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Feasible schemes for preparation of all five-atom graph states

Zhang Jin(张 瑾)

Department of Modern Physics, University of Science and Technology of China, Hefei 230026, *China*

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We propose feasible schemes for preparation of all five-atom graph states by cavity quantum electrodynamics (QED). Our schemes require only the atom–cavity interaction with a large detuning which is available in current experiment so that these schemes are within the reach of the current technology.

Keywords: entanglement, graph states, cavity QED
PACC: 0365, 4250

1. Introduction

Graph states are special multiparticle entangled states that can be represented by mathematical graphs, where the vertices of the graphs take the role of quantum spin systems and edges represent Ising interactions.[1−4] They not only provide new insights into fundamental tests of nonlocality[5] and decoherence[6] but also serve as essential resources for quantum error correction,[7] quantum communications[8] and, most prominently, one-way quantum computation.[9]

Recently, for their important applications, graph states have aroused much interest. Hein *et al*[1,4] have shown some properties of graph states. In their investigation, connected graphs with up to four-vertices are shown to be equivalent to the star-graphs corresponding to Greenberger–Horne–Zeilinger (GHZ) states, or the linear graphs corresponding to the linear cluster states by local unitary (LU) transformations and graph isomorphisms, while for five-vertices, besides these two common types, there exist other two types of connected graphs. Meanwhile a special feature of the graph states is that their entangled properties and potential use in quantum information processing are closely related to their underlying graphs.[1] Thus the preparation of different types of graph states may provide new models to characterize multiparticle entangled properties and introduce some applications in multiparty quantum communications. So far, the GHZ states and the four-photon linear cluster state have been reported experimentally.[10,11] However, the preparation of other types of graph states still remains a challenge.

As one of the promising candidates, the cavity quantum electrodynamics system has been used for various quantum information tasks.[12,13] Ye *et al* have pointed out that this is due to the fact that cold and localized atoms are not only the important resource of entanglement but also well suited for storing quantum information in long-lived internal states.[14] Here we propose schemes for preparing all types of five-atom graph states by the atom–cavity interaction in cavity QED. During the preparation, the cavities do not act as memories and only the atom–cavity interaction with a large detuning is required, which makes our schemes insensitive to the cavity decay and hence feasible under the current experimental technology.

The paper is structured as follows. We start in Section 2 by introducing the atom–cavity interaction with a large detuning. Then in Section 3 we proceed to describe the preparation of four types of five-atom graph states in cavity QED and give a brief discussion on the experimental particulars. Finally, concluding remarks are made in Section 4.

2. The atom–cavity interaction with a large detuning

Assume that N identical two-level atoms simultaneously interact with a single mode cavity and the detuning δ between the atomic transition frequency and the cavity frequency is far greater than the atom–

cavity coupling strength g. Especially, when considering the case that the cavity field is initially in the vacuum state (through our paper the cavities are all assumed to be initially in the vacuum state), the effective interaction Hamiltonian in the interaction picture can be written as[15−17]

$$H_{\rm e}=\lambda\left(\sum_{j=1}^{N}|1\rangle_{j\ j}\langle 1|+\sum_{j,k=1,j\neq k}^{N}S_j^+S_k^-\right), \quad (1)$$

where $\lambda=g^2/\delta$, $S_j^+=|1\rangle_{j\ j}\langle 0|$ and $S_j^-=|0\rangle_{j\ j}\langle 1|$ with $|0\rangle_j$ and $|1\rangle_j$ being the ground and the excited states of the jth atom, respectively.

We first consider the simplest case that two atoms simultaneously pass through the cavity. After an interaction time t, we obtain various output states corresponding to different input states. With the chosen interaction time $t=\pi/(2\lambda)$, we have

$$|0\rangle_1|0\rangle_2\rightarrow|0\rangle_1|0\rangle_2\rightarrow|0\rangle_1|0\rangle_2, \quad (2)$$

$$|1\rangle_1|0\rangle_2\rightarrow\frac{1}{2}(e^{-2i\lambda t}+1)|1\rangle_1|0\rangle_2+\frac{1}{2}(e^{-2i\lambda t}-1)|0\rangle_1|1\rangle_2\rightarrow-|0\rangle_1|1\rangle_2, \quad (3)$$

$$|0\rangle_1|1\rangle_2\rightarrow\frac{1}{2}(e^{-2i\lambda t}+1)|0\rangle_1|1\rangle_2+\frac{1}{2}(e^{-2i\lambda t}-1)|1\rangle_1|0\rangle_2\rightarrow-|1\rangle_1|0\rangle_2, \quad (4)$$

$$|1\rangle_1|1\rangle_2\rightarrow e^{-2i\lambda t}|1\rangle_1|1\rangle_2\rightarrow-|1\rangle_1|1\rangle_2, \quad (5)$$

while the cavity still remains in the vacuum state.

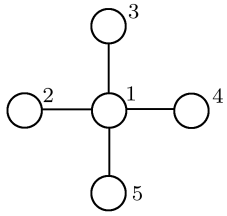
If N atoms are simultaneously sent through the cavity and each of them is initially in the state $(1/\sqrt{2})(|0\rangle+i|1\rangle)$, after the interaction time $t=\pi/(2\lambda)$, the state of the atomic system evolves as follows:[16]

$$|\psi\rangle=\frac{1}{\sqrt{2^{N+1}}}e^{i\pi/4}\left\{\prod_{j=1}^{N}[|0\rangle_j+(-i)^N|1\rangle_j]-i\prod_{j=1}^{N}[|0\rangle_j-(-i)^N|1\rangle_j]\right\}, \quad (6)$$

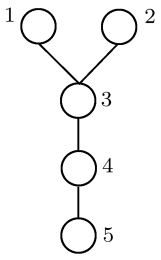
while the cavity field is also virtually excited.

3. Preparation of all five-atom graph states

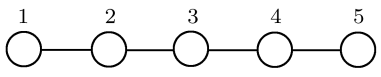
From Ref.[1], as shown in Fig.1, there exist four inequivalent types of graphs and any connected graph with five vertices is equivalent to one of them under LU transformations and graph isomorphisms.



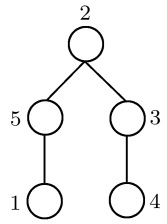
No.1 The five-atom star-graph state



No.2 The five-atom Y-shaped graph state



No.3 The five-atom linear cluster state



No.4 The five-atom circle-shaped graph state

Fig.1. Four types of connected graphs with five vertices that are not equivalent under LU transformation and graph isomorphisms.

The states corresponding to these graphs can be respectively written as

$$\begin{aligned}
\text{No.1}\quad & \frac{1}{\sqrt{2}}(|0\rangle_1|+\rangle_2|+\rangle_3|+\rangle_4|+\rangle_5 \\
& +|1\rangle_1|-\rangle_2|-\rangle_3|-\rangle_4|-\rangle_5), \qquad (7)\\
\text{No.2}\quad & \frac{1}{2}(|+\rangle_1|+\rangle_2|0\rangle_3|0\rangle_4|+\rangle_5 \\
& +|+\rangle_1|+\rangle_2|0\rangle_3|1\rangle_4|-\rangle_5 \\
& +|-\rangle_1|-\rangle_2|1\rangle_3|0\rangle_4|+\rangle_5 \\
& -|-\rangle_1|-\rangle_2|1\rangle_3|1\rangle_4|-\rangle_5), \qquad (8)\\
\text{No.3}\quad & \frac{1}{2}(|+\rangle_1|0\rangle_2|+\rangle_3|0\rangle_4|+\rangle_5 \\
& +|+\rangle_1|0\rangle_2|-\rangle_3|1\rangle_4|-\rangle_5 \\
& +|-\rangle_1|1\rangle_2|-\rangle_3|0\rangle_4|+\rangle_5 \\
& +|-\rangle_1|1\rangle_2|+\rangle_3|1\rangle_4|-\rangle_5), \qquad (9)\\
\text{No.4}\quad & \frac{1}{2\sqrt{2}}(|0\rangle_1|0\rangle_2|+\rangle_3|0\rangle_4|+\rangle_5 \\
& +|1\rangle_1|0\rangle_2|+\rangle_3|0\rangle_4|-\rangle_5 \\
& +|0\rangle_1|0\rangle_2|-\rangle_3|1\rangle_4|+\rangle_5 \\
& -|1\rangle_1|0\rangle_2|-\rangle_3|1\rangle_4|-\rangle_5 \\
& +|0\rangle_1|1\rangle_2|-\rangle_3|0\rangle_4|-\rangle_5 \\
& +|1\rangle_1|1\rangle_2|-\rangle_3|0\rangle_4|+\rangle_5 \\
& +|0\rangle_1|1\rangle_2|+\rangle_3|1\rangle_4|-\rangle_5 \\
& -|1\rangle_1|1\rangle_2|+\rangle_3|1\rangle_4|+\rangle_5), \qquad (10)
\end{aligned}$$

where $|+\rangle_i=(1/\sqrt{2})(|0\rangle_i+|1\rangle_i)$, $|-\rangle_i=(1/\sqrt{2})(|0\rangle_i-|1\rangle_i)$.

Now, let us describe the details of our schemes for preparation of these states. In order to construct the five-atom star-graph state, five atoms are all initially prepared in the state $(1/\sqrt{2})(|0\rangle+\mathrm{i}|1\rangle)$. They simultaneously interact with a cavity and the interaction time $t=\pi/(2\lambda)$. From Eq.(6), after they fly out of the cavity, these five atoms are in the state

$$|\varphi\rangle_1=\frac{1}{8}\left[\prod_{j=1}^{5}(|0\rangle_j-\mathrm{i}|1\rangle_j)-\mathrm{i}\prod_{j=1}^{5}(|0\rangle_j+\mathrm{i}|1\rangle_j)\right]. \qquad (11)$$

By single qubit rotations $(1/\sqrt{2})(|0\rangle_1-\mathrm{i}|1\rangle_1)\rightarrow|0\rangle_1$, $(1/\sqrt{2})(|0\rangle_1+\mathrm{i}|1\rangle_1)\rightarrow\mathrm{i}|1\rangle_1$, $(1/\sqrt{2})(|0\rangle_k\pm\mathrm{i}|1\rangle_k)\rightarrow|\mu\rangle_k$ $(k=2,3,4,5)$, the state $|\varphi\rangle_1$ can be exactly converted to the star-graph state in Eq.(7).

For the construction of the five-atom Y-shaped graph state, the four atoms 1, 2, 3 and 5 are initially prepared in the state $(1/\sqrt{2})(|0\rangle+\mathrm{i}|1\rangle)$ and the fourth atom is in the state $|+\rangle_4$. First, the atoms 1, 2, 3 and 5 are simultaneously sent into a cavity and the interaction time $t=\pi/(2\lambda)$. Then let atoms 4 and 5 interact with the cavity field and also choose $t=\pi/(2\lambda)$. From equations (2)–(6), the evolution of the whole state is

$$\begin{aligned}
|\varphi\rangle_2 &= \frac{1}{4}(|0\rangle_1+\mathrm{i}|1\rangle_1)\otimes(|0\rangle_2+\mathrm{i}|1\rangle_2) \\
&\quad\otimes(|0\rangle_3+\mathrm{i}|1\rangle_3)\otimes(|0\rangle_5+\mathrm{i}|1\rangle_5)|+\rangle_4 \\
&\rightarrow \frac{1}{\sqrt{2}}(|+\rangle_1|+\rangle_2|+\rangle_3|+\rangle_5|+\rangle_4 \\
&\quad -\mathrm{i}|-\rangle_1|-\rangle_2|-\rangle_3|-\rangle_5|+\rangle_4) \\
&\rightarrow \frac{1}{2}(|+\rangle_1|+\rangle_2|+\rangle_3|0\rangle_4|-\rangle_5 \\
&\quad -|+\rangle_1|+\rangle_2|+\rangle_3|1\rangle_4|+\rangle_5 \\
&\quad -\mathrm{i}|-\rangle_1|-\rangle_2|-\rangle_3|0\rangle_4|-\rangle_5 \\
&\quad -\mathrm{i}|-\rangle_1|-\rangle_2|-\rangle_3|1\rangle_4|+\rangle_5), \qquad (12)
\end{aligned}$$

which is converted to the state in Eq.(8) under local transformations $|+\rangle_3\rightarrow|0\rangle_3$, $|-\rangle_3\rightarrow\mathrm{i}|1\rangle_3$, $|-\rangle_5\rightarrow|+\rangle_5$, $|+\rangle_5\rightarrow-|-\rangle_5$.

Next, let us consider the generation of five-atom linear cluster state. The atoms 1, 2 and 5 are initially in the state $(1/\sqrt{2})(|0\rangle+\mathrm{i}|1\rangle)$ and the atoms 3 and 4 are in the state $|+\rangle_3|+\rangle_4$. Atoms 1, 2 and 5 are simultaneously sent into a cavity and $t=\pi/(2\lambda)$. When they fly out, these three atoms are respectively driven by classical fields and undergo the transitions $(1/\sqrt{2})(|0\rangle_1\pm\mathrm{i}|1\rangle_1)\rightarrow|\pm\rangle_1$, $(1/\sqrt{2})(|0\rangle_2+\mathrm{i}|1\rangle_2)\rightarrow|0\rangle_2$, $(1/\sqrt{2})(|0\rangle_2-\mathrm{i}|1\rangle_2)\rightarrow\mathrm{i}|1\rangle_2$, $(1/\sqrt{2})(|0\rangle_5\pm\mathrm{i}|1\rangle_5)\rightarrow|\pm\rangle_5$. Then let atoms 3 and 5 interact with the cavity with $t=\pi/(2\lambda)$. Finally, after they fly out, atoms 4 and 5 are sent into the cavity with $t=\pi/(2\lambda)$, then we obtain

$$\begin{aligned}
|\varphi\rangle_3 &= \frac{1}{2\sqrt{2}}(|0\rangle_1+\mathrm{i}|1\rangle_1)\otimes(|0\rangle_2+\mathrm{i}|1\rangle_2) \\
&\quad\otimes(|0\rangle_5+\mathrm{i}|1\rangle_5)|+\rangle_3|+\rangle_4 \\
&\rightarrow \frac{1}{4}[(|0\rangle_1+\mathrm{i}|1\rangle_1)(|0\rangle_2+\mathrm{i}|1\rangle_2) \\
&\quad\times(|0\rangle_5+\mathrm{i}|1\rangle_5)|+\rangle_3|+\rangle_4-\mathrm{i}(|0\rangle_1-\mathrm{i}|1\rangle_1) \\
&\quad\times(|0\rangle_2-\mathrm{i}|1\rangle_2)(|0\rangle_5-\mathrm{i}|1\rangle_5)|+\rangle_3|+\rangle_4] \\
&\rightarrow \frac{1}{\sqrt{2}}(|+\rangle_1|0\rangle_2|+\rangle_5|+\rangle_3|+\rangle_4 \\
&\quad +|-\rangle_1|1\rangle_2|-\rangle_5|+\rangle_3|+\rangle_4) \\
&\rightarrow \frac{1}{2}(|+\rangle_1|0\rangle_2|0\rangle_3|-\rangle_5|+\rangle_4 \\
&\quad -|+\rangle_1|0\rangle_2|1\rangle_3|+\rangle_5|+\rangle_4 \\
&\quad +|-\rangle_1|1\rangle_2|0\rangle_3|-\rangle_5|+\rangle_4 \\
&\quad +|-\rangle_1|1\rangle_2|1\rangle_3|+\rangle_5|+\rangle_4) \\
&\rightarrow \frac{1}{2}(|+\rangle_1|0\rangle_2|-\rangle_3|0\rangle_4|-\rangle_5 \\
&\quad +|+\rangle_1|0\rangle_2|+\rangle_3|1\rangle_4|+\rangle_5 \\
&\quad +|-\rangle_1|1\rangle_2|+\rangle_3|0\rangle_4|-\rangle_5 \\
&\quad +|-\rangle_1|1\rangle_2|-\rangle_3|1\rangle_4|+\rangle_5). \qquad (13)
\end{aligned}$$

Under local transformations $|\pm\rangle_3 \rightarrow |\mu\rangle_3$ and $|\pm\rangle_5 \rightarrow |\mu\rangle_5$, the state $|\varphi\rangle_3$ becomes

$$\begin{aligned}|\varphi\rangle_3 = \frac{1}{2}(&|+\rangle_1|0\rangle_2|+\rangle_3|0\rangle_4|+\rangle_5\\ &+|+\rangle_1|0\rangle_2|-\rangle_3|1\rangle_4|-\rangle_5\\ &+|-\rangle_1|1\rangle_2|-\rangle_3|0\rangle_4|+\rangle_5\\ &+|-\rangle_1|1\rangle_2|+\rangle_3|1\rangle_4|-\rangle_5),\end{aligned} \tag{14}$$

which is exactly the five-atom linear cluster state given by Eq.(9).

If we proceed by sending atoms 1 and 5 through the cavity with $t = \pi/(2\lambda)$, we have

$$\begin{aligned}|\varphi\rangle_4 = \frac{1}{2\sqrt{2}}(&|0\rangle_1|0\rangle_2|+\rangle_3|0\rangle_4|-\rangle_5\\ &-|1\rangle_1|0\rangle_2|+\rangle_3|0\rangle_4|+\rangle_5\\ &+|0\rangle_1|0\rangle_2|-\rangle_3|1\rangle_4|-\rangle_5\\ &+|1\rangle_1|0\rangle_2|-\rangle_3|1\rangle_4|+\rangle_5\\ &+|0\rangle_1|1\rangle_2|-\rangle_3|0\rangle_4|+\rangle_5\\ &-|1\rangle_1|1\rangle_2|-\rangle_3|0\rangle_4|-\rangle_5\\ &+|0\rangle_1|1\rangle_2|+\rangle_3|0\rangle_4|+\rangle_5\\ &+|1\rangle_1|1\rangle_2|+\rangle_3|1\rangle_4|-\rangle_5),\end{aligned} \tag{15}$$

which can be converted to the state given by Eq.(16) by utilizing transitions $|1\rangle_1 \rightarrow -|1\rangle_1$ and $|\pm\rangle_5 \rightarrow |\mu\rangle_5$.

So far, we have presented schemes for preparation of all types of five-atom graph states by using the atom–cavity interaction in cavity QED. Now let us briefly discuss the experimental feasibility of our schemes. For the Rydberg atoms, the radiative time is about $T_r = 3 \times 10^{-2}$ s and with the choice of a proper cavity, the efficient decay time of the cavity can also be limited to the order of $T_c \approx 10^{-2}$ s.[15] When $\delta = 10$ g and the coupling constant is $g = 2\pi \times 24$ kHz, the required atom–cavity–field interaction time is of the order of $T \approx 10^{-4}$ s, then the time needed to complete the whole procedure is much shorter than T_r and T_c. Thus the influence of spontaneous atomic emission and cavity decay is negligible. Meanwhile there can be no exchange of quantum information between the atoms and the cavity field, and the cavity field is virtually excited, which greatly reduces the requirement on the quality factor of the cavity. In fact, such an atom–cavity field interaction has been successfully used to experimentally prepare the two-atom EPR states by Osnaghi *et al.*[18] Therefore, our schemes are feasible under current experimental technological conditions.

4. Conclusions

In summary, we have proposed feasible schemes to prepare all types of five-atom graph states using cavity QED technique. We have briefly discussed the experimental feasibility of our schemes and shown our proposals are within the reach of the current cavity QED technology. In addition, we would like to point out that besides potential applications of the graph states in multiparty quantum communications and quantum computation, the study of their characteristic features is important for better understanding the entanglement of many parties. So we believe that our schemes have profound implications for the development of quantum information.

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