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The dynamics of triple-well trapped Bose–Einstein condensates with atoms feeding and loss effects*

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In this paper, we consider the macroscopic quantum tunnelling and self-trapping phenomena of Bose–Einstein condensates (BECs) with three-body recombination losses and atoms feeding from thermal cloud in triple-well potential. Using the three-mode approximation, three coupled Gross–Pitaevskii equations (GPEs), which describe the dynamics of the system, are obtained. The corresponding numerical results reveal some interesting characteristics of BECs for different scattering lengths. The self-trapping and quantum tunnelling both are found in zero-phase and π-phase modes. Furthermore, we observe the quantum beating phenomenon and the resonance character during the self-trapping and quantum tunnelling. It is also shown that the initial phase has a significant effect on the dynamics of the system.

Keywords: Bose–Einstein condensates, three-body recombination and atoms feeding, quantum self-trapping, quantum tunnelling

PACC: 0365, 0155, 7335

1. Introduction

Since Bose–Einstein condensates (BECs) were observed in experiment, the study of their dynamical properties has been a very active field of research. Much attention has been paid to the investigation of tunnelling phenomena and macroscopic quantum self-trapping between two-coupled BECs.[1−8] It is found that there are two types of interesting effects, one is the periodic variation of relative atomic population between two condensates and the other is the so-called macroscopic self-trapping, with localization of atoms in one of the condensates and a periodically varying population around a constant value. In recent experiments, a far off-resonant laser barrier has been used to split a trapped condensate into three or even more well-trapped condensates, lowering the laser intensity such that atom tunnelling through the barrier takes place. So the interference is developed, somewhat like Josephson effect in superconductivity.[2] The tunnelling dynamics of triple-well trapped condensates is investigated theoretically.[9−16] The self-trapping of BECs in a multi-well has been observed experimentally, however, behind which the relevant physics is not fully understood yet.[17,18] Thus, it is important to know how the nonlinear Josephson oscillation and self-trapping phenomenon in multi-well system, the simplest triple-well trapped BECs, can show more interesting behaviours and provide the convenience for the study of BECs in the optical lattice.

To understand the behaviours of BECs, several proposals have been made to generalize the Gross–Pitaevskii equation (GPE) including the effects of the thermal cloud and two- or three-body recombination terms.[8,19−21] So in this paper, we will combine analytic method with numerical method to investigate the new effects of atoms feeding from thermal cloud and atom loss by three-body recombination in triple-well trapped BECs. The numerical results of the relative atomic population show many interesting properties, such as macroscopic quantum self-trappings and tunnellings in zero-phase mode and π-phase mode. At the same time, the quantum beating phenomenon in the self-trapping process and the resonance character in π-phase mode manifest themselves.

2. The three-mode approximation

By considering triple-well trapped BECs with nonconservative effects, the attractive interactions be-

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tween BECs is thus rewritten by the time-dependent GPE including the three-body recombination and atoms feeding contributions to describe the dynamics of the BECs.[8,19−21]

$$\mathrm{i}\hbar\partial_t \Psi(\boldsymbol{r},t) = \Bigg[-\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{ext}}(\boldsymbol{r},t) - g|\Psi(\boldsymbol{r},t)|^2 - \mathrm{i}\xi'|\Psi(\boldsymbol{r},t)|^4 + \mathrm{i}\gamma \Bigg] \Psi(\boldsymbol{r},t), \tag{1}$$

where Ψ is the wavefunction of the condensate in triple-well which is normalized to the total number of atoms $N \equiv \int_{-\infty}^{\infty} \Psi(\boldsymbol{r},t)|^2 \mathrm{d}\boldsymbol{r}$; g is coupling constant in direct proportional to the scattering length a and expressed as $g = 4\pi\hbar^2|a|/m$ with m being the atomic mass; nonzero ξ' denotes the probability of ejection of atoms from the condensates due to three-body recombination, $\xi' = C\hbar^3/(2m)a^4$; nonzero γ is the probability of the atoms feeding from the thermal cloud into the condensates. (Note that ξ' and γ are two small positive constants in a practical situation, and the total number of atoms in the condensates is not conserved for nonzero ξ' and γ); V_{ext} is the confining potential and represents anisotropic trap. Here we choose an experimental 'cigar-shape' trap with an elongated axis in the x-direction. The dimensionless quasi-one-dimensional equation of Eq.(1) is

$$\mathrm{i}\partial_\tau \tilde{\Psi}(x,\tau) = [-\nabla^2 + V_{\mathrm{ext}}(x,\tau) - A|\tilde{\Psi}(x,\tau)|^2 - \mathrm{i}\xi|\tilde{\Psi}(x,\tau)|^4 + \mathrm{i}\gamma]\tilde{\Psi}(x,\tau) \tag{2}$$

with dimensionless variables $x = \boldsymbol{r}/l_x$, $\tau = t\omega_x$, $\tilde{\Psi} = \Psi/\tilde{n}^{1/2}$, and $A = a/a_{\mathrm{s}}$. Here, x, τ, $\tilde{\Psi}$, and A are the dimensionless coordinate, time, wavefunction and scattering length of the BECs, respectively; $l_x = \sqrt{\hbar/(m\omega_x)}$ with ω_x being the trap frequency along the x-axis corresponding to the harmonic frequency of the triple-well potential; $\tilde{n} = (4|a_{\mathrm{s}}|\omega_r/(l_x\omega_x))^{-1}$ with ω_r being the frequency in the transverse r-direction, and parameters ω_x, ω_r and $|a_{\mathrm{s}}|$ being on the order of $10^2\mathrm{s}^{-1}$, $10^3\mathrm{s}^{-1}$ and 10^{-9}m, respectively; a_{s} is the characteristic s-wave scattering length. We select the values for ξ and γ such that they receive a physical justification: $\gamma = 1.6\times10^{-4}$ and $\xi = CA^4|a_{\mathrm{s}}|^2/(48\pi^2 l_x^2)$ with $C = 1000$ that was used in Borca's work.[22]

Assuming that the nonlinear interaction between the atoms is not too large but the potential barrier is sufficiently high, such that the atom interactions between wells are weak. The word 'weak' here means that the spatial distribution functions $\phi_i(x)$ are localized in each well with a very weak overlap and there is no coupling between the first and the third wells. As in the double-well case, in order to investigate the dynamics of BECs in the triple-well potential, we look for the solution of Eq.(2) in the form of the three-mode approximation $\tilde{\Psi}(x,\tau) = \psi_1(\tau)\phi_1(x) + \psi_2(\tau)\phi_2(x) + \psi_3(\tau)\phi_3(x)$, where $\psi_i(\tau)$ and $\phi_i(x)$ are approximately single-particle mode functions which each are a localized single-particle ground state for each of the three wells, and $\phi_i(x)$ is an appropriate set of orthonormal mode functions for triple-well potential, thus $\int \phi_i^*(x)\phi_j(x)\mathrm{d}x = \delta_{ij}(i,j=1,2,3)$. Substituting these approximations into Eq.(2), we obtain

$$\mathrm{i}\partial_\tau\psi_1 = (E_1 - U_1|\psi_1|^2)\psi_1 - K_{12}\psi_2 + \mathrm{i}(\gamma - U_1'|\psi_1|^4)\psi_1, \tag{3}$$

$$\mathrm{i}\partial_\tau\psi_2 = (E_2 - U_2|\psi_2|^2)\psi_2 - K_{12}\psi_1 - K_{23}\psi_3 + \mathrm{i}(\gamma - U_2'|\psi_2|^4)\psi_2, \tag{4}$$

$$\mathrm{i}\partial_\tau\psi_3 = (E_3 - U_3|\psi_3|^2)\psi_3 - K_{23}\psi_2 + \mathrm{i}(\gamma - U_3'|\psi_3|^4)\psi_3, \tag{5}$$

where E_i, U_i, U'_i and K for $i = 1,2,3$ denote the overlap integrations $E_i = \int(|\nabla\phi_i|^2 + |\phi_i|^2 V_{\mathrm{ext}})\mathrm{d}x$, $U_i = A\int|\phi_i|^4\mathrm{d}x$, $U_i' = \xi\int|\phi_i|^6\mathrm{d}x$, $K_{12} = -\int(\nabla\phi_1\nabla\phi_2 + \phi_1 V_{\mathrm{ext}}\phi_2)\mathrm{d}x$, and $K_{23} = -\int(\nabla\phi_2\nabla\phi_3 + \phi_2 V_{\mathrm{ext}}\phi_3)\mathrm{d}x$. Here, E_i are the zero-point energies in each well, U_i are directly proportional to the mean-field interaction between atoms, U'_i describe the tunnelling dynamics of the three-body recombination interactions, which are directly proportional to ξ, the tunnelling probability K_{ij} describe the amplitude of the tunnelling between condensates. E_i, U_i, U'_i, K_{ij} can be written in terms of $\phi_i(x)$ wavefunction overlap. Let

$$\psi_i(\tau) = \sqrt{N_i(\tau)}\mathrm{e}^{\mathrm{i}\theta_i(\tau)}, \qquad (i=1,2,3), \tag{6}$$

so that the time-dependence of the wavefunction $\psi_i(\tau)$ is embodied in N_i and θ_i, where $N_i(\tau)$ is the number of particles in the i well, in units of $(4a_{\mathrm{s}}\omega_{\mathrm{r}}/(l_x\omega_x))^{-1}$, and $\theta_i(\tau)$ is the phase of state ψ_i. The normalization condition is $N_1 + N_2 + N_3 = N$, where N is the total number of atoms of the triple-well trapped condensates in the same units as N_i's. Substituting Eq.(6) into Eqs.(3)–(5), for simplicity, we first consider the case in which the tunnelling probability between the first and the second wells is the same as that between the second and the third wells, i.e. $K_{12} = K_{23} = K$, and in order to understand the novel phenomena of interference effects between triple-well trapped BECs, we consider an ideal case,[9,10] i.e. the totally symmetric case, in which $U_1 = U_2 = U_3 = U$, $E_1 = E_2 = E_3 = E$, and $U'_1 = U'_2 = U'_3 = U'$, so

$N_1 - N_2 = N_3 - N_2$ or $N_1 = N_3$, then, we can obtain

$$\frac{\mathrm{d}N}{\mathrm{d}\tau} = 2\gamma N - 2U'N\frac{(N^2+6Z^2)}{9} + \frac{4U'Z^3}{9}, \quad (7)$$

$$\frac{\mathrm{d}Z}{\mathrm{d}\tau} = -2K\sqrt{N^2 - ZN - 2Z^2}\sin\phi + 2\gamma Z - 2U'Z\frac{(N^2+Z^2)}{3} + \frac{2U'NZ^2}{3}, \quad (8)$$

$$\frac{\mathrm{d}\phi}{\mathrm{d}\tau} = -UZ + K\cos\phi\frac{4Z+N}{\sqrt{N^2 - ZN - 2Z^2}}, \quad (9)$$

where $Z = N_1 - N_2$, and $\phi = \theta_2 - \theta_1$. We can find that the dynamical equations given by Eqs.(7)–(9) are different from those for the double-well case.[8] Equations (7)–(9) have some coupling terms different from those in Eqs.(16)–(18) in Ref.[8]. Thus, the middle well is closely related to the character of the triple-well system. We expect that the transition to self-trapping in a triple-well system can show distinct character from that of doubled-well system. Their corresponding results will be discussed in detail in the next section. The effect of atoms feeding from the thermal cloud (γ) can be modelled as a linear amplification, and ξ is also important due to the three-body recombination. Because of these two terms, the total number of N is not conserved but varies with time. These equations are non-integrable, and analytical solutions can be obtained only by using approximate methods like the variational approach. In the following, we will analyse the stability of stationary solutions, then give the corresponding numerical illustrations. We can find some of characteristics through the adjustments of the three-body losses, i.e. by changing the scattering length.

3. The stationary states

In this section, we study the stability of stationary solutions. From Eqs.(7)–(9) with $\dot{N} = 0$, $\dot{Z} = 0$, and $\dot{\phi} = 0$, we can obtain the stationary state equations of (Z_0, N_0, ϕ_0). But these equations are complex, and they are difficult to solve analytically. On the other hand, we can obtain the stationary solutions (Z_0, N_0, ϕ_0) numerically, with the values of parameters K, γ, U' and U fixed. The nonzero stationary solution Z_0 corresponds to a macroscopic quantum self-trapping state. In experiment, tunnelling probability K can be modified by reducing the height of the trapped potential, the three-body loss parameter γ can be adjusted by using the Feshbach resonance, and the three-body parameter U' and two-body parameter U can be varied simultaneously by changing the value of scattering length A. As an example, we choose the physical parameters in a region allowable experimentally as $U = 0.7A$, $K = 3.9$, $\gamma = 1.6 \times 10^{-4}$, and $U' = 0.05 \times 10^{-4}A^4$.

Table 1 shows some numerical stationary solutions (ϕ_0, N_0, Z_0) at different values of A. The results show that there are at least two real stationary solutions for a fixed value of A. The number of the stationary solutions increases with the decreasing of the value of A. We can also find that the phases of the stationary states are always 0, $\pm\pi/2$, $\pm\pi$ in value.

Table 1. The stationary solutions for different scattering lengths.

A	ϕ_0	N_0	Z_0	A	ϕ_0	N_0	Z_0
3	0	1.46	−0.46	0.49	−1.57	23.56	−23.56
	3.14	1.68	−0.34		0	33.46	−22.93
0.62	0	22.92	−14.04		1.57	47.12	23.56
	3.14	42.52	−4.76		3.14	68.69	−6.69
0.55	0	29.09	−19.58	0.206	−1.57	133.30	−133.30
	1.57	37.40	18.70		0	304.55	113.02
	1.57	18.70	−18.70		0	387.30	41.38
	3.14	54.29	−5.67		0	147.79	−133.04
0.506	−1.57	44.19	22.09		1.57	266.59	133.30
	0	32.08	−21.42		3.14	396.55	−20.87
	1.57	22.09	−22.09				
	3.14	64.29	−6.48				

We can expect that some of the stationary solutions are stable and some are unstable. We can investigate the stability of these stationary solutions numerically. Taking $A = 0.49$ and $A = 0.55$ for example, the numerical results of Eqs.(7)–(9) are shown in Fig.1. In Fig.1, the initial conditions $N(0) = 29$, $Z(0) = 10$, $\phi(0) = 0$ and $\phi(0) = \pi$ have been used.

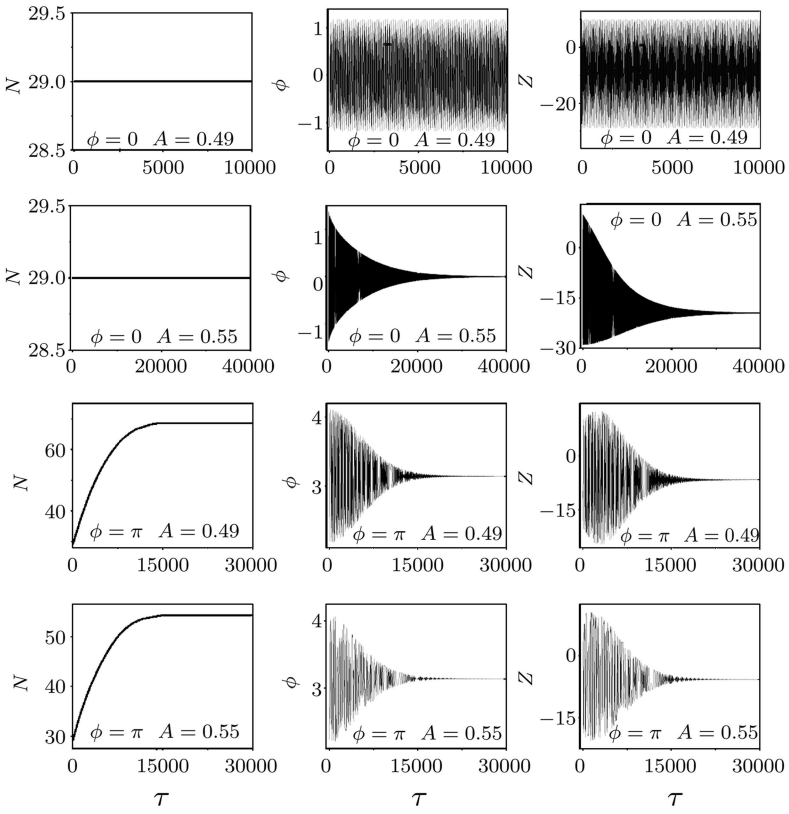


Fig.1. The time evolutions of N, ϕ, Z at the different initial phases for $A = 0.49$ and 0.55.

We find that, for $A = 0.49$ and $\phi(0) = 0$, ϕ and Z oscillate in a large region and do not tend to be stationary solutions. Thus, these stationary solutions are unstable. But for $A = 0.55$ and $\phi(0) = 0$, N, ϕ and Z finally tend to be the stationary solutions ($N_0 = 29$, $\phi_0 = 0$ and $Z_0 = -19.58$), which are in good agreement with the stationary solutions given in Table 1. Thus, these stationary solutions are stable. For $A = 0.49$ and $A = 0.55$ with $\phi(0) = \pi$, N, ϕ, Z finally tend to be the stationary solutions ($N_0 = 68.48$, $\phi_0 = 3.14$, $Z_0 = -6.69$, and $N_0 = 54.27$, $\phi_0 = 3.14$, $Z_0 = -5.67$, respectively), which are also in good agreement with the stationary solutions given in Table 1. Thus, these solutions corresponding to the stationary solutions given in Table 1 are stable. So, we can find that the initial phase has a significant effect on stability of the stationary solution. In addition, the stability of stationary solution can be modified by parameters K, γ, U', and U. To analytically investigate the effects of all parameters on the stability of stationary solution is a formidable work. Hence, in the next section, the non-stationary features of the system will be illustrated numerically. We will show that most of the stationary states given in Table 1 are stable and the system can always tend to enter these stationary states. On the other hand, the system can always reach a self-trapping state.

4. Effect of scattering length on tunnelling dynamics

In this section, we use numerical analysis to investigate the dynamical characteristics such as macroscopic quantum self-trapping and π-phase oscillations

of BECs in a triple-well. It is well known that for ^{85}Rb the effective interaction between the atoms is attractive with scattering length $a = -400a_0$, where a_0 is the Bohr radius. In the present numerical results, we take $a_s = -100a_0$ and adjust the s-wave scattering length $a = Aa_s$ by changing the value of A. We choose the initial conditions and physical parameters in a region allowable experimentally as $N(0) = 29$, $Z(0) = 10$, $U = 0.7A$, $K = 3.9$, $\gamma = 1.6 \times 10^{-4}$, and $U' = 0.05 \times 10^{-4}A^4$.

4.1. Zero-phase mode

The numerical results of Eqs.(7)–(9) for $\phi(0) = 0$ with the value of A reducing gradually are shown in Fig.2.

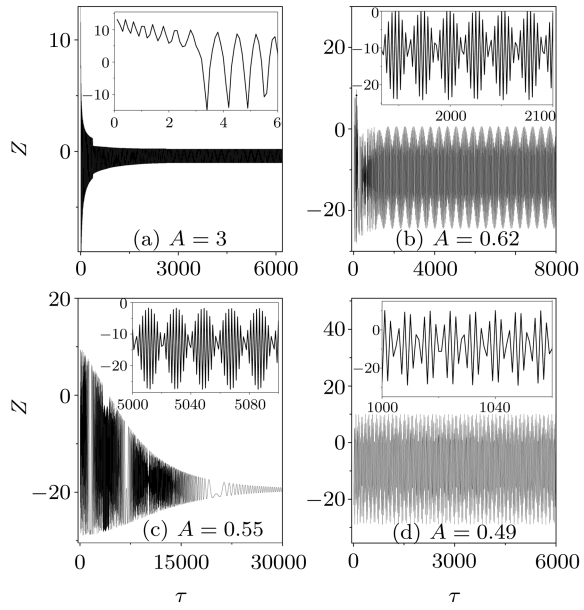


Fig.2. The time evolutions of relative population Z for the different values of s-wave scattering length A: (a) 3.0, (b) 0.62, (c) 0.55, and (d) 0.49.

In Fig.2(a) where $A = 3$, the relative population Z varies periodically around a small negative constant value. A short-lived self-trapping can be observed before $\tau = 3.5$ which is shown in the inset of Fig.2(a). The relative population will reach the stationary state ($Z_0 = -0.46$, as shown in Table 1) after a long time evolution. This phenomenon can also be seen in a region of $0.62 < A < 3$. In this region, Z tends to be a negative constant with a small absolute magnitude and the system enters a self-trapping state. In Fig.2(b) where $A = 0.62$, Z oscillates around a constant value of -10, which is in agreement with the result given in Table 1. We can also find that Z oscillates with a quantum beating character as shown in the inset of Fig.2(b). The average relative population only with a constant ripple implies that the system is in a self-trapping state. This phenomenon can also be seen in a region of $0.585 < A < 0.62$. However, when $A < 0.585$, the system displays several new properties. For $A = 0.55$, as shown in Fig.2(c), the amplitude of population Z decreases with time, and finally Z tends to be a negative constant but with a larger absolute magnitude and the system enters a strong macroscopic self-trapping state ($Z_0 = -19.58$, as shown in Table 1). A similar phenomenon can be observed for $0.505 < A < 0.55$, but the time for condensates to enter a complete self-trapping state is delayed as the value of A decreases . The quantum beating phenomenon still appears in this case as shown in the inset of Fig.2(c). When we continue decreasing A as shown in Fig.2(d) for $A = 0.49$, the system oscillates periodically again around $Z \approx -10$ as shown in Table 1. This stationary state is unstable. But as we will show in the next subsection, the π-phase mode with $A = 0.49$ will be stable. The atoms tunnelling between the wells and quantum beating phenomenon also appear. Figure 2 shows very rich properties of the condensates in three wells with atoms feeding and loss effects. It is indicated that we can adjust the three-body interaction parameter by Feshbach resonances to control the system between periodic and stationary states.

4.2. π-phase mode

Now we consider π-phase case, i.e. $\phi(0) = \pi$. As shown below, the tunnelling of condensates in a π-phase mode has some different properties from those in a zero-phase mode. The time variation of Z with A is shown in Fig.3. For $A = 3$, the amplitude of the relative population Z is small and finally Z decays to a small negative constant value (see Fig.3(a)). The total number of particles and relative phase each reach a stationary state ($N_0 = 1.67$, ϕ_0=3.14, as shown in Table 1) as Z tends to the stable stationary state ($Z_0 = -0.34$, as shown in Table 1). The similar phenomenon can also be seen in a region of $A > 3$. When we continue decreasing A (see Fig.3(b)), the nonzero relative population Z oscillates between the wells and its amplitude decays with time, finally the relative population Z reaches a finite negative value

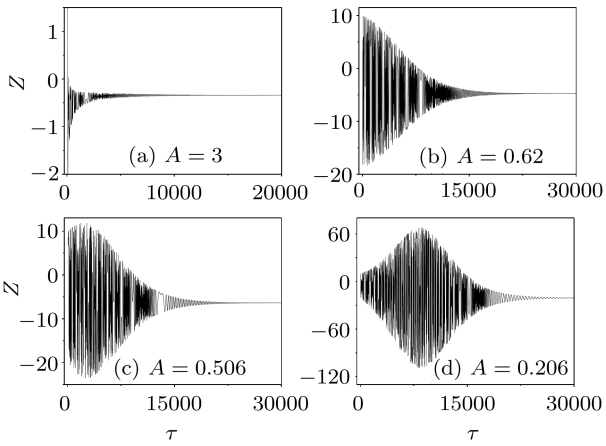


Fig.3. The time evolutions of relative population Z for different values of s-wave scattering length A: (a) 3, (b) 0.62, (c) 0.506, (d) 0.206.

with a small absolute magnitude and the system reaches a strong self-trapping state ($Z_0 = -4.76$, as shown in Table 1). These can be seen in a region of $0.62 < A < 3$. However, when $A < 0.62$, the results show an interesting character. As A decreases, such as the cases of $A = 0.506$ and 0.206 shown in Figs.3(c) and 3(d), the relative population finally tends to be a negative value and the system enters a macroscopic self-trapping state ($Z_0 = -6.48$ and $Z_0 = -20.86$, as shown in Table 1). In addition, Figs.3(c) and 3(d) each reveal a resonance character, i.e. the oscillation of Z has a maximal amplitude. We can also find that with A decreasing, the occurrence of the resonance is postponed and the strength of the resonance, i.e. the oscillation amplitude, is enhanced. This resonance phenomenon indicates that the system experiences a strong tunnelling when it enters a macroscopic self-trapping state. We can expect that the resonance character should be related to the physical processes of atom feeding and losing. The detailed analysis of the physics of the resonance is beyond the scope of this paper and we will do it in our further study.

4.3. The general case with zero-phase

Generally, we consider the effect induced by the weak interaction between the first and the third wells. Consider the relative atomic populations $Z_1 = N_1 - N_2$, and $Z_2 = N_1 - N_3$, let $K_{12} = K_{21} = K_{23} = K_{32} = K$, $\phi_1 = \theta_2 - \theta_1$, and $\phi_2 = \theta_3 - \theta_1$, and substitute Eq.(6) into Eqs.(3)–(5), then we will obtain the following five coupled dynamical equations:

$$\frac{\partial\phi_1}{\partial\tau} = \frac{3KZ_1\cos\phi_1}{\sqrt{(Z_1+Z_2+N)(Z_2-2Z_1+N)}} + UZ_1 + K\cos(\phi_1-\phi_2)\sqrt{\frac{Z_1-2Z_2+N}{Z_2-2Z_1+N}}, \tag{10}$$

$$\frac{\partial\phi_2}{\partial\tau} = K\cos(\phi_1-\phi_2)\sqrt{\frac{Z_2-2Z_1+N}{Z_1-2Z_2+N}} + UZ_2 - K\cos\phi_1\sqrt{\frac{Z_2-2Z_1+N}{Z_1+Z_2+N}}, \tag{11}$$

$$\begin{aligned}\frac{\partial Z_1}{\partial\tau} = {} & -\frac{4}{3}K\sin\phi_1\times\sqrt{(Z_1+Z_2+N)(Z_2-2Z_1+N)} - \frac{2}{3}K\sin(\phi_1-\phi_2)\times\sqrt{(Z_2-2Z_1+N)(Z_1-2Z_2+N)} \\ & -\frac{2}{3}\gamma(Z_2-2Z_1+N) + \frac{2\gamma}{3}(Z_1+Z_2+N) + \frac{U'}{27}(Z_2-2Z_1+N)^3 - \frac{U'}{27}(Z_1+Z_2+N)^3,\end{aligned} \tag{12}$$

$$\begin{aligned}\frac{\partial Z_2}{\partial\tau} = {} & -\frac{2}{3}K\sin\phi_1\times\sqrt{(Z_1+Z_2+N)(Z_2-2Z_1+N)} + \frac{2}{3}K\sin(\phi_1-\phi_2)\times\sqrt{(Z_2-2Z_1+N)(Z_1-2Z_2+N)} \\ & -\frac{2}{3}\gamma(Z_1-2Z_2+N) + \frac{2\gamma}{3}(Z_1+Z_2+N) + \frac{2U'}{27}(Z_1-2Z_2+N)^3 - \frac{2U'}{27}(Z_1+Z_2+N)^3,\end{aligned} \tag{13}$$

$$\frac{\partial N}{\partial\tau} = 2\gamma N - 2U'\left[\frac{(Z_1+Z_2+N)^3}{27} + \frac{(Z_2-2Z_1+N)^3}{27} + \frac{(Z_1-2Z_2+N)^3}{27}\right]. \tag{14}$$

The variations of relative populations Z_1 and Z_2 with time are shown in the first and second columns of Fig.4, respectively. We choose the same conditions as those used in Figs.2 and 3, but with $\phi_1 = 0$, $\phi_2 = 0$, and $K = 3.9$ adopted here. It is clear from Fig.4 that in a large range of the value of A, Z_1 finally tends to be a negative constant value, but Z_2 oscillates around zero and the average relative population of Z_2 is zero too. This indicates that the atoms tunnelling takes place among three wells and finally most of atoms are trapped in the middle well but the numbers of atoms in the other two wells are still equal. We also find that, as A decreases, the oscillation amplitudes of Z_1 and Z_2 increase and resonance phenomenon also takes place.

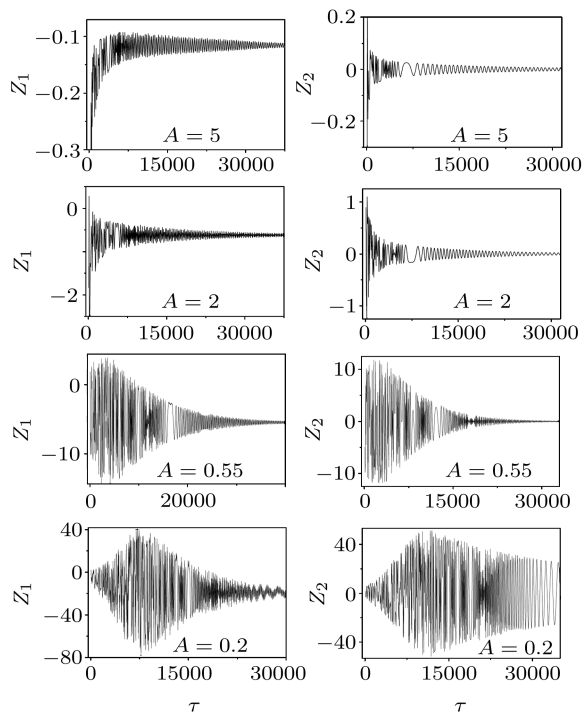


Fig.4. The time evolutions of relative populations Z_1 and Z_2 for different values of s-wave scattering length A: (a) 5, (b) 2, (c) 0.55, and (d) 0.2.

5. Conclusions

We have analysed the effects of three-body recombination and atoms feeding from thermal cloud on a triple-well trapped condensates. Our approach is based on a simplified three-mode approximation. The stationary and non-stationary features of the system have been discussed analytically and numerically. At first, we identify the stability of the stationary solutions and find that the effect of initial phase on the stability of stationary solutions is important. Furthermore, the non-stationary tunnelling of atoms among three wells and the self-trapping state in one well are revealed numerically. It is shown that with the decrease of scattering length A, the tunnelling of atoms between wells presents a highly asymmetric character and the oscillation of relative population manifests the quantum beating and resonance phenomenon.

It is well known that self-trapping is caused by nonlinear interactions between atoms. The BEC atoms in a symmetric double-well potential show a highly asymmetric distribution as if most atoms are trapped in one well. But for triple-well trapped BECs, the atoms can be trapped in any well and the system finally reaches a self-trapping state. Furthermore, we have observed the quantum beating phenomenon and the resonance character during the self-trapping and quantum tunnelling, but these behaviours do not emerge in a double-well system.

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