Vol 17 No 3, March 2008
1674-1056/2008/17(03)/0847-05
Chinese Physics B
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Chaotic attractor transforming control of hybrid Lorenz–Chen system*

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(Received 5 April 2007; revised manuscript received 17 August 2007)

Based on passive theory, this paper studies a hybrid chaotic dynamical system from the mathematics perspective to implement the control of system stabilization. According to the Jacobian matrix of the nonlinear system, the stabilization control region is gotten. The controller is designed to stabilize fast the minimum phase Lorenz–Chen chaotic system after equivalently transforming from chaotic system to passive system. The simulation results show that the system not only can be controlled at the different equilibria, but also can be transformed between the different chaotic attractors.

Keywords: hybrid Lorenz–Chen system, chaotic attractor, transforming control
PACC: 0545

1. Introduction

Chaos has been applied in many fields such as physics, chemistry, economics and so on. The sensitivity of chaotic system outputs to state initial values, together with the complexity and singularity of chaotic systems and phenomena, makes the study of chaotic control theory challenging.[1] In the early 1990s, American physicists Ott, Grebogi and Yorke proposed a control method using inherent chaotic characters.[2] Afterward, some scholars developed this control method and proved its validity with many experiments, which resulted in an upsurge in chaotic control and its applications.[3−6] However, these chaotic control methods were proposed with no regard to the inherent characters of chaotic dynamical systems. They therefore ignored controlling and understanding chaos from the mathematics perspective, as well as including no stable analyses of controlled systems.

Passive systems show a kind of characteristics of dissipative network.[7,8] Using the passive network theory, we can analyse the dynamical system characters of nonlinear system, such as stabilization, dynamic characteristics. This paper mainly researches into hybrid Lorenz–Chen chaotic dynamical system from the mathematics perspective to implement the control of system stabilization. In the stabilization control region, the chaotic system can be stabilized by the state feedback.

2. Passive system

Consider a continuous chaotic system given by

$$\begin{aligned} \dot{x} &= f(x) + g(x)u, \\ y &= h(x), \end{aligned} \tag{1}$$

where $x \in R^n$ is the state variable, $u \in R^m$ is the input, $y \in R^p$ is the output, f and g are smooth vector fields. Function h is smooth mapping. It is supposed that the vector field f has at least one equilibrium. Without loss of generality, it can be assumed that the equilibrium is $x = 0$. If the equilibrium is not $x = 0$, through coordinate shift we can transform the equilibrium to $x = 0$.

A system (1) is said to be passive if function $f(x)$ and $g(x)$ exist and there is a real-valued constant β, such that for $\forall t \geq 0$[6]

$$\int_0^t u^{\mathrm{T}}(\tau) y(\tau) \mathrm{d}\tau \geq \beta, \tag{2}$$

or if there is a constant $\rho > 0$ and real-valued constant

*Project supported by the National Natural Science Foundation of China (Grant No 60702023) and Natural Science Foundation of Zhejiang Province, China (Grant No Y104414).
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β, such that for $\forall t \geq 0$

$$\int_0^t u^{\mathrm{T}}(\tau)y(\tau)\mathrm{d}\tau + \beta \geq \int_0^t \rho y^{\mathrm{T}}(\tau)y(\tau)\mathrm{d}\tau. \quad (3)$$

In other words, a system (1) is said to be passive if there is a non-negative function $V(x)$: $X \to R, V(0) = 0$, named storage function, such that for $\forall x \in X$, $\forall t \geq 0$

$$V(x) - V(x_0) \leq \int_0^t y^{\mathrm{T}}(s)u(s)\mathrm{d}s. \quad (4)$$

A passive system with storage function $V(x)$ is said to be strictly passive if there exists a positive definite function $S(x)$, such that for $\forall x \in X$, $\forall t \geq 0$

$$V(x) - V(x_0) = \int_0^t y^{\mathrm{T}}(s)u(s)\mathrm{d}s - \int_0^t S(x(s))\mathrm{d}s. \quad (5)$$

Theorem 1[8] Suppose system is passive and locally zero-state detectable with positive definite storage function $V(x)$. Let φ be any smooth function such that $y^{\mathrm{T}}\varphi(y) > 0$ and $\varphi(0) = 0$ for each nonzero y. The control law

$$u(t) = -\varphi(y) \quad (6)$$

asymptotically stabilizes at the equilibrium point $x = 0$.

Non-passive chaotic systems can be equivalent to passive systems through designing system controllers, and then can be stabilized at the equilibrium point $x = 0$.

3. The transforming control of Lorenz–Chen chaotic attractors

In 1963, Lorenz firstly found a chaotic attractor in a simple three-dimensional autonomous system.[9] Then Chen and Ueta found a similar but non-equivalent chaotic attractor,[10] which is known to be the dual of the Lorenz system. Recently the Lü system is reported which bridges the gap between the Lorenz system and the Chen system.[11] In a recent work of Lü *et al*,[12,13] they introduced a unified chaotic system which contains the Lorenz and the Chen systems as two dual systems at the two extremes of its parameter spectrum. The new system represents the continued transition from the Lorenz to Chen system and is chaotic over the entire spectrum of the key system parameter. Via this unified chaotic system, the transforming control of different chaotic attractors becomes easy.

The differential equations of Lorenz–Chen chaotic dynamical system are as follows:[12]

$$\begin{aligned} \dot{x}_1 &= (25\theta + 10)(x_2 - x_1), \\ \dot{x}_2 &= (28 - 35\theta)x_1 + (29\theta - 1)x_2 - x_1x_3, \\ \dot{x}_3 &= x_1x_2 - \frac{8+\theta}{3}x_3, \end{aligned} \quad (7)$$

where $\theta \in [0, 1]$. Obviously, the system is Lorenz system when $\theta = 0$, and it is Chen system when $\theta = 1$. It is a continuous chaotic system when θ varies from 0 to 1.

The equilibria of the Lorenz–Chen chaotic system are (0,0,0), $(\sqrt{72 - 7\theta - 2\theta^2}, \sqrt{72 - 7\theta - 2\theta^2}, 27 - 6\theta)$ and $(-\sqrt{72 - 7\theta - 2\theta^2}, -\sqrt{72 - 7\theta - 2\theta^2}, 27 - 6\theta)$. The equilibria are different when θ varies from 0 to 1.

Theorem 2 For the system (7), suppose $x = 0$ is an equilibrium point of this system, and $\theta \in [0, 1]$. The system (7) can be equivalent to a passive system using state feedback, and therefore can be globally asymptotically stabilized.

Proof Add the controller u to the second equation of system (7). Suppose that state vector x_2 is the system output and suppose $z_1 = x_1$, $z_2 = x_3$, $y = x_2$, then the system can be expressed in normal form

$$\begin{aligned} \dot{z}_1 &= -(25\theta + 10)z_1 + (25\theta + 10)y, \\ \dot{z}_2 &= -\frac{8+\theta}{3}z_2 + z_1y, \\ \dot{y} &= (28 - 35\theta)z_1 + (29\theta - 1)y - z_1z_2 + u. \end{aligned} \quad (8)$$

We have

$$\begin{aligned} \dot{z} &= f_0(z) + p(z, y)y, \\ \dot{y} &= b(z, y) + a(z, y)u, \end{aligned} \quad (9)$$

where

$$f_0(z) = \left[-(25\theta + 10)z_1 \ -\frac{8+\theta}{3}z_2\right]^{\mathrm{T}},$$

$$p(z, y) = \left[25\theta + 10\, z_1\right]^T,$$

$a(z, y) = 1$, $b(z, y) = (28 - 35\theta)z_1 + (29\theta - 1)y - z_1z_2$.

Choose a storage function

$$V(z, y) = W(z) + \frac{1}{2}y^2, \quad (10)$$

where $W(z)$ is the Lyapunov function of $f_0(z)$, and $W(0) = 0$.

$$W(z) = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2, \quad (11)$$

According to Eq.(11), we have

$$\dot{W} = \frac{\mathrm{d}W}{\mathrm{d}t} = z_1\dot{z}_1 + z_2\dot{z}_2 = [z_1\ z_2]\begin{bmatrix} -(25\theta+10)z_1 \\ -\dfrac{8+\theta}{3}z_2 \end{bmatrix}. \quad (12)$$

Because $\dfrac{\mathrm{d}W}{\mathrm{d}t} \le 0$, it can be concluded that $W(z)$ is the Lyapunov function of $f_0(z)$ and that $f_0(z)$ is globally asymptotically stable.

According to the above analyses, zero dynamical character of hybrid chaotic dynamical system is Lyapunov stable and it is a minimum phase system.

We choase

$$u = a^{-1}(z,y)\Big[-b^{\mathrm{T}}(z,y) - \frac{\partial}{\partial z}W(z)p(z,y) - \alpha y + v\Big], \quad (13)$$

where α is a positive real constant, v is the external input signal. That is,

$$u = (1-29\theta-\alpha)y + v + (10\theta-38)z_1, \quad (14)$$

Enter Eq.(14) into Eq.(7),

$$\begin{aligned} \dot{x}_1 &= (25\theta+10)(x_2-x_1), \\ \dot{x}_2 &= (-10-25\theta)x_1 - \alpha x_2 - x_1x_3 + v, \\ \dot{x}_3 &= x_1x_2 - \frac{8+\theta}{3}x_3. \end{aligned} \quad (15)$$

From Eq.(10)

$$\dot{V} = \frac{\partial}{\partial z}W(z)\cdot f_0(z) + \frac{\partial}{\partial z}W(z)\cdot p(z,y)\cdot y + [b(z,y)+a(z,y)u]y. \quad (16)$$

Because the equivalent system is minimum phase and has KYP property, we have $\dfrac{\partial}{\partial z}W(z)f_0(z) \le 0$, and then

$$\frac{\mathrm{d}V}{\mathrm{d}t} \le \frac{\partial}{\partial z}W(z)\cdot p(z,y)\cdot y + [b(z,y)+a(z,y)u]y. \quad (17)$$

Enter Eq.(13) into Eq.(17),

$$\frac{\mathrm{d}V}{\mathrm{d}t} \le -\alpha y^2 + vy. \quad (18)$$

Let $\beta = V(z_0,y_0)$, we have

$$V(z,y) + \int_0^t \alpha y^2(s)\mathrm{d}s \le \int_0^t v(s)y(s)\mathrm{d}s + \beta, \quad (19)$$

Because storage function $V(z,y)$ is positive definite, $V(z,y) \ge 0$, and thus we have

$$\int_0^t \alpha y^2(s)\mathrm{d}s \le \int_0^t v(s)y(s)\mathrm{d}s + \beta. \quad (20)$$

So the equivalent system is a passive system which follows the definition of passive system.

4. The estimation of stabilization control region

According to the Jacobian matrix of nonlinear system, the stabilization control region can be estimated. That is, the choosing value of the parameters α and v in controller can be obtained via the Jacobian matrix.

The Jacobian matrix of system (15) at (0,0,0) is

$$J = \begin{pmatrix} -25\theta-10 & 25\theta+10 & 0 \\ -10-25\theta-x_3 & -\alpha & -x_1 \\ x_2 & x_1 & -\dfrac{8+\theta}{3} \end{pmatrix}_{(0,0,0)} = \begin{pmatrix} -25\theta-10 & 25\theta+10 & 0 \\ -10-25\theta & -\alpha & 0 \\ 0 & 0 & -\dfrac{8+\theta}{3} \end{pmatrix}$$

The sufficiency condition which the system is of asymptotically stabilization at origin is that all of the eigenvalues of Jacobian matrix have negative real parts. According to Jacobian matrix J, we have $\alpha > -(25\theta+10)$.

During the design of controller, it has been defined that $\alpha > 0$. Considering $\alpha > -(25\theta+10)$, we find that the stabilization control region is $\alpha > 0$.

5. Simulation results

Suppose the initial point of hybrid chaotic system is $(1,-1,1)$, and then the controller $u(t)$ is

$$u = (1-29\theta-\alpha)x_2 + v + (10\theta-38)x_1.$$

Enter $v = 0$ into the controller and choose the equilibrium point (0,0,0), and let $\theta = 0.3$. Figure 1 plots the curves on the results that Lorenz–Chen chaotic system is driven from $(1,-1,1)$ to (0,0,0) when $\alpha = 0.02$, $\alpha = 20$ and $\theta = 0.3$.

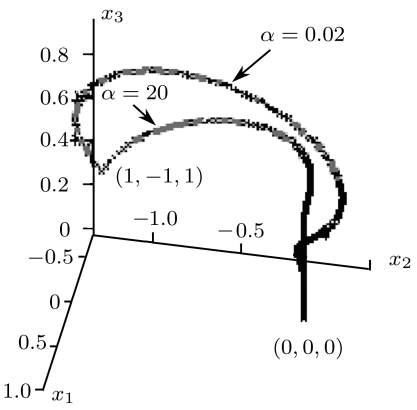


Fig.1. The Lorenz–Chen chaotic system is driven from $(1, -1, 1)$ to (0,0,0).

We will study how to design the controller to stabilize the hybrid chaotic system at any equilibrium (x_1^*, x_2^*, x_3^*) which is of nontrivial equilibrium.

Let $\dot{x} = 0$ and substituting (x_1^*, x_2^*, x_3^*) into (15), we have $x_1^* = x_2^*$, $x_3^* = \dfrac{3}{8+\theta} x_1^* x_2^*$, $v = (10 + 25\theta)x_1^* + \alpha x_2^* + x_1^* x_3^*$.

Choose the new equilibrium point (2,2,1.5) and put $v = 23 + 2\alpha$ into Eq.(15) when $\theta = 0$. Suppose the initial point of hybrid chaotic system is $(1, -1, 1)$. Figure 2 plots the curves on the results that Lorenz–Chen chaotic system is driven from $(1, -1, 1)$ to (2,2,1.5) and then to (0,0,0) when $\alpha = 20$ and $\theta = 0$.

From the above figures, we can find that it takes only very short time for the system to be stabilized at equilibria (0,0,0) and (2,2,1.5). Therefore we can stabilize the hybrid chaotic system at any equilibrium (x_1^*, x_2^*, x_3^*) through varying external input v.

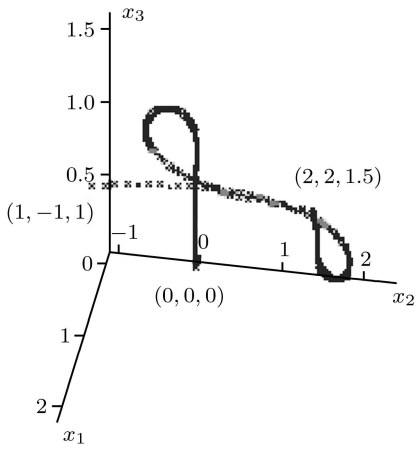


Fig.2. The Lorenz–Chen chaotic system is driven from $(1, -1, 1)$ to (2,2,1.5) and then to (0,0,0).

Figure 3 plots the curves on the transforming control results of different chaotic attractors. In those figures, the initial point is $(1, -1, 1)$, and the system is of chaotic attractor where factor $\theta = 0.3$, then the chaotic attractor is switched to equilibrium (0,0,0) when $t = 20\,\mathrm{s}$. The system is switched to the Chen attractor when $t = 40\,\mathrm{s}$. Finally, the Chen chaotic attractor is stabilized at equilibrium (10,10,100/3) when $t = 60\,\mathrm{s}$. Through varying equilibrium, the transforming control of different chaotic attractors can be realized. We can transform the Lorenz chaotic attractor to the Lü chaotic attractor through varying factor θ, then to the Chen chaotic attractor, and so on.

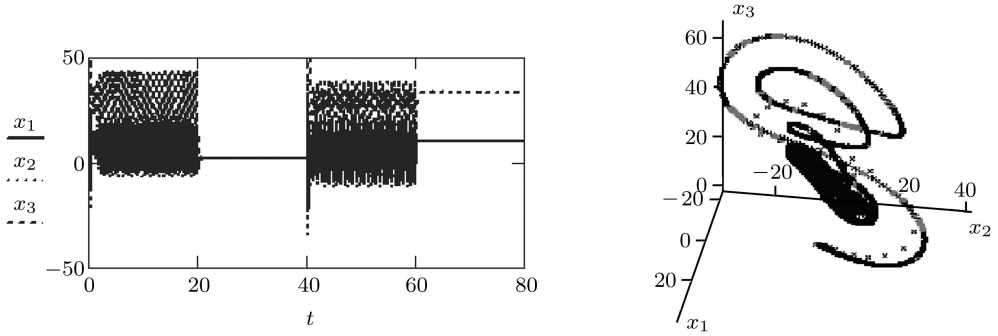


Fig.3. The transforming control results between different chaotic attractors.

6. Conclusion

Passive control theory can be used to control one kind of hybrid chaotic dynamical system. This control method combines chaotic systems with passive system. By using the passive system characteristic, the minimum phase nonlinear system transformed by Lorenz–Chen chaotic system can be globally asymp-

totically stabilized at the desired point by state feedback and therefore implement the control of closed-loop system stabilization. The different chaotic attractors can be transformed by passive controller between different chaotic systems.

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