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Probabilistic teleportation of multi-particle partially entangled state*

Chen Xiu-Bo(陈秀波)[a)b)†], Du Jian-Zhong(杜建忠)[c)],
Wen Qiao-Yan(温巧燕)[a)b)], and Zhu Fu-Chen(朱甫臣)[d)]

[a)] *State Key Laboratory of Networking and Switching Technology, Beijing University of Posts and Telecommunications, Beijing* 100876, *China*

[b)] *School of Science, Beijing University of Posts and Telecommunications, Beijing* 100876, *China*

[c)] *School of Information Engineering, Beijing University of Science and Technology, Beijing* 100083, *China*

[d)] *National Laboratory for Modern Communications, P O Box* 810, *Chengdu* 610041, *China*

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Utilizing the generalized measurement described by positive operator-valued measure, this paper comes up with a protocol for teleportation of an unknown multi-particle entangled (GHZ) state with a certain probability. The feature of the present protocol is to weaken requirement for the quantum channel initially shared by sender and receiver. All unitary transformations performed by receiver are summarized into a formula. On the other hand, this paper explicitly constructs the efficient quantum circuits for implementing the proposed teleportation by means of universal quantum logic operations in quantum computation.

Keywords: quantum teleportation, quantum circuits, positive operator-valued measure, unitary transformations

PACC: 0365, 0367

1. Introduction

In the burgeoning field of quantum communication and quantum calculation, quantum teleportation[1] has been considered to be one of the singular fruits. The new methods of teleportation have interested many research groups, and research work for quantum teleportation has been greatly developed in theory and experiment.[2−9] Quantum entanglement is one of the most crucial resources, and hence is studied intensively in various ways. Although due attention has been paid to generate entanglement, quantum entanglement has to be transported among spatially separated spots in some applications.[10]

It is significant that quantum entanglement is directly teleported between two spatially separated parties. Entanglement teleportation[11,12] transfers the entanglement initially imposed on an unknown multipartite state to a multipartite state at a remote place. Entanglement teleportation transfers not only the amount of entanglement but also the entanglement structure. In the past few years, many researchers have extensively studied Greenberger–Horne–Zeilinger (GHZ) states due to their important role in the quantum information processing and communication. Many theoretical proposals have appeared for the generation of multiqubit GHZ states. Recently, it has been reported that GHZ state up to six-photon has been created in experiment.[13] As especially relevant to this work, we propose a novel protocol for the entanglement teleportation of multi-particle GHZ state $|\varphi\rangle = x|0\rangle^{\otimes(N+1)} + y|1\rangle^{\otimes(N+1)}$.

In this paper, utilizing the generalized measurement described by positive operator-valued measure (POVM), we propose a protocol for probabilistic teleportation of multi-particle partially entangled GHZ state. Generally, any state of N-partite system can be teleported by using N pairs of entangled particles as quantum channel and the transmission of classical message.[14] However, in this protocol the quantum

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†E-mail: flyover100@163.com

channel is only a partially entangled pair which is used as a bridge between two spatially separated parties, traditionally called Alice (sender) and Bob (receiver). Based on the number of classical message transmitted by Alice, Bob introduces appropriate number of additional particles and performs a set of unitary operations to complete the entanglement teleportation with a certain probability.

The POVM is the most general physical measurement in quantum mechanics which may allow the extraction of more information than the usual von Neumann-type projective measurement. It is valuable to obtain a physical means to carry out the POVM. It deserves mentioning that the quantum circuit, i.e. quantum logic network, is essential for practical realization of the POVM in experiment. There are two usual methods to construct quantum circuits. One is to decompose the unitary operation into universal quantum logic operation.[15,16] Recently, following the line of Jensen and Schack,[17] based on the theory of majorization, Gu *et al*[18] showed the second method to construct the quantum circuits in realizing deterministic entanglement concentration. In their method, the quantum circuits are obtained by decomposing the process of state evolution from initial states to final states. Brassard *et al*[19] proposed quantum logic networks to carry out teleportation in 1998. Enlightened by the above works, we construct the quantum circuits for the POVM with quantum Toffoli gates in probabilistically distinguishing two non-orthogonal states. Moreover, the efficient quantum circuits for implementing the proposed probabilistic teleportation are constructed by means of universal quantum logic operations in quantum computation.

The structure of this paper is as follows. In Section 2, using the generalized measurement described by POVM, we propose a protocol for teleporting an unknown multi-particle entangled GHZ state via a two-particle partially entangled pair. The essential purpose of the present paper is to cut down the number of entangled particle initially shared by sender and receiver. The probability for obtain the successful teleportation is calculated and all kinds of transformations performed by the parties are analysed in detail. In Section 3, we construct the efficient quantum circuits for implementing the proposed teleportation. A brief discussion and the concluding summary are given in Section 4.

2. Probabilistic teleportation of a multi-particle entangled state

Alice wants to teleport an unknown $(N+1)$-particle entangled GHZ state to distant receiver Bob:

$$|\varphi\rangle = x|0\rangle^{\otimes(N+1)} + y|1\rangle^{\otimes(N+1)}, \tag{1}$$

where x and y are non-negative real numbers satisfying $x^2 + y^2 = 1$, $N \geq 2$. When N equals to 2, the state is the important three-particle GHZ state $x|000\rangle + y|111\rangle$. In contrast to the previous protocol for teleportation of three-particle GHZ state,[8,9] the present protocol can economize more entanglement resources utilized as quantum channel and classical messages transmitted.

The quantum channel is a two-particle partially entangled pair.

$$|\phi\rangle_{(N+2)(N+3)} = a|00\rangle_{(N+2)(N+3)} + b|11\rangle_{(N+2)(N+3)}, \tag{2}$$

here, a and b are positive real numbers and satisfy $a^2 + b^2 = 1$, $a > b$. Alice is in possession of particles $1, 2, \ldots, (N+2)$, while particle $(N+3)$ is in Bob's possession. At the beginning of the protocol, the state of the whole system composed of particles $1, 2, \ldots, (N+1)$ and quantum channel is $|\Phi\rangle = |\varphi\rangle \otimes |\phi\rangle_{(N+2)(N+3)}$.

To begin with, Alice performs joint measurement on particles $(N+1)$ and $(N+2)$ in Bell basis. As a result, we can get the following states at random

$$\begin{aligned} &\langle \Phi^{\pm}|_{(N+1)(N+2)}\Phi\rangle \\ &= (ax|0\rangle^{\otimes N}|0\rangle_{(N+3)} \pm by|1\rangle^{\otimes N}|1\rangle_{(N+3)})/\sqrt{2}, \end{aligned} \tag{3}$$

$$\begin{aligned} &\langle \Psi^{\pm}|_{(N+1)(N+2)}\Phi\rangle \\ &= (bx|0\rangle^{\otimes N}|1\rangle_{(N+3)} \pm ay|1\rangle^{\otimes N}|0\rangle_{(N+3)})/\sqrt{2}. \end{aligned} \tag{4}$$

The four Bell states can be expressed as $|\Phi^{\pm}\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$, $|\Psi^{\pm}\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$. For convenience, Alice's Bell-state measurement results Φ^+, Φ^-, Ψ^+, Ψ^- are expressed as classical bit strings M_1M_2=00, 10, 01, 11, respectively.

Then, Alice rotates every qubit in $1, 2, \ldots, N$ by Hadamard transformation in the form $H|0\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, $H|1\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$. The following state can be obtained:

$$(H^{\otimes N}\otimes I_{(N+3)})\langle\Phi^{\pm}|_{(N+1)(N+2)}\Phi\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2^N}}\Big[ax|0\rangle_{(N+3)}\otimes\sum_{i_1,i_2,\ldots i_N}|i_1,i_2,\ldots i_N\rangle + (-1)^{M_1}by|1\rangle_{(N+3)} \otimes\sum_{i_1,i_2,\ldots i_N}(-1)^{\sum_{j=1}^{N} i_j}|i_1,i_2,\ldots i_N\rangle\Big], \tag{5}$$

$$(H^{\otimes N}\otimes I_{(N+3)})\langle\Psi^{\pm}|_{(N+1)(N+2)}\Phi\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2^N}}\Big[bx|1\rangle_{(N+3)}\otimes\sum_{i_1,i_2,\ldots i_N}|i_1,i_2,\ldots i_N\rangle + (-1)^{M_1}ay|0\rangle_{(N+3)} \otimes\sum_{i_1,i_2,\ldots i_N}(-1)^{\sum_{j=1}^{N} i_j}|i_1,i_2,\ldots i_N\rangle\Big], \tag{6}$$

here, $i_j \in \{0,1\}$ $j = 1,2,\ldots,N$.

Alice measures every qubit $1,2,\ldots,N$ in the basis $\{|0\rangle,|1\rangle\}$. For qubit j, we denote the measurement result $|0\rangle,|1\rangle$ as classical bit $i_j = 0$, $i_j = 1$, respectively. To make Bob know the collapsed state with certainty, Alice communicates N+2 classical bits to Bob, including measurement outcomes on particles $1,2,\ldots,N$ and Bell-state measurement results on particles $(N+1)$ and $(N+2)$.

With these classical bits in hand, Bob introduces an auxiliary particle A with the state $|0\rangle$, and performs CNOT operation with particle $(N+3)$ as the control qubit and the particle A as the target qubit. Bob obtains the state of particles $A(N+3)$, and rewrites it as the following combined state:

$$ax|00\rangle_{A(N+3)} + (-1)^{M_1}(-1)^{\sum_{j=1}^{N} i_j}by|11\rangle_{A(N+3)} = \frac{1}{2}[(x|0\rangle_{(N+3)} + (-1)^{M_1}(-1)^{\sum_{j=1}^{N} i_j}y|1\rangle_{(N+3)})(a|0\rangle_A + b|1\rangle_A) + (x|0\rangle_{(N+3)} - (-1)^{M_1}(-1)^{\sum_{j=1}^{N} i_j}y|1\rangle_{(N+3)})(a|0\rangle_A - b|1\rangle_A)], \tag{7}$$

$$bx|11\rangle_{A(N+3)} + (-1)^{M_1}(-1)^{\sum_{j=1}^{N} i_j}ay|00\rangle_{A(N+3)} = \frac{1}{2}[(x|1\rangle_{(N+3)} + (-1)^{M_1}(-1)^{\sum_{j=1}^{N} i_j}y|0\rangle_{(N+3)})(a|0\rangle_A + b|1\rangle_A) - (x|1\rangle_{(N+3)} - (-1)^{M_1}(-1)^{\sum_{j=1}^{N} i_j}y|0\rangle_{(N+3)})(a|0\rangle_A - b|1\rangle_A)]. \tag{8}$$

Bob performs the measurement described by the following operators on particle A:

$$A_1 = \frac{1}{2\,a^2}\begin{pmatrix} b^2 & ab \\ ab & a^2 \end{pmatrix}, \quad A_2 = \frac{1}{2\,a^2}\begin{pmatrix} b^2 & -ab \\ -ab & a^2 \end{pmatrix}, \quad A_3 = \begin{pmatrix} 1-\dfrac{b^2}{a^2} & 0 \\ 0 & 0 \end{pmatrix}. \tag{9}$$

It is straightforward to verify that $\{A_1,\ A_2,\ A_3\}$ form a legitimate POVM. It is impossible for Bob to determine whether he has been given $a|0\rangle_A + b|1\rangle_A$ or $a|0\rangle_A - b|1\rangle_A$ with perfect reliability. However, he can utilize the above POVM to optimally distinguish the two non-orthogonal states. For convenience, if the measurement result is A_1, we denote the result as m=1. Similarly, if the measurement outcome A_2 occurs, we denote it as m=2. Some of the time, Bob will obtain the measurement outcome A_3, and he can infer nothing about the identity of the state. In this case, the exact teleportation fails.

Based on the number of the classical bits sent by Alice, Bob introduces N additional qubits $|0\rangle_{(N+4)},|0\rangle_{(N+5)},\ldots,|0\rangle_{(2N+3)}$. He performs a set of CNOT operations with particle $(N+3)$ as the control qubit and every additional qubit as the target qubit. Afterwards, we can get

$$\frac{1}{\sqrt{2}}\frac{b}{\sqrt{2^N}}[x|0\rangle_{(N+3)}|0\rangle_{(N+4)}|0\rangle_{(N+5)}\cdots|0\rangle_{(2N+3)}$$
$$+(-1)^{M_1}(-1)^{\sum_{j=1}^{N} i_j}(-1)^{m-1}y|1\rangle_{(N+3)}|1\rangle_{(N+4)}|1\rangle_{(N+5)}\cdots|1\rangle_{(2N+3)}], \quad (10)$$
$$\frac{1}{\sqrt{2}}\frac{b}{\sqrt{2^N}}[x|1\rangle_{(N+3)}|1\rangle_{(N+4)}|1\rangle_{(N+5)}\cdots|1\rangle_{(2N+3)}$$
$$+(-1)^{M_1}(-1)^{\sum_{j=1}^{N} i_j}(-1)^{m-1}y|0\rangle_{(N+3)}|0\rangle_{(N+4)}|0\rangle_{(N+5)}\cdots|0\rangle_{(2N+3)}]. \quad (11)$$

According to Alice's $N+2$ classical bits measurement results and the measurement result of POVM, Bob needs to apply the following transformation to recover the transmitted state:

$$Z_{(2N+3)}^{i_N} Z_{(2N+2)}^{i_{N-1}} \cdots Z_{(N+4)}^{i_1} X_{(2N+3)}^{M_2} X_{(2N+2)}^{M_2} \cdots X_{(N+4)}^{M_2} Z_{(N+3)}^{M_1} Z_{(N+3)}^{m-1} X_{(N+3)}^{M_2}. \quad (12)$$

As a result, particles $(N+3), (N+4), \ldots, (2N+3)$ successfully collapse into the following state $x|0\rangle^{\otimes(N+1)} + y|1\rangle^{\otimes(N+1)}$, which is just a replica of Alice's original state. At the same time, no trace of identity of the unknown quantum state remains in Alice's region. The total probability of successful teleportation is $4 \times 2^N \times (b^2/2^{N+1}) = 2b^2$.

3. Quantum circuits for teleportation

So far, we have proposed a novel protocol for teleportation of multi-particle entangled state. The next highly desirable problem is to realize the teleportation by physical means. It is well known that any operation in quantum mechanics can be represented by a unitary evolution together with a measurement. At the same time, any unitary evolution can be accomplished via universal quantum logic gates.[20,21] In this section, we firstly decompose the POVM for probabilistically distinguishing the non-orthogonal states, and then on the basis of the Eq.(12), we construct the efficient quantum circuit for implementing the teleportation. The formalism we use, which is called a quantum gate array, was introduced by Deutsch.[21]

Without loss of generality, suppose that Alice's measurement results are M_1M_2=01, i_1=1, $i_2 = i_3 = \cdots = i_N = 0$, the state of particles $A(N+3)$ in Eq.(8) is

$$bx|11\rangle_{A(N+3)} - ay|00\rangle_{A(N+3)}$$
$$= \frac{1}{2}[(x|1\rangle_{(N+3)} - y|0\rangle_{(N+3)})(a|0\rangle_A + b|1\rangle_A)$$
$$-(x|1\rangle_{(N+3)} + y|0\rangle_{(N+3)})(a|0\rangle_A - b|1\rangle_A)]. \quad (13)$$

We introduce two auxiliary qubits C and C' with the state $|11\rangle_{CC'}$. Define a unitary operator P acting on particles $CC'A(N+3)$ by

$$P(bx|11\rangle_{A(N+3)} - ay|00\rangle_{A(N+3)})|11\rangle_{CC'}$$
$$=\sum_m \sqrt{A_m}(bx|11\rangle_{A(N+3)} - ay|00\rangle_{A(N+3)})|m\rangle_{CC'}. \quad (14)$$

Here, m in kets represents a binary string, e.g. $|2\rangle_{CC'} = |10\rangle_{CC'}$. The results $\{|m\rangle_{CC'},\ m = 1, 2, 3\}$ correspond to the possible measurement results of the POVM $\{A_1,\ A_2,\ A_3\}$, respectively. From the above equation, it is straightforward to derive a key relation that the unitary evolution P together with the conventional orthogonal measurement is equivalent to the required POVM measurement. So, our task changes into obtaining the quantum circuits for unitary operator P in Eq.(14). The components of the state $bx|11\rangle_{A(N+3)} - ay|00\rangle_{A(N+3)}$ evolve in the following manner:

$$bx|1111\rangle_{CC'A(N+3)} \Rightarrow \frac{abbx}{\sqrt{2}\,a}|0101\rangle + \frac{a^2bx}{\sqrt{2}\,a}|0111\rangle$$
$$-\frac{abbx}{\sqrt{2}\,a}|1001\rangle + \frac{a^2bx}{\sqrt{2}\,a}|1011\rangle, \quad (15)$$
$$-ay|1100\rangle_{CC'A(N+3)} \Rightarrow -\frac{b^2ay}{\sqrt{2}\,a}|0100\rangle - \frac{abay}{\sqrt{2}\,a}|0110\rangle$$
$$-\frac{b^2ay}{\sqrt{2}\,a}|1000\rangle + \frac{abay}{\sqrt{2}\,a}|1010\rangle$$
$$-ay\sqrt{1-\frac{b^2}{a^2}}|1100\rangle. \quad (16)$$

The particles are arranged in the order $CC'A(N+3)$, and we omit the subscripts. As a result, $(bx|11\rangle_{A(N+3)} - ay|00\rangle_{A(N+3)})|11\rangle_{CC'}$ changes into

$$bx|1111\rangle_{CC'A(N+3)} - ay|1100\rangle_{CC'A(N+3)} \Rightarrow \frac{abbx}{\sqrt{2}\,a}|0101\rangle - \frac{b^2ay}{\sqrt{2}\,a}|0100\rangle + \frac{a^2bx}{\sqrt{2}\,a}|0111\rangle - \frac{abay}{\sqrt{2}\,a}|0110\rangle - \frac{abbx}{\sqrt{2}\,a}|1001\rangle - \frac{b^2ay}{\sqrt{2}\,a}|1000\rangle + \frac{a^2bx}{\sqrt{2}\,a}|1011\rangle + \frac{abay}{\sqrt{2}\,a}|1010\rangle - ay\sqrt{1-\frac{b^2}{a^2}}|1100\rangle. \quad (17)$$

From the Eq.(17), we can see clearly that after measurement on particle A in the basis $\{|0\rangle, |1\rangle\}$, if $CC' = 01$, i.e. m=1, Bob can get the state $x|1\rangle_{(N+3)} - y|0\rangle_{(N+3)}$. After the projective measurement on the particle A, if $CC' = 10$, i.e. m=2, the state $x|1\rangle_{(N+3)} + y|0\rangle_{(N+3)}$ is obtained. Certainly, if $CC' = 11$, the teleportation fails.

Now, we construct the concrete quantum circuit for the POVM, i.e. the quantum circuit for unitary operator P in Eq.(14). Here, our goal is to decompose the unitary operator P in Eq.(14) into the quantum Toffoli gate. The quantum Toffoli gate is an (n+1)-bit gate that applies a one-bit unitary operation to the last (n+1)th qubit (target qubit), if and only if the first n qubits (control qubits) are all equal to one. Deutsch[21] showed that a simple generalization of the Toffoli gate is sufficient as an universal gate.

Firstly, we construct the quantum circuit for the evolution in Eq.(15). According to the initial state and the final state in Eq.(15), the evolution can be realized by the following operation.

(1) Apply a unitary operation U_1 on $C'A$, if and only if (N+3) is in $|1\rangle$.

$$\begin{aligned} & U_1 bx|1111\rangle_{CC'A(N+3)} \\ =& \frac{abbx}{\sqrt{2}\,a}|1101\rangle + \frac{a^2bx}{\sqrt{2}\,a}|1111\rangle \\ & - \frac{abbx}{\sqrt{2}\,a}|1001\rangle + \frac{a^2bx}{\sqrt{2}\,a}|1011\rangle. \end{aligned} \quad (18)$$

(2) Perform NOT operation on particle C, if and only if $C'(N+3)$ is in $|11\rangle$.

It is not difficult to obtain the decomposition for U_1 depicted by the quantum Toffoli gate U_{11}, U_{12} and U_{13}. That is, the evolution in Eq.(18) can be realized by firstly performing U_{11} on A if and only if $C'(N+3)$ is in $|11\rangle$, then U_{12} on C' if and only if $A(N+3)$ is in $|11\rangle$, finally U_{13} on C' if and only if $A(N+3)$ is in $|01\rangle$.

The quantum circuit for the evolution in Eq.(15) is shown in Fig.1.

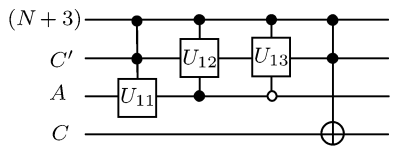


Fig.1. Quantum circuit for the evolution in Eq.(15).

Where

$$U_{11} = \begin{pmatrix} -a & b \\ b & a \end{pmatrix},$$

$$U_{12} = \begin{pmatrix} \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

$$U_{13} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (19)$$

Secondly, the evolution in Eq.(16) is realized by decomposing the process of state evolution from initial states to final states.

(1) Apply a unitary operation U_2 on CA if and only if $C'(N+3)$ is in $|10\rangle$.

$$\begin{aligned} & U_2(-ay)|1100\rangle_{CC'A(N+3)} \\ =& -\frac{b^2ay}{a}|0100\rangle - \frac{abay}{a}|0110\rangle \\ & - ay\sqrt{1-\frac{b^2}{a^2}}|1100\rangle. \end{aligned} \quad (20)$$

(2) Perform NOT operation on C, if and only if $C'(N+3)$ is in $|10\rangle$.

$$\begin{aligned} & -\frac{b^2ay}{a}|1100\rangle_{CC'A(N+3)} - \frac{abay}{a}|1110\rangle_{CC'A(N+3)} \\ & - ay\sqrt{1-\frac{b^2}{a^2}}|0100\rangle_{CC'A(N+3)}. \end{aligned} \quad (21)$$

(3) Apply to C' a unitary operation U_3 if and only if $CA(N+3)$ is in $|100\rangle$, and U_{13} in Eq.(19) if and only if $CA(N+3)$ is in $|110\rangle$.

(4) Perform NOT operation on C, if and only if $C'(N+3)$ is in $|10\rangle$.

It is easy to decompose the U_2 into the quantum Toffoli gate depicted by the U_{21} and U_{22}. That is, the evolution in Eq.(20) can be realized by firstly performing U_{21} on C if and only if $C'A(N+3)$ is in $|100\rangle$, then U_{22} on A if and only if $CC'(N+3)$ is in $|010\rangle$.

The quantum circuit for the evolution in Eq.(16) is shown in Fig.2.

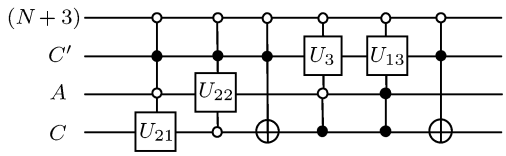


Fig.2. Quantum circuit for the evolution in Eq.(16).

Where

$$U_{21} = \begin{pmatrix} -\sqrt{1-\frac{b^2}{a^2}} & \frac{b}{a} \\ \frac{b}{a} & \sqrt{1-\frac{b^2}{a^2}} \end{pmatrix},$$

$$U_{22} = \begin{pmatrix} b & a \\ a & -b \end{pmatrix},$$

$$U_3 = \begin{pmatrix} \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{22}$$

The final quantum circuit for the POVM in distinguishing two non-orthogonal states, i.e. the quantum circuit for the unitary operation P in Eq.(14), is shown in Fig.3.

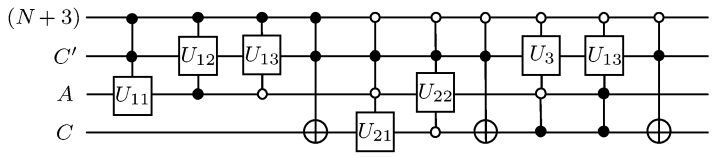


Fig.3. Quantum circuit for the POVM, i.e. quantum circuit for the unitary operation P in Eq.(14).

Barenco *et al*[20] showed that all unitary operations on arbitrarily many bits can be decomposed into the combination of a set of one-bit quantum gates and two-bit CNOT gates. So, an array of quantum Toffoli gates above can further be decomposed. For saving space, we do not depict out their quantum circuits by means of one-bit quantum gates and two-bit CNOT gates.

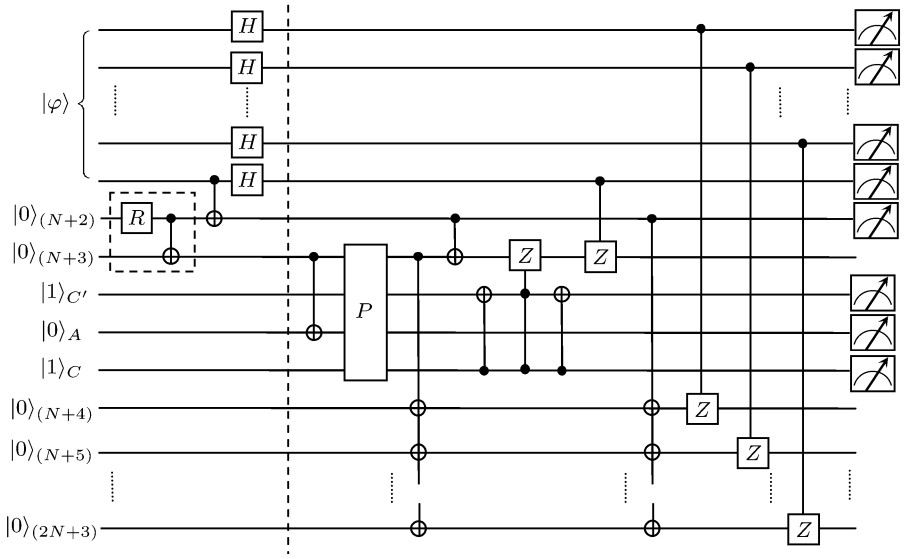


Fig.4. Quantum circuit for the teleportation based on the POVM. Here, the quantum circuit for P acting on the particles $(N+3)C'AC$ is shown by Fig.3. In the dashed rectangle, the state of particles $(N+2)$ and $(N+3)$ is the quantum channel. The $N+2$ top lines represent Alice's system, while the $(N+3)$th line is Bob's system. The qubits $C'AC$ are auxiliary qubits. The N bottom qubits are the additional qubits. The initially entangled state owned by Alice is restored on particles $(N+3)(N+4)\ldots(2N+3)$.

Based on the above decomposition for the POVM and Bob's operations in Eq.(12), the quantum circuit for implementing the presented protocol is shown in Fig.4.

4. Conclusions

In this paper, we propose a protocol for quantum entanglement teleportation. The main feature is that via only a pair of partially entangled particle, the teleportation of a $(N+1)$-particle partially entangled GHZ state is obtained with a certain probability. The present scheme weakens the requirement for quantum channel initially shared by the parties. The probability for successful teleportation is calculated, and all kinds of unitary transformations performed by sender and receiver are given in detail. One of the most important works is that we summarize the Bob's various unitary operations into a formula.

On the other hand, it is valuable to obtain the quantum circuit for realizing the teleportation in experiment. Lately, increasing attention has been paid to it.[16,22,23] In this paper, the quantum circuits of the POVM required to probabilistically distinguish the two non-orthogonal states are explicitly constructed. In addition, we construct the efficient quantum circuits for implementing the proposed teleportation protocol by means of universal quantum logic operations in quantum computation.

Recently, Shao *et al* [24] has proposed the efficient scheme for implementing an N-qubit Toffoli gate by a single resonant interaction with cavity quantum electrodynamics. An approach is proposed to realize an N-qubit controlled-U gate with superconducting quantum interference devices in cavity QED.[25] Zou *et al* [26] have proposed an experimental scheme to implement a two-qubit conditional Z gate with atoms trapped in a high-Q optical cavity. Bell measurement on two photons has been considered.[27] The von Neumann measurement could be implemented with high-efficiency photon detectors[28] and quantum CNOT gate has been demonstrated successfully.[29] Therefore, we believe that the proposed protocol in our paper will be realized in experiment.

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