The RF photoinjector can produce high quality beams, and it is possible to maintain the quality along the beam line by carefully matching between the components.

In the RF photoinjector, the transverse force acting on the electrons comes from space charge, RF and solenoid focusing fields. The dominant part of these forces is linear, but the forces acting on different slices of the beam are different, resulting in different space phases of slices, so there appears an overlap of slice phase spaces after the electrons have been emitted from the cathode, and the projected emittance is also larger than the slice emittance.

Because the movement of the electrons is governed by the linear part of forces, it is possible to calculate the difference between the slice phase spaces. According to the difference, we can obtain the relation between the slice emittance and the projected emittance, which is

$$\varepsilon = \sqrt{\varepsilon_0^2 + \Delta\varepsilon^2}. \tag{1}$$

where ε_0 is the slice emittance,[5,6] and $\Delta\varepsilon$ is the emittance growth due to the overlap of the slice phase spaces.[7]

The growth of the slice emittance mainly comes from the nonlinear part of forces and the thermal emittance. Carefully adjusting the parameters of the RF photoinjector to make $\Delta\varepsilon = 0$, the projected emittance can reach a minimum value, which is the value of the slice emittance.

2. Movement of electrons

Considering the linear part of forces which are due to space charge, RF field and the solenoid field, the transverse movement of the electrons in the cavity can be expressed as[8]

$$\frac{\mathrm{d}p_r}{\mathrm{d}z} = N(z,\varphi)r, \tag{2}$$

where

$$N(z,\varphi) = \frac{e}{m_0\beta c^2}\left(E_r - \beta B_\theta\right) - \left(\frac{eB_z(z)}{m_0 c^2}\right)^2 \frac{1}{4\beta\gamma} + \frac{I(\zeta)}{2I_0}\frac{1}{R^2}\frac{1}{\beta^2\gamma^2};$$

$$p_r = \beta\gamma r';$$

$$E_r = -\frac{1}{2}\frac{\mathrm{d}E_z(z)}{\mathrm{d}z}\sin(\omega t + \varphi)$$

with $E_z(z)$ being the accelerating field of the cavity and φ being the injecting phase; B_θ =

$-\frac{1}{2}E_z(z)\cos(\omega t+\varphi)$; $B_z(z)$ is the z-component of the solenoid field;

$$I(\xi)=I_{\rm peak}\left(1-\frac{2A^2}{\gamma^2}\left[1+12\left(\frac{\xi}{L}\right)^2+80\left(\frac{\xi}{L}\right)^4\right]\right)$$

is the current for the uniformly distributed beam with $I_{\rm peak}$ being the peak current of the beam, ξ being the longitudinal position of the slice inside the bunch, $A=R/L$, L being the beam length, ξ being the position of the slice inside the bunch, I_0=17kA, R being the beam radius.

Considering different slices, Eq.(2) can be written as

$$\frac{{\rm d}p_r}{{\rm d}z}=N(z,\varphi_0+\Delta)(r_0+\delta), \tag{3}$$

where φ_0 is the injecting phase of the central slice, Δ is the difference in injecting phase between the slices and the central slice, r_0 is the movement position of electron in the central slice, δ is the difference in movement position between the slices and the central slice. The first term of the difference in the force between slices can be expressed as

$$\frac{{\rm d}\beta\gamma r_0'}{{\rm d}z}=N(z,\varphi_0)r_0, \tag{4a}$$

$$\frac{{\rm d}\beta\gamma\delta'}{{\rm d}z}=\frac{\partial N(z,\varphi_0)}{\partial\varphi}\delta r_0. \tag{4b}$$

3. Projected emittance

The movement of the electron is governed by the linear part of forces, therefore Eq.(2) can be expressed in the matrix form

$$\begin{pmatrix} r\\ p_r\end{pmatrix}=\begin{pmatrix} M_{11} & M_{12}\\ M_{21} & M_{22}\end{pmatrix}\begin{pmatrix} r_0\\ p_{r0}\end{pmatrix}. \tag{5}$$

The slice phase space of the bunch can be obtained as follows:[9]

$$\begin{pmatrix}\beta & \alpha\\ \alpha & \gamma\end{pmatrix}=\begin{pmatrix} M_{11} & M_{12}\\ M_{21} & M_{22}\end{pmatrix}*\begin{pmatrix}\beta_0 & 0\\ 0 & \gamma_0\end{pmatrix}*\begin{pmatrix} M_{11} & M_{21}\\ M_{12} & M_{22}\end{pmatrix}, \tag{6}$$

where $\beta_0=R_0^2$ and $\gamma_0=\varepsilon_t^2/\beta_0$, $\varepsilon_t=\sqrt{\beta_0\gamma_0}$ (ε_t is the thermal emittance, R_0 is the initial radius).

Then it follows that

$$\beta=\beta_0M_{11}^2+\gamma_0M_{12}^2, \tag{7a}$$

$$\gamma=\beta_0M_{21}^2+\gamma_0M_{22}^2, \tag{7b}$$

$$\alpha=\beta_0M_{11}*M_{21}+\gamma_0M_{12}*M_{22}. \tag{7c}$$

For the bunch, according to the phase space, they are:

$$\beta=\langle r^2\rangle,\quad \gamma=\langle p_r^2\rangle,\quad \text{and}\quad \alpha=\langle rp_r\rangle. \tag{8}$$

According to the definition of root-mean-square (RMS) emittance, the transverse emittance of the beam can be expressed as

$$\begin{aligned}\varepsilon^2=&\beta_0^2\left(\langle M_{11}^2\rangle*\langle M_{21}^2\rangle-\langle M_{11}*M_{21}\rangle^2\right)\\&+\gamma_0^2\left(\langle M_{22}^2\rangle*\langle M_{12}^2\rangle-\langle M_{22}*M_{12}\rangle^2\right)\\&+\beta_0\gamma_0[\langle M_{11}^2\rangle*\langle M_{22}^2\rangle+\langle M_{12}^2\rangle*\langle M_{21}^2\rangle\\&-2\langle M_{11}*M_{12}*M_{21}*M_{22}\rangle].\end{aligned}$$

From Eq.(5), it follows that $M_{12}\ll M_{11}$ and $M_{22}\ll M_{12}$, so

$$\varepsilon^2\approx\beta_0^2\left(\langle M_{11}^2\rangle*\langle M_{21}^2\rangle-\langle M_{11}*M_{21}\rangle^2\right). \tag{9}$$

We define

$$k_1=\langle M_{11}\rangle\frac{\partial M_{21}}{\partial\varphi}-\langle M_{21}\rangle\frac{\partial M_{11}}{\partial\varphi}. \tag{10}$$

From Eq.(9), the projected emittance can be expressed as

$$\varepsilon\approx\sqrt{R_0^4k_1^2(\omega T)^2+\varepsilon_0^2}, \tag{11}$$

where $T=\sqrt{\langle t_0^2\rangle}$ is the RMS beam pulse width at the cathode, ε_0 is the slice emittance which is larger than the thermal emittance ε_t because of the nonlinear force and other factors.

From Eq.(11) it follows that the projected emittance consists of two parts: one is the slice emittance along the beam line, and the other is the emittance growth from the overlap of the slice phase spaces.

When $k_1=0$, the second term vanishes and the projected emittance achieves a minimum value equal to the slice emittance. The second term is independent of the slice emittance but proportional to the square of the initial beam radius and the beam pulse width at the cathode. The relation between the slice emittance and projected emittance is shown in Eq.(11).

4. Emittance function

k_1 behaves as a chromatic aberration function in the transport line and can be obtained from Eqs.(4a) and (4b) with the initial condition

$$r_0=1,\quad \text{and}\quad \beta\gamma r_0'=0. \tag{12}$$

And $\frac{\partial M_{11}}{\partial t}$ and $\frac{\partial M_{21}}{\partial t}$ can be obtained from the solution of Eq.(4):

$$\frac{{\rm d}\beta\gamma\delta'}{{\rm d}z}=f(z)r_0, \tag{13}$$

where $\delta = \dfrac{\partial M_{11}}{\partial_t}$, $\beta\gamma\delta' = \dfrac{\partial M_{21}}{\partial_t}$, and $f(z) = \left.\dfrac{\partial N(z,\varphi)}{\partial\varphi}\right|_{\varphi=\varphi_0}$ which means the difference in phase space between the slices.

From Eqs.(10), (12), (12) and (13), we can obtain k_1 as follows:

$$\frac{\mathrm{d}k_1}{\mathrm{d}z} = \left\langle M_{11}^2 \right\rangle \left(f(z) - N(z,\varphi_0) \int \frac{\mathrm{d}\xi}{\beta\gamma} \int f(\zeta)\,\mathrm{d}\zeta \right). \tag{14}$$

where M_{11} is related to beam radius R by

$$R^2 = \langle M_{11} \rangle^2 R_0^2. \tag{15}$$

Defining $\eta = k_1 R_0$, which is called the emittance function in the present paper, then Eq.(14) can be written as

$$\frac{\mathrm{d}\eta}{\mathrm{d}z} = \left(f(z) - N(z,\varphi_0) \int \frac{\mathrm{d}\xi}{\beta\gamma} \int f(\zeta)\,\mathrm{d}\zeta \right) R^2. \tag{16}$$

And from Eq.(11), the projected emittance can be expressed as

$$\varepsilon \approx \sqrt{\eta^2 (\omega T)^2 + \varepsilon_0^2}. \tag{17}$$

From Eqs.(16) and (17), it can be concluded that the emittance function η, which means an overlap of the slice phase spaces, shows the relation between the slice emittance and projected emittance. When $\eta = 0$, the emittance growth from the overlap of the slice phase spaces disappears, and the projected emittance reaches a minimum value.

From Eq.(16) it can be found that even when $f(z) = 0$, the emittance function is not zero ($\eta \neq 0$), but it keeps constant. Only when $f(z) \neq 0$, will η change. And the variation of emittance function η is proportional to the square of the beam radius. If the beam radius is small, the emittance changes a little, but for a large beam radius, the emittance changes significantly.

For the case in which the electron energy is not high, effects of space charges on the different slices are different. Because $f(z) \neq 0$, η changes even in the drift space, and the projected emittance also changes. The space charge effect cannot only make the emittance rise, but also cause emittance to decrease. After the booster, the space charge effects should be very low, and the differences in phase space between slices are almost zero with $f(z) = 0$. Then η keeps constant in the drift space after a booster, and the emittance also keeps constant. And if the emittance after this booster is not optimized, it can be optimized at the next acceleration cell when $f(z) \neq 0$ in order to make $\eta = 0$ and realize the emittance compensation.

5. Analysis of emittance function

The projected emittance of the RF gun connected with the booster can be obtained from Eqs.(16) and (17). The following discussion is based on the electron movement from the cathode to the end of the booster. The movement is divided into three stages: inside the RF gun, the drift space behind the gun, and inside the booster. For each stage, the emittance function and the corresponding projected emittance are discussed under the optimized condition for emittance compensation.

5.1. In the RF gun

In the RF gun, the electrons are emitted from the surface of the cathode and immediately accelerated to a speed close to the relativistic speed. Near the surface of the cathode, the speed of the electrons changes significantly. The transverse force acting on the electrons is mainly due to the space charge effect, and the beam radius increases. Far from the cathode, the space charge effect drops significantly, and the beam radius undergoes a slow change.

In order to simplify the analysis, the beam radius change is taken into account only in the place close to the surface of the cathode, and it can be expressed as

$$R \approx \left[1 + K \frac{Q}{R_0^2 T} \right]^2 R_0, \tag{18}$$

where Q is the beam charge, and K is a function of the gradient at the cathode.

The z-component distribution of the RF field in the cavity is generally expressed as[10]

$$E_z = E_0 \sum_{n=1}^{\infty} a_n \cos(nkz). \tag{19}$$

where $k = 2\pi/\lambda = \omega/c$.

Considering the fundamental harmonic, $N(z,\varphi)$ can be given by

$$N(z,\varphi) = \alpha k^2 \cos(2kz + \varphi) - \left(\frac{eB_z(z)}{m_0 c^2} \right)^2 \frac{1}{4\beta\gamma} + \frac{I(\zeta)}{2I_0} \frac{1}{R^2} \frac{1}{\beta^2\gamma^2}, \tag{20}$$

where $\alpha = \dfrac{eE_0}{2m_0 c^2 k}$.

So $N(z,\varphi_0)$ and $f(z)$ can be obtained respectively as

$$N(z,\varphi_0) = \alpha k^2\cos(2kz+\varphi_0) - \left(\frac{eB_z(z)}{m_0c^2}\right)^2\frac{1}{4\beta\gamma} + \frac{I_{\rm peak}}{2I_0}\frac{1}{R^2}\frac{1}{\beta^2\gamma^2}, \tag{21}$$

$$f(z) \approx \alpha k^2\sin(2kz+\varphi_0). \tag{22}$$

If the electron energy is not very high, by using the approximation $\gamma \approx 1+2\alpha kz\sin(\varphi_0)$, then Eq.(16) becomes

$$\frac{{\rm d}\eta}{{\rm d}z} = \left(\alpha k^2\sin(2kz+\varphi_0) - \frac{1}{2}\left(\left(\frac{eB_z(z)}{m_0c^2}\right)^2\frac{1}{4\beta\gamma} - \frac{I_{\rm peak}}{2I_0}\frac{1}{R^2}\frac{1}{\beta^2\gamma^2}\right)\ln z\right)R^2. \tag{23}$$

It can be seen that the oscillation of the emittance in the RF gun results from the transverse force of RF field. Neglecting the change of the radius in the injector yields

$$\eta/R^2 \approx \alpha k(1-\ln z)[\cos(\varphi_0)-\cos(2kz+\varphi_0)] + \frac{1}{2}\left(\frac{I_{\rm peak}}{2I_0}\frac{1}{R^2}\int_0^z\frac{\ln z{\rm d}z}{\beta^2\gamma^2} - \int_0^z\left(\frac{eB_z(z)}{m_0c^2}\right)^2\frac{\ln z{\rm d}z}{4\beta\gamma}\right). \tag{24}$$

From Eq.(24), it is easy to see how the solenoid field compensates the projected emittance outside the gun. In a certain range, the solenoid field will reduce the projected emittance. But if it is too high, it will also increase the projected emittance just as the space charge effect does.

5.2. Drift space after the gun

In the drift space behind the gun, only the space charges exert an influence on the electrons, so $N(z,\varphi)$ is expressed as

$$N(z,\varphi) = \frac{I(\zeta)}{2I_0}\frac{1}{R^2}\frac{1}{\gamma^2}. \tag{25}$$

Because of energy spread, the space charge effects on the different slices are different. With only the first term taken into consideration, $N(z,\varphi_0)$ and $f(z)$ can be expressed respectively as

$$N(z,\varphi_0) = \frac{I_{\rm peak}}{2I_0}\frac{1}{R^2}\frac{1}{\gamma^2}, \tag{26}$$

$$f(z) = -\frac{I_{\rm peak}}{I_0}\frac{1}{R^2}\frac{1}{\gamma^3}\frac{\partial\gamma}{\partial\varphi}, \tag{27}$$

So

$$\frac{{\rm d}\eta}{{\rm d}z} = -\frac{I_{\rm peak}}{I_0\gamma^3}\left(1-\frac{z^2}{2}\left(\frac{I_{\rm peak}}{2I_0}\frac{1}{R^2\gamma^2}\right)\right)\frac{\partial\gamma}{\partial\varphi}. \tag{28}$$

Ignoring the second term in Eq.(28) gives

$$\frac{{\rm d}\eta}{{\rm d}z} \approx -\frac{I_{\rm peak}}{I_0\gamma^3}\frac{\partial\gamma}{\partial\varphi}. \tag{29}$$

Ignoring the change of energy spread in the drift space yields

$$\eta \approx \eta_1 - \frac{I_{\rm peak}}{I_0\gamma^3}\frac{\partial\gamma}{\partial\varphi}z. \tag{30}$$

where η_1 is the value of the emittance function at the exit of gun.

According to Eq.(18), the projected emittance behind the gun can be expressed as

$$\varepsilon \approx \sqrt{\left(\eta_1 - \frac{I_{\rm peak}\Delta\gamma}{I_0\gamma^3}z\right)^2+\varepsilon_0^2}. \tag{31}$$

where $\Delta\gamma$ is the energy spread.

Usually $\dfrac{\partial\gamma}{\partial\varphi} > 0$, so that the emittance will decrease until it reaches a minimum value (i.e. slice emittance), and after that it will increase when $\eta_1 > 0$. When $\eta_1 \le 0$, it will always increase till electrons arrive at the booster.

As for the Gaussian distribution beam, the situation will be much more complicated, and the second term must be considered.

5.3. In the booster

The z-component of RF field in the booster can be expressed as

$$E_z(z,t) = E_0\cos(kz)\sin(\omega t+\varphi). \tag{32}$$

When the electron energy becomes high, the space charge effect will decrease. Ignoring the space charge effect, according to the RF field in the RF cavity, $N(z,\varphi)$ can be given by

$$N(z,\varphi) = \alpha_1 k^2\sin(2kz+\varphi), \tag{33}$$

where $\alpha_1 = \dfrac{eE_1}{2m_0c^2k}$ with E_1 being the gradient in the booster. So $N(z,\varphi_0)$ and $f(z)$ can be written respectively as

$$N(z,\varphi_0) = \alpha_1 k^2\sin(2kz+\varphi_0), \tag{34}$$

$$f(z) = \alpha_1 k^2\cos(2kz+\varphi_0). \tag{35}$$

If the electron energy is high, the second term in Eq.(17) will be very small relatively. And then we can have

$$\frac{\mathrm{d}\eta}{\mathrm{d}z} \approx f(z)\,R^2 \approx \alpha_1 k^2 \cos(2kz+\varphi_0)R^2. \tag{36}$$

This equation indicates that for a small beam radius, the emittance in the booster undergoes a slow change. For a large beam radius it changes significantly in the booster.

Because of the focusing effects of the RF field, there usually exists a small change of the beam radius in the booster. Ignoring this, Eq.(36) becomes

$$\eta \approx \eta_2 + \alpha_1 k\left[\sin(2kz+\varphi_0) - \sin(\varphi_0)\right]R^2, \tag{37}$$

where η_2 is the value of the emittance function in front of the booster.

From Eq.(38), the value of the emittance function behind the booster is obtained to be

$$\eta \approx \eta_2 - 2\alpha_1 k \sin(\varphi_0)R^2. \tag{38}$$

From this equation, it is easy to make the value of the emittance function equal to zero after the booster and to realize the emittance compensation (when the projected emittance is equal to the slice emittance).

The injecting phase for the booster is

$$-\frac{\pi}{2} \le \varphi_0 \le \frac{\pi}{2}. \tag{39}$$

When $\eta_2 = 0$, then injecting phase is $\varphi_0 = 0$, which is for the maximum gain of electron energy in the booster. When $\eta_2 > 0$, the injecting phase is $\varphi_0 > 0$. When $\eta_2 < 0$, the injecting phase is $\varphi_0 < 0$.

If the emittance compensation is not realized after the booster, it can be realized in the next accelerating module by adjusting the injecting phase according to Eq.(38).

6. Simulation based on the configuration of PITZ

Based on the configuration of PITZ (Photoinjector Test Facility in DESY Zeuthen), calculation and simulation are carried out under the following condition: the bunch charge of 1nC, flat time of 10ps, rising time of 2ps, a beam radius of 0.8mm on the cathode, gradient of 40MV/m at the cathode, and injecting phase of 40 degree. The booster is a 9-cell cavity placed 1.0m away from the cathode and its average accelerating field is 20MV/m.

Figure 1 displays the emittance function versus beam position in an unmatched case when the peak field of the solenoid magnet for emittance compensation is 0.1T. Figure 2 displays the corresponding projected emittance versus beam position based on Eqs.(16), (17) and (18).

The simulated results by using ASTRA(where ASTRA stands for A Space Charge Tracking Algorithm, which is mainly developed for the photoinjector in DESY Hamburg) are shown in Figs.3 and 4, which shows the projected emittance (transverse emittance) versus the beam position and the slice emittance versus the beam position. Comparing Fig.2 with Fig.3, one can see a small difference between them, which is because the longitudinal movement and the interaction between slices are not included in the calculation.

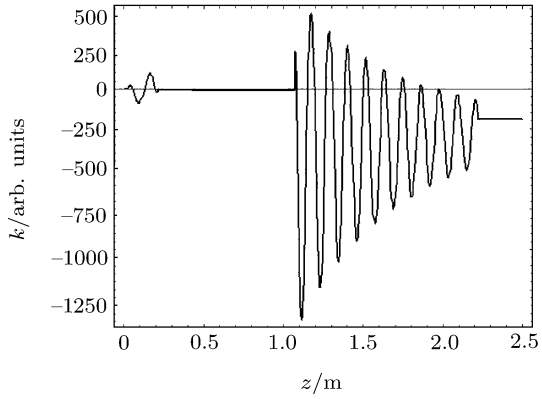


Fig.1. Emittance function k versus beam position in an unmatched case.

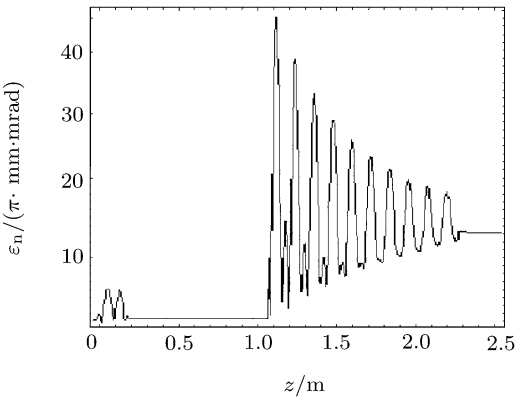


Fig.2. Calculated emittance versus beam position in an unmatched case.

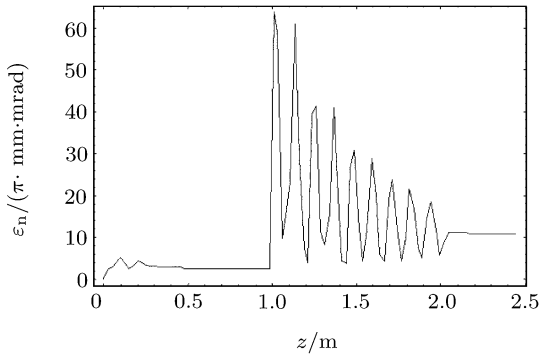


Fig.3. Simulated emittance versus beam position in an unmatched case.

In Fig.4, the slice emittance seems to change significantly just as the projected emittance does, which is because that the slice emittance determined by using ASTRA is not that of the 'real' slice with a zero pulse width. Such a slice emittance is just the projected emittance of one-tenth width of the whole bunch. The contribution from the overlap of the phase space between 'real' slices is ten times smaller than that from the whole bunch. According to Eq.(17), when the emittance changes significantly, such a contribution makes the slice emittance determined by using the ASTRA quite different from the 'real' slice emittance.

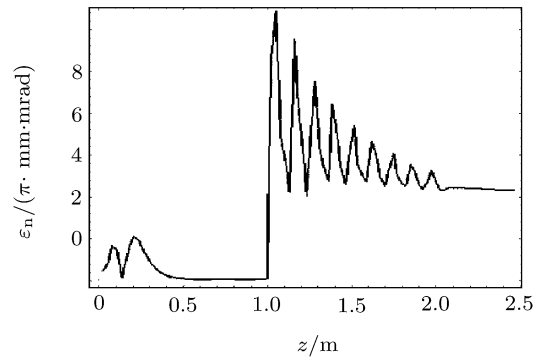


Fig.4. Simulated slice emittance versus beam position in an unmatched case.

Figure 5 shows the emittance function versus position under the optimized condition when the solenoid field is 0.175T. It can be seen that the emittance function is almost zero after the booster, so under such a condition the emittance after the booster should be close to the slice emittance and reaches a minimum value (this is verified by the simulation, see Fig.6). Figure 6 shows the emittance simulated on ASTRA. Figure 7 shows the simulated slice emittance. In Fig.6, the emittance changes insignificantly after the injector. According to Eq.(17), the contribution from the overlap of the phase space between 'real' slices in the slice is very small in this case, and the slice emittance is also very close to the 'real' slice emittance. So the slice emittance after the injector in Fig.7 can be considered as the 'real' slice emittance. The simulated emittance in Fig.6 after the booster reaches a minimum value of 1.28π·mm·mrad. It is close to the slice emittance after the injector in Fig.6, which is 1.10π·mm·mrad. This means that the emittance function after the booster is zero, just as shown in the Fig.5.

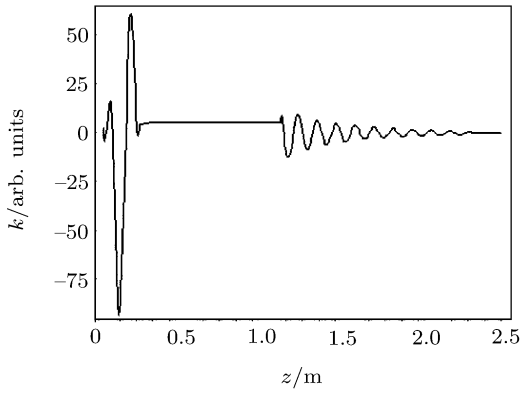


Fig.5. Emittance function k versus beam position in a matched case.

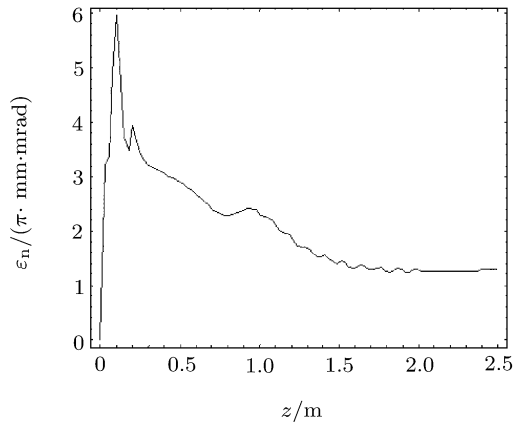


Fig.6. Simulated emittance versus beam position in a matched case.

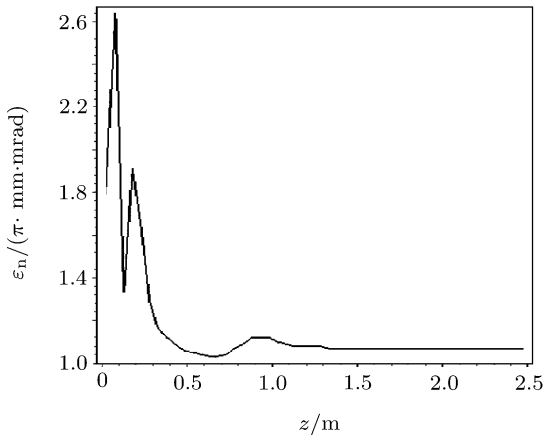


Fig.7. Simulated slice emittance versus beam position in a matched case.

7. Conclusion

In the RF photoinjector, the slice emittance is very important, and it undergoes a much slower change than the projected emittance along the beam line. The careful matching of the configuration of the injector (especially matching before the booster) can make the projected emittance[11] very close to the slice emittance and maintain a constant value after the booster. According to the emittance function, there are many options to lower the emittance, but if other parameters (such as energy spread and longitudinal emittance) are included, the optional ranges will decrease.

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