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# Teleportation of an arbitrary mixture of diagonal states of multiqudit*

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This paper proposes a scheme to teleport an arbitrary mixture of diagonal states of multiqutrit via classical correlation and classical communication. To teleport an arbitrary mixture of diagonal states of $N$ qutrits, $N$ classically correlated pairs of two qutrits are used as channel. The sender (Alice) makes Fourier transform and conditional gate (i.e., $\mathrm{XOR}^{(3)}$ gate) on her qutrits and does measurement in appropriate computation bases. Then she sends $N$ ctrits to the receiver (Bob). Based on the received information, Bob performs the corresponding unitary transformation on his qutrits and obtains the teleported state. Teleportation of an arbitrary mixture of diagonal states of multiqudit is also discussed.

**Keywords:** teleportation, classical correlation, Fourier transform, conditional gate
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## 1. Introduction

Teleportation, one of the most important branches of quantum information processing, is originally proposed by Bennett *et al*[1] and is experimentally realized by Bouwmeester *et al.*[2] Teleportation is a process that the sender (Alice) transmits an unknown quantum state to a remote receiver (Bob) via a quantum channel with the help of some classical communications without transmission of the carrier. Recently, more and more attentions have been paid to implement teleportation both in theory[3−10] and experiment.[11−15]

In the standard teleportation scheme,[1] an entangled pair is needed for teleporting an unknown quantum state. Recently, there has been interest in the action of classical correlation in quantum information processing. Reference [16] used classically correlated pair to distribute entanglement. Reference [17] showed that a classically correlated channel is sufficient to teleport any set of commuting qubits, and Ref.[18] proposed a protocol for teleporting an arbitrary mixture of diagonal states of multiqubit via a classically correlated channel.

The conventional building blocks for quantum processors are qubits, i.e., the two-level physical systems. To carry more information, one requires more particles. However, as the number of involved particles increases, the entanglement among particles decreases sharply due to the coupling between system and environment. In order to overcome this problem, we can use particle in high-dimensional system, i.e., qudit. There are several reasons for that. Firstly, a qudit can carry more information than a qubit, so it can increase information flux in quantum communication. Secondly, a high-dimensional system has many advantages in fault-tolerant quantum computation and quantum error-correction.[19] Moreover, the high-dimensional system can increase security in quantum cryptography.[20] Therefore, some proposals have been put forward to implement quantum information processing in high-dimensional Hilbert space.[1,21,22]

Here, we generalize the scheme of Ref.[18] to the high-dimensional case and present a protocol to teleport an arbitrary mixture of diagonal states of multiqutrit via classical correlation and classical communication. Similar to Ref.[18], Alice makes some unitary transformation and measurement on her qutrits. Based on Alice's measurement result, Bob performs the corresponding unitary operation to make his qutrits being in the teleported state. In addition, teleportation of an arbitrary mixture of diagonal states of

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multiqudit is also discussed.

## 2. Teleportation of a mixture of diagonal states of one qutrit

We assume that an arbitrary mixture of diagonal states of one qutrit to be teleported is in the following form:

$$\rho_X(1) = a_0 \left|0\right\rangle_X \left\langle 0\right| + a_1 \left|1\right\rangle_X \left\langle 1\right| + a_2 \left|2\right\rangle_X \left\langle 2\right|, \quad (1)$$

where $a_0$, $a_1$, and $a_2$ are all real and positive, and satisfy $a_0 + a_1 + a_2 = 1$. A classically correlated pair of two qutrits shared by Alice and Bob can be written as

$$\rho_{AB} = \frac{1}{3}(\left|00\right\rangle_{AB} \left\langle 00\right| + \left|11\right\rangle_{AB} \left\langle 11\right| + \left|22\right\rangle_{AB} \left\langle 22\right|), \quad (2)$$

where qutrits $A$ and $B$ belong to Alice and Bob, respectively. Thus, the initial state of the whole system is

$$\begin{aligned} &\rho_{\rm ini}(1) \\ &= \rho_X(1) \otimes \rho_{AB} \\ &= \frac{1}{3}(a_0 \left|0\right\rangle_X \left\langle 0\right| + a_1 \left|1\right\rangle_X \left\langle 1\right| + a_2 \left|2\right\rangle_X \left\langle 2\right|) \\ &\quad \otimes(\left|00\right\rangle_{AB} \left\langle 00\right| + \left|11\right\rangle_{AB} \left\langle 11\right| + \left|22\right\rangle_{AB} \left\langle 22\right|). \end{aligned} \quad (3)$$

The process of teleportation can be completed by two stages. In the first stage, Alice performs an operation as follows:

$$O(1) = F^A \cdot {\rm XOR}^{(3)}_{AX}, \quad (4)$$

where $F^A$ is a Fourier transform on qutrit $A$ and ${\rm XOR}^{(3)}_{AX}$ is a conditional gate between two qutrits $A$ and $X$. $F^A$ and ${\rm XOR}^{(3)}_{AX}$ are unitary operations defined by

$$F\left|j\right\rangle = \frac{1}{\sqrt{3}} \sum_{l=0}^{2} \exp[2{\rm i}\pi lj/3] \left|l\right\rangle \quad (5)$$

and

$${\rm XOR}^{(3)}_{AX} \left|m\right\rangle_A \left|n\right\rangle_X = \left|m\right\rangle_A \left|m \ominus n\right\rangle_X, \quad (6)$$

respectively, where the '$\ominus$' operation indicates difference $(m - n)$ modulo 3. After above operations, the state of the system is transformed into

$$\begin{aligned} &\rho_1(1) \\ &= \frac{1}{9}[(\left|00\right\rangle_{XA} \left\langle 00\right| + \left|01\right\rangle_{XA} \left\langle 01\right| + \left|02\right\rangle_{XA} \left\langle 02\right|)(a_0 \left|0\right\rangle_B \left\langle 0\right| + a_1 \left|1\right\rangle_B \left\langle 1\right| + a_2 \left|2\right\rangle_B \left\langle 2\right|) \\ &\quad +(\left|10\right\rangle_{XA} \left\langle 10\right| + \left|11\right\rangle_{XA} \left\langle 11\right| + \left|12\right\rangle_{XA} \left\langle 12\right|)(a_2 \left|0\right\rangle_B \left\langle 0\right| + a_0 \left|1\right\rangle_B \left\langle 1\right| + a_1 \left|2\right\rangle_B \left\langle 2\right|) \\ &\quad +(\left|20\right\rangle_{XA} \left\langle 20\right| + \left|21\right\rangle_{XA} \left\langle 21\right| + \left|22\right\rangle_{XA} \left\langle 22\right|)(a_1 \left|0\right\rangle_B \left\langle 0\right| + a_2 \left|1\right\rangle_B \left\langle 1\right| + a_0 \left|2\right\rangle_B \left\langle 2\right|) \\ &\quad +(\left|00\right\rangle_{XA} \left\langle 01\right| + \left|01\right\rangle_{XA} \left\langle 02\right| + \left|02\right\rangle_{XA} \left\langle 00\right|)(a_0 \left|0\right\rangle_B \left\langle 0\right| + a_1 \exp\left(\frac{4}{3}{\rm i}\pi\right) \left|1\right\rangle_B \left\langle 1\right| + a_2 \exp\left(\frac{2}{3}{\rm i}\pi\right) \left|2\right\rangle_B \left\langle 2\right|) \\ &\quad +(\left|01\right\rangle_{XA} \left\langle 00\right| + \left|02\right\rangle_{XA} \left\langle 01\right| + \left|00\right\rangle_{XA} \left\langle 02\right|)(a_0 \left|0\right\rangle_B \left\langle 0\right| + a_1 \exp\left(\frac{2}{3}{\rm i}\pi\right) \left|1\right\rangle_B \left\langle 1\right| + a_2 \exp\left(\frac{4}{3}{\rm i}\pi\right) \left|2\right\rangle_B \left\langle 2\right|) \\ &\quad +(\left|10\right\rangle_{XA} \left\langle 11\right| + \left|11\right\rangle_{XA} \left\langle 12\right| + \left|12\right\rangle_{XA} \left\langle 10\right|)(a_2 \left|0\right\rangle_B \left\langle 0\right| + a_0 \exp\left(\frac{4}{3}{\rm i}\pi\right) \left|1\right\rangle_B \left\langle 1\right| + a_1 \exp\left(\frac{2}{3}{\rm i}\pi\right) \left|2\right\rangle_B \left\langle 2\right|) \\ &\quad +(\left|11\right\rangle_{XA} \left\langle 10\right| + \left|12\right\rangle_{XA} \left\langle 11\right| + \left|10\right\rangle_{XA} \left\langle 12\right|)(a_2 \left|0\right\rangle_B \left\langle 0\right| + a_0 \exp\left(\frac{2}{3}{\rm i}\pi\right) \left|1\right\rangle_B \left\langle 1\right| + a_1 \exp\left(\frac{4}{3}{\rm i}\pi\right) \left|2\right\rangle_B \left\langle 2\right|) \\ &\quad +(\left|20\right\rangle_{XA} \left\langle 21\right| + \left|21\right\rangle_{XA} \left\langle 22\right| + \left|22\right\rangle_{XA} \left\langle 20\right|)(a_1 \left|0\right\rangle_B \left\langle 0\right| + a_2 \exp\left(\frac{4}{3}{\rm i}\pi\right) \left|1\right\rangle_B \left\langle 1\right| + a_0 \exp\left(\frac{2}{3}{\rm i}\pi\right) \left|2\right\rangle_B \left\langle 2\right|) \\ &\quad +(\left|21\right\rangle_{XA} \left\langle 20\right| + \left|22\right\rangle_{XA} \left\langle 21\right| + \left|20\right\rangle_{XA} \left\langle 22\right|)(a_1 \left|0\right\rangle_B \left\langle 0\right| + a_2 \exp\left(\frac{2}{3}{\rm i}\pi\right) \left|1\right\rangle_B \left\langle 1\right| + a_0 \exp\left(\frac{4}{3}{\rm i}\pi\right) \left|2\right\rangle_B \left\langle 2\right|)]. \end{aligned} \quad (7)$$

Then, Alice makes the measurement on qutrits $X$ and $A$ in the computation bases: $\{\left|00\right\rangle_{XA} \left\langle 00\right|, \left|01\right\rangle_{XA} \left\langle 01\right|, \left|02\right\rangle_{XA} \left\langle 02\right|, \left|10\right\rangle_{XA} \left\langle 10\right|, \left|11\right\rangle_{XA} \left\langle 11\right|, \left|12\right\rangle_{XA} \left\langle 12\right|, \left|20\right\rangle_{XA} \left\langle 20\right|, \left|21\right\rangle_{XA} \left\langle 21\right|, \left|22\right\rangle_{XA} \left\langle 22\right|\}$. For the first three bases, Alice sends a ctrit 0 to Bob; for the second three bases, Alice sends a ctrit 1 to Bob; for the last three bases, Alice sends a ctrit 2 to Bob. Based on the classical information sent by Alice, Bob performs an appropriate unitary transformation $\lambda_i^B$ given by Table 1 to make his qutrit being in the teleported state, where $\lambda_i$ is defined in the restricted three-dimensional space $\{\left|0\right\rangle, \left|1\right\rangle, \left|2\right\rangle\}$ as follows:

$$\lambda_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

**Table 1.** The relationship between unitary transformation $\lambda_i^B$, corresponding classical information, and Alice's measurement result.

| measurement result | classical information | unitary transformation $\lambda_i^B$ |
|---|---|---|
| $\|00\rangle_{XA}\langle 00\|\,;\|01\rangle_{XA}\langle 01\|\,;\|02\rangle_{XA}\langle 02\|$ | 0 | $\lambda_0$ |
| $\|10\rangle_{XA}\langle 10\|\,;\|11\rangle_{XA}\langle 11\|\,;\|12\rangle_{XA}\langle 12\|$ | 1 | $\lambda_1$ |
| $\|20\rangle_{XA}\langle 20\|\,;\|21\rangle_{XA}\langle 21\|\,;\|22\rangle_{XA}\langle 22\|$ | 2 | $\lambda_2$ |

## 3. Teleportation of a mixture of diagonal states of two qutrits

In this section, we turn to discuss the teleportation of a mixture of diagonal states of two qutrits, which can be expressed by

$$\rho_{X_1X_2}(2) = \sum_{x_1,x_2=0}^{2} a_{x_1x_2} |x_1x_2\rangle_{X_1X_2}\langle x_1x_2|\,. \tag{8}$$

Alice and Bob now share two classically correlated pairs of two qutrits which has the same form as Eq.(2). In this case, the initial state of the system constituted by qutrits $X_1, X_2, A_1, B_1, A_2$ and $B_2$ is

$$\begin{aligned}\rho_{\rm ini}(2) &= \rho_{X_1X_2}(2)\otimes\rho_{A_1B_1}\otimes\rho_{A_2B_2}\\ &= \frac{1}{9}\sum_{x_1,x_2=0}^{2} a_{x_1x_2}|x_1x_2\rangle_{X_1X_2}\langle x_1x_2|\\ &\quad\otimes(|00\rangle_{A_1B_1}\langle 00| + |11\rangle_{A_1B_1}\langle 11|\\ &\quad + |22\rangle_{A_1B_1}\langle 22|)\\ &\quad\otimes(|00\rangle_{A_2B_2}\langle 00| + |11\rangle_{A_2B_2}\langle 11|\\ &\quad + |22\rangle_{A_2B_2}\langle 22|),\end{aligned} \tag{9}$$

where qutrits $X_1, X_2, A_1, A_2$ belong to Alice while $B_1, B_2$ are at Bob's side.

Analogously, Alice first executes an operation

$$O(2) = F^{A_2}\cdot F^{A_1}\cdot {\rm XOR}^{(3)}_{A_2X_2}\cdot {\rm XOR}^{(3)}_{A_1X_1}, \tag{10}$$

i.e., after Alice applies two conditional gates ${\rm XOR}^{(3)}_{A_1X_1}$ and ${\rm XOR}^{(3)}_{A_2X_2}$, she again performs two Fourier transforms $F^{A_1}$ and $F^{A_2}$ on qutrits $A_1$ and $A_2$. For the later measurement taken by Alice, only 81 diagonal terms make sense and each diagonal term can be described by

$$\begin{aligned}&\frac{1}{81}|x_1x_2\rangle_{X_1X_2}\langle x_1x_2|\otimes|y_1\rangle_{A_1}\langle y_1|\otimes|y_2\rangle_{A_2}\langle y_2|\\ &\times\sum_{\alpha_1,\alpha_2=0}^{2} a_{\alpha_1\alpha_2}(\lambda_{x_1}|\alpha_1\rangle_{B_1}\langle\alpha_1|\,\lambda_{x_1})\\ &\otimes(\lambda_{x_2}|\alpha_2\rangle_{B_2}\langle\alpha_2|\,\lambda_{x_2}),\end{aligned} \tag{11}$$

where $x_1, x_2, y_1, y_2 = 0, 1, 2$. Then, Alice makes the measurement on qutrits $X_1, X_2, A_1$ and $A_2$ in the computation bases $\{|x_1x_2\,\rangle_{X_1X_2}\langle x_1x_2|\otimes|y_1\rangle_{A_1}\langle y_1|\otimes|y_2\rangle_{A_2}\langle y_2|, x_1, x_2, y_1, y_2 = 0, 1, 2\}$, and sends 2 ctrits to Bob. Based on the received information, Bob performs an appropriate unitary transformation $\lambda_{x_1}^{B_1}\otimes\lambda_{x_2}^{B_2}$ given by Table 2 to make his two qutrits being in the teleported state.

**Table 2.** The relationship between unitary transformation $\lambda_{x_1}^{B_1}\otimes\lambda_{x_2}^{B_2}$, corresponding classical information, and Alice's measurement result.

| measurement result | corresponding two ctrits | unitary transformation $\lambda_{x_1}^{B_1}\otimes\lambda_{x_2}^{B_2}$ |
|---|---|---|
| $\|00\rangle_{X_1X_2}\langle 00\|\otimes\|y_1\rangle_{A_1}\langle y_1\|\otimes\|y_2\rangle_{A_2}\langle y_2\|$ | 00 | $\lambda_0\otimes\lambda_0$ |
| $\|01\rangle_{X_1X_2}\langle 01\|\otimes\|y_1\rangle_{A_1}\langle y_1\|\otimes\|y_2\rangle_{A_2}\langle y_2\|$ | 01 | $\lambda_0\otimes\lambda_1$ |
| $\|02\rangle_{X_1X_2}\langle 02\|\otimes\|y_1\rangle_{A_1}\langle y_1\|\otimes\|y_2\rangle_{A_2}\langle y_2\|$ | 02 | $\lambda_0\otimes\lambda_2$ |
| $\|10\rangle_{X_1X_2}\langle 10\|\otimes\|y_1\rangle_{A_1}\langle y_1\|\otimes\|y_2\rangle_{A_2}\langle y_2\|$ | 10 | $\lambda_1\otimes\lambda_0$ |
| $\|11\rangle_{X_1X_2}\langle 11\|\otimes\|y_1\rangle_{A_1}\langle y_1\|\otimes\|y_2\rangle_{A_2}\langle y_2\|$ | 11 | $\lambda_1\otimes\lambda_1$ |
| $\|12\rangle_{X_1X_2}\langle 12\|\otimes\|y_1\rangle_{A_1}\langle y_1\|\otimes\|y_2\rangle_{A_2}\langle y_2\|$ | 12 | $\lambda_1\otimes\lambda_2$ |
| $\|20\rangle_{X_1X_2}\langle 20\|\otimes\|y_1\rangle_{A_1}\langle y_1\|\otimes\|y_2\rangle_{A_2}\langle y_2\|$ | 20 | $\lambda_2\otimes\lambda_0$ |
| $\|21\rangle_{X_1X_2}\langle 21\|\otimes\|y_1\rangle_{A_1}\langle y_1\|\otimes\|y_2\rangle_{A_2}\langle y_2\|$ | 21 | $\lambda_2\otimes\lambda_1$ |
| $\|22\rangle_{X_1X_2}\langle 22\|\otimes\|y_1\rangle_{A_1}\langle y_1\|\otimes\|y_2\rangle_{A_2}\langle y_2\|$ | 22 | $\lambda_2\otimes\lambda_2$ |

## 4. Teleportation of a mixture of diagonal states of $N$ qutrits

This scheme can be directly generalized to teleport an arbitrary mixture of diagonal states of $N$ qutrits, which can be denoted by

$$\begin{aligned}&\rho_{X_1X_2\ldots X_N}(N)\\&=\sum_{x_1,x_2,\ldots,x_N=0}^{2}a_{x_1x_2\ldots x_N}\left|x_1x_2\ldots x_N\right\rangle_{X_1X_2\ldots X_N}\\&\quad\times\left\langle x_1x_2\ldots x_N\right|.\end{aligned}\tag{12}$$

Obviously, Alice and Bob should share $N$ classically correlated pairs of two qutrits. The initial state of the whole system is

$$\begin{aligned}&\rho_{\rm ini}(N)\\&=\rho_{X_1X_2\ldots X_N}(N)\otimes\prod_{k=1}^{N}\rho_{A_kB_k}\\&=\frac{1}{3^N}\sum_{x_1,x_2,\ldots,x_N=0}^{2}a_{x_1x_2\ldots x_N}\left|x_1x_2\ldots x_N\right\rangle_{X_1X_2\ldots X_N}\\&\quad\times\left\langle x_1x_2\ldots x_N\right|\otimes\prod_{k=1}^{N}(\left|00\right\rangle_{A_kB_k}\left\langle 00\right|+\left|11\right\rangle_{A_kB_k}\\&\quad\times\left\langle 11\right|+\left|22\right\rangle_{A_kB_k}\left\langle 22\right|),\end{aligned}\tag{13}$$

where qutrits $X_1, X_2, \ldots, X_N, A_1, A_2, \ldots, A_N$ are at Alice's side while $B_1, B_2, \ldots, B_N$ are at Bob's side.

Firstly, Alice manipulates an operation

$$O(N)=\prod_{k=1}^{N}F^{A_k}\cdot\prod_{k=1}^{N}{\rm XOR}_{A_kX_k}^{(3)}\tag{14}$$

on her qutrits. It is not difficult to gain each diagonal term of transformed state with the following form:

$$\begin{aligned}&\frac{1}{9^N}\left|x_1x_2\ldots x_N\right\rangle_{X_1X_2\ldots X_N}\left\langle x_1x_2\ldots x_N\right|\\&\times\sum_{\alpha_1,\alpha_2,\ldots,\alpha_N=0}^{2}a_{\alpha_1\alpha_2\ldots\alpha_N}\\&\times\prod_{k=1}^{N}(\left|y_k\right\rangle_{A_k}\left\langle y_k\right|(\lambda_{x_k}\left|\alpha_k\right\rangle_{B_k}\left\langle\alpha_k\right|\lambda_{x_k}),\end{aligned}\tag{15}$$

where $x_1, \ldots, x_N, y_1, \ldots, y_N = 0, 1, 2$. Secondly, Alice makes the measurement on her qutrits $X_1, X_2, \ldots, X_N, A_1, A_2, \ldots, A_N$ in the computation bases $\{\left|x_1x_2\ldots x_N\right\rangle_{X_1X_2\ldots X_N}\left\langle x_1x_2\ldots x_N\right|\otimes\left|y_1y_2\ldots y_N\right\rangle_{A_1A_2\ldots A_N}\left\langle y_1y_2\ldots y_N\right|, x_1, x_2, \ldots, x_N, y_1, y_2, \ldots, y_N = 0, 1, 2\}$, and sends $N$ ctrits to Bob. Based on the received information, Bob performs a corresponding unitary transformation $\prod_{i=1}^{N}\lambda_{x_i}^{B_i}$ to make his $N$ qutrits being in the teleported state, i.e., $\rho_{B_1B_2\ldots B_N}(N)=\sum_{x_1,x_2,\ldots,x_N=0}^{2}a_{x_1x_2\ldots x_N}\left|x_1x_2\ldots x_N\right\rangle_{B_1B_2\ldots B_N}\left\langle x_1x_2\ldots x_N\right|$.

## 5. Teleportation of a mixture of diagonal states of $N$ qudits

The scheme can further be extended to teleport a mixture of diagonal states of $N$ qudits. The teleported state in $D$-dimensional Hilbert space can be expressed by $\rho^D_{Z_1Z_2\ldots Z_N}(N)=\sum_{d_1,d_2,\ldots,d_N=0}^{D-1}a_{d_1d_2\ldots d_N}\left|d_1d_2\ldots d_N\right\rangle_{Z_1Z_2\ldots Z_N}\left\langle d_1d_2\ldots d_N\right|$. $N$ classically correlated pairs of two qudits shared by Alice and Bob are in the state $\prod_{p=1}^{N}\rho_{E_pF_p}$, where $\rho_{E_pF_p}=\frac{1}{D}\sum_{\varepsilon_p=0}^{D-1}\left|\varepsilon_p\varepsilon_p\right\rangle_{E_pF_p}\left\langle\varepsilon_p\varepsilon_p\right|$. Now, the initial state of the whole system is

$$\begin{aligned}&\rho^D_{\rm ini}(N)\\&=\rho^D_{Z_1Z_2\ldots Z_N}(N)\otimes\prod_{p=1}^{N}\rho_{E_pF_p}\\&=\frac{1}{D^N}\sum_{d_1,d_2,\ldots,d_N=0}^{D-1}a_{d_1d_2\ldots d_N}\left|d_1d_2\ldots d_N\right\rangle_{Z_1Z_2\ldots Z_N}\\&\quad\times\left\langle d_1d_2\ldots d_N\right|\otimes\prod_{p=1}^{N}\sum_{\varepsilon_p=0}^{D-1}\left|\varepsilon_p\varepsilon_p\right\rangle_{E_pF_p}\left\langle\varepsilon_p\varepsilon_p\right|,\end{aligned}\tag{16}$$

where qudits $Z_1, Z_2, \ldots, Z_N, E_1, E_2, \ldots, E_N$ are at Alice's side while $F_1, F_2, \ldots, F_N$ are at Bob's side.

Similarly, Alice executes an operation $O^D(N)=\prod_{p=1}^{N}F^{E_p}\cdot\prod_{p=1}^{N}{\rm GXOR}^{(D)}_{E_pZ_p}$, where $F^{E_p}$ and ${\rm GXOR}^{(D)}_{E_pZ_p}$ are Fourier transform and generalized quantum XOR gate,[21] respectively, and both are defined in $D$-dimension Hilbert space. $F^{E_p}$ and ${\rm GXOR}^{(D)}_{E_pZ_p}$ are concretely denoted by

$$F^{E_p}\left|j\right\rangle_{E_p}=\frac{1}{\sqrt{D}}\sum_{l=0}^{D-1}\exp[2{\rm i}\pi lj/D]\left|l\right\rangle_{E_p}\tag{17}$$

and

$${\rm GXOR}^{(D)}_{E_pZ_p}\left|m\right\rangle_{E_p}\left|n\right\rangle_{Z_p}=\left|m\right\rangle_{E_p}\left|m\ominus n\right\rangle_{Z_p},\tag{18}$$

respectively, where the '$\ominus$' operation indicates difference $(m-n)$ modulo $D$. After that, diagonal terms

of transformed state are

$$\sum_{d_1,d_2,\ldots,d_N=0}^{D-1} a_{d_1d_2\ldots d_N} \prod_{p=1}^{N} \Big( \left|\Psi_{l_p}\right\rangle_{Z_pE_p} \left\langle \Psi_{l_p}\right| \otimes \frac{1}{D^N} U_{l_p} \left|d_p\right\rangle_{F_p} \left\langle d_p\right| U_{l_p} \Big),$$

where $\left|\Psi_{l_p}\right\rangle_{Z_pE_p} \left\langle \Psi_{l_p}\right| = \frac{1}{D} \sum_{f_p=0}^{D-1} \left|l_p\right\rangle_{Z_p} \left\langle l_p\right| \otimes \left|f_p\right\rangle_{E_p} \left\langle f_p\right|$ and $U_{l_pm_p}$ satisfies $U_{l_p} \left|d_p\right\rangle_{F_p} \left\langle d_p\right| U_{l_p} = \left|(D-1)\, l_p \oplus d_p\right\rangle_{Fp} \left\langle (D-1)l_p \oplus d_p\right|$ $(l_p,\ d_p = 0, 1, \ldots, D-1)$. Alice makes the measurement on her qudits $Z_1, Z_2, \ldots, Z_N, E_1, E_2, \ldots, E_N$, in the computation bases $\left\{ \prod_{p=1}^{N} \left|l_p\right\rangle_{Z_p} \left\langle l_p\right| \otimes \left|f_p\right\rangle_{E_p} \left\langle f_p\right|, p = 1, 2, \ldots, N; l_p, f_p = 0, 1, \ldots, D-1 \right\}$. Conditioned on the measurement result of Alice, Bob has to perform a unitary transformation $\left\{ \prod_{p=1}^{N} U_{l_p}, p = 1, 2, \ldots, N; l_p = 0, 1, \ldots, D-1 \right\}$ to make his qudits being in the teleported state $\rho^D_{F_1F_2\ldots F_N}(N) = \sum_{d_1,d_2,\ldots,d_N=0}^{D-1} a_{d_1d_2\ldots d_N} \left|d_1d_2\ldots d_N\right\rangle_{F_1F_2\ldots F_N} \left\langle d_1d_2\ldots d_N\right|$.

# 6. Conclusion

In summary, we propose a protocol to teleport an arbitrary mixture of diagonal states of one qutrit. A classically correlated pair of two qutrits is used as channel. After Alice makes Fourier transform and conditional gate $\mathrm{XOR}^{(3)}$ on her qutrits, Bob performs an appropriate unitary transformation corresponding to the measurement results taken by Alice to make his qutrit being in the teleported state. We also generalize the teleportation scheme to the cases of two qutrits, $N$ qutrits and multiqudit. The remarkable feature of this scheme is that the channel is composed of some classically correlated pairs of two qutrits (or two qudits in the case of multiqudit). The reason for doing teleportation without quantum entanglement is that the teleported state in our proposal is not fully unknown but partially known. Our result shows that classical correlation is important and useful in quantum information processing.

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