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Cooper-pair number-phase quantization analysis in double-Josephson-junction mesoscopic circuit coupled by a capacitor*

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This paper examines the quantization of mesoscopic circuit including Josephson junctions. Following Feynman's assumption, via the Hamilton dynamic approach and by virtue of the entangled state representation, it constructs Hamiltonian operator for the double-Josephson-junction mesoscopic circuit coupled by a capacitor. Then it uses the Heisenberg equation of motion to derive the induction voltage across each Josephson junction. The result manifestly shows how the voltage is affected by the capacitance coupling.

Keywords: Josephson-junction, Cooper-pair, entangled state representation, phase squeezing
PACC: 7335, 0367

1. Introduction

In recent years the miniaturization of integrated circuits tends to the atomic-scale dimension gradually, so much attention has been focused on mesoscopic physics because of the growth of nanometre techniques and rapid development of microelectronics.[1−5] Mesoscopic devices with typical size 0.1μm at low temperature interacting with non-classical microwaves exhibit interesting quantum phenomena.[6] Thus the quantum effects must be taken into account in mesoscopic circuits, and a correct quantum theory is necessary for the device physics in integrated circuits of nanoelectronics. In the mesoscopic scale, the circuit must be quantized and the pioneer of taking quantization of a simple LC mesoscopic circuit (L stands for the inductance, and C, the capacitance) is Louisell.[7] In Refs.[8, 9] by noticing the Coulomb blockade effect arising from the discreteness of electric charge, Li and Chen proposed a quantum theory of electric circuits which is based on the fact that electronic charge takes discrete values. In Ref.[10] based on finding the correct explicit form of charge ascending and lowering operators we proposed a concise quantization scheme for mesoscopic circuits which is in accord with the discreteness of electric charge. The charge eigenstates are derived from the appropriate entangled state representation.[11,12] When a mesoscopic circuit includes Josephson junctions,[13] charge qubit can be achieved in a Cooper-pair island. Then the quantization theory for Josephson junction circuits is needed. At this point, we mention that Vourdas is the first who pointed out that the phase difference across the junction should be considered as an operator.[14] In Ref.[15] following Feynman's assumption[16] that "Since electron pairs are bosons, when there are a lot of them in a given state there is an especially large amplitude for other pairs to go to the same state. So nearly all of the pairs will be locked down at the lowest energy in exactly the same state", "A bound pair would act as a Bose particle", and by analogy with the number-difference-phase theory in quantum optics, we have introduced nonlinear Bose operator to represent the phase across Josephson junction, and built a bosonic operator Hamiltonian model, which established itself firmly on the ground of entangled state representation. The operator version of Josephson equations for a single Josephson junction has also been obtained.

In this work we consider quantization of electric

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circuit including Josephson junction, to be concrete, we shall make Cooper-pair number-phase quantization analysis for a double Josephson-junction circuit coupled by a capacitor, since for designing quantum computers, miscellaneous compositions of elementary circuit blocks including Josephson junction should be studied. In this particular circuit, there exists an elementary two-Josephson-junction structure, i.e. two Josephson charge qubits structure are coupled by a capacitor $C_{\rm c}$. In each single-qubit structure, the Cooper-pair island is connected to a gate voltage $V_{\rm i}$ (used as external control parameter) through a gate capacitor $C_{{\rm g}i}$, $i = 1, 2$. We want to investigate how the voltage across each Josephson junction changes due to the existence of coupling. For this aim, we hope to propose a bosonic operator model for describing the double Josephson-junction circuit coupled by a capacitor, i.e. construct a quantum mechanical Hamiltonian, then we use the entangled state representation to propose Cooper-pair number-phase quantization scheme. Further, we can use the Heisenberg equation of motion to re-derive two Josephson equations: the current equation and the induction voltage equation across each Josephson junction. The result manifestly shows the role of the capacitance coupling between the two qubits structures. Before deriving this quantization scheme we should start with classical Hamilton dynamics, i.e. consider the mesoscopic circuit drawn in Fig.1 as a dynamic Hamilton system.

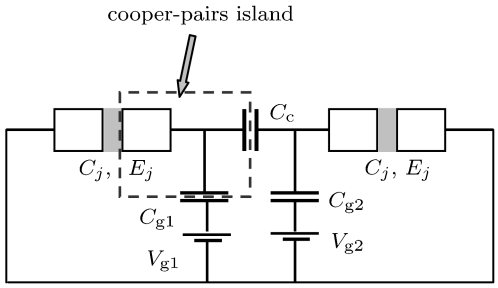


Fig.1. The mesoscopic circuit with two Josephson junctions.

2. Hamilton analysis of double-Josephson-junction mesoscopic circuit coupled by a capacitor

A single Josephson junction[17] is characterized by two Josephson equations (the current equation and the induction voltage equation)

$$I_i = I_{\rm c}\sin\varphi_i, \quad (i = 1, 2), \tag{1}$$

$$\dot{\varphi}_i = \frac{2eu_i}{\hbar}, \tag{2}$$

where $I_{\rm c} = 2eE_j/\hbar$ is the critical electric current of the Josephson junction, u_i is the voltage drop across the ith Josephson junction, and φ_i is the phase difference between the two wave functions across the ith Josephson junction. So when tunnelling happens, the work done across the junction is

$$\int_0^t u_i I_{\rm c}\sin\varphi_i {\rm d}t = E_j(\cos\varphi_i - 1). \tag{3}$$

We employ the Hamilton dynamics to analyze the circuit drawn in Fig.1 by defining φ_1 and φ_2 as generalized coordinates, the Cooper-pair's potential energy is

$$v = \sum_{i=1}^{2} E_j(1 - \cos\varphi_i), \tag{4}$$

on the other hand, the charging energy in all the capacitors is

$$T = \sum_{i=1}^{2}\left(\frac{1}{2}C_i u_i^2 + \frac{1}{2}C_{{\rm g}i}u_{{\rm g}i}^2\right) + \frac{1}{2}C_{\rm c}u_{\rm c}^2, \tag{5}$$

where C_i is the capacitance of ith junction, T is considered as the kinetic energy, since from Eq.(2) one can see that u_i is related to the generalized velocity $\dot{\varphi}_i$, and from the simple analysis of voltage at the nodes of the island we see

$$u_{{\rm g}i} = u_i - V_i, \quad u_{\rm c} = u_1 - u_2. \tag{6}$$

Substituting Eqs.(2) and (6) into Eq.(5) we have

$$T = \sum_{i=1}^{2}\left[\frac{1}{2}C_{\Sigma i}\left(\frac{\hbar}{2e}\right)^2\dot{\varphi}_i^2 + \hbar N_{{\rm g}i}\dot{\varphi}_i + \frac{2N_{{\rm g}i}^2e^2}{C_{{\rm g}i}}\right] - C_{\rm c}\left(\frac{\hbar}{2e}\right)^2\dot{\varphi}_1\dot{\varphi}_2, \tag{7}$$

where $N_{{\rm g}i} = -\dfrac{C_{{\rm g}i}V_i}{2e}$ is named as gate charge number (usually, when $N_{{\rm g}i} \cong 1$, the Cooper-pairs island is referred as a Josephson charge qubit) and

$$C_{\Sigma i} = C_i + C_{\rm c} + C_{{\rm g}i}. \tag{8}$$

Then the Lagrangian function of the system is

$$\ell = T - v = \sum_{i=1}^{2}\left[\frac{1}{2}C_{\Sigma i}\left(\frac{\hbar}{2e}\right)^2\dot{\varphi}_i^2 + \hbar N_{{\rm g}i}\dot{\varphi}_i + \frac{2N_{{\rm g}i}^2e^2}{C_{{\rm g}i}} - E_j(1 - \cos\varphi_i)\right] - C_{\rm c}\left(\frac{\hbar}{2e}\right)^2\dot{\varphi}_1\dot{\varphi}_2. \tag{9}$$

The tunnelling through the Josephson junction causes Cooper-pairs to land the island. The excess charge $2en_1$ of the first island, which can be quantized, is determined by the number n_1 of excess Cooper-pairs. Analysing some nodes on the first island, one can obtain the following relation: $u_1C_1 - 2en_1 = -(u_{\rm g1}C_{\rm g1} + u_{\rm c}C_{\rm c})$, thus from Eqs.(2) and (6) we have

$$\begin{aligned} 2en_1 =& u_{\rm g1}C_{\rm g1} + u_{\rm c}C_{\rm c} + u_1C_1 \\ =& C_{\rm g1}\frac{\hbar}{2e}\dot{\varphi}_1 + 2eN_{\rm g1} + C_{\rm c}\frac{\hbar}{2e}(\dot{\varphi}_1 - \dot{\varphi}_2) + C_1\frac{\hbar}{2e}\dot{\varphi}_1 \\ =& \frac{\hbar}{2e}(C_{\sum 1}\dot{\varphi}_1 - C_{\rm c}\dot{\varphi}_2) + 2eN_{\rm g1}. \end{aligned} \tag{10}$$

Note that for the second Cooper-pair island the excess charge is

$$\begin{aligned} 2en_2 =& u_{\rm g2}C_{\rm g1} - u_{\rm c}C_{\rm c} + u_2C_2 \\ =& \frac{\hbar}{2e}(C_{\sum 2}\dot{\varphi}_2 - C_{\rm c}\dot{\varphi}_1) + 2eN_{\rm g2}. \end{aligned} \tag{11}$$

From Eq.(9) we calculate the generalized momenta conjugated to φ_1 and φ_2,

$$\begin{aligned} p_1 \equiv \frac{\partial \ell}{\partial \dot{\varphi}_1} =& \frac{\hbar}{2e}\left[\frac{\hbar}{2e}(C_{\Sigma 1}\dot{\varphi}_1 - C_{\rm c}\dot{\varphi}_2) + 2N_{\rm g1}e\right] \\ =& n_1\hbar, \end{aligned} \tag{12}$$

$$\begin{aligned} p_2 \equiv \frac{\partial \ell}{\partial \dot{\varphi}_2} =& \frac{\hbar}{2e}\left[\frac{\hbar}{2e}(C_{\Sigma 2}\dot{\varphi}_2 - C_{\rm c}\dot{\varphi}_1) + 2N_{\rm g2}e\right] \\ =& n_2\hbar, \end{aligned} \tag{13}$$

and it turns out that the generalized momentum p_i is proportional to the number n_i of Cooper-pairs, which implies the possibility of number-phase quantization, as we will do in the next section. From Eqs.(10) and (11) we can express φ_i by n_i

$$\dot{\varphi}_1 = \left(\frac{\hbar}{2e}\right)^2 \frac{C_{\sum 2}(n_1 - N_{\rm g1})\hbar + C_{\rm c}(n_2 - N_{\rm g2})\hbar}{C_{\sum 1}C_{\sum 2} - C_{\rm c}^2}, \tag{14}$$

$$\dot{\varphi}_2 = \left(\frac{\hbar}{2e}\right)^2 \frac{C_{\sum 1}(n_2 - N_{\rm g2})\hbar + C_{\rm c}(n_1 - N_{\rm g1})\hbar}{C_{\sum 1}C_{\sum 2} - C_{\rm c}^2}. \tag{15}$$

Thus, using Legendre transformation and Eqs.(14) and (15) we can obtain the Hamiltonian of the system

$$\begin{aligned} H =& p_1\dot{\varphi}_1 + p_2\dot{\varphi}_2 - \ell \\ =& n_1\hbar\dot{\varphi}_1 + n_2\hbar\dot{\varphi}_2 - \ell \\ \equiv& H_1 + H_2 + H_{\rm int}, \end{aligned} \tag{16}$$

where

$$\begin{aligned} H_i =& E_{\rm c}^{(i)}(n_i - N_{{\rm g}i})^2 + E_j(1 - \cos\varphi_i) \\ & - 2N_{{\rm g}i}^2e^2/C_{{\rm g}i} \end{aligned} \tag{17}$$

is the effective Hamiltonian of the single qubit structure, and

$$H_{\rm int} = E_{\rm c,c}(n_1 - N_{\rm g1})(n_2 - N_{\rm g2}) \tag{18}$$

is the effective interacting Hamiltonian, where

$$E_{\rm c}^{(i)} = \frac{(2e)^2}{2C_{\rm eff}^{(i)}}, \quad E_{\rm c,c} = \frac{(2e)^2}{2C_{\rm c,eff}}, \tag{19}$$

$$\begin{aligned} & C_{\rm eff}^{(1)} = \frac{W}{C_{\sum 2}}, \quad C_{\rm eff}^{(2)} = \frac{W}{C_{\sum 1}}, \\ & C_{\rm c,eff} = \frac{W}{C_{\rm c}}, \quad W \equiv C_{\sum 1}C_{\sum 2} - C_{\rm c}^2, \end{aligned} \tag{20}$$

and $E_{\rm c}^{(i)} = \dfrac{(2e)^2}{2C_{\rm eff}^{(i)}}$ is the single Cooper-pair charging energy on the ith island.

3. Quantization of the double-Josephson-junction circuit

According to Feynman's assumption[16] that "A bound pair acts as a Bose particle", we are naturally led to provide the bosonic operator model to quantize the classical Hamiltonian given by Eq.(16). As we mentioned in the above section that the excess charge $2en_i$ should be quantized, so we let

$$n_i \to \hat{n}_i \equiv \hat{a}_i^+\hat{a}_i - \hat{b}_i^+\hat{b}_i \tag{21}$$

be Cooper-pair number operator in each island, here, $\hat{a}_i^+$ and $\hat{b}_i^+$ are, respectively, Bose positive particle creation operator and Bose antiparticle creation operator. To embody the number-phase quantization scheme we introduce the entangled state $|\eta\rangle_i$[11,12]

$$|\eta\rangle_i = \exp\left[-\frac{1}{2}|\eta_i|^2 + \eta_i\hat{a}_i^+ - \eta_i^*\hat{b}_i^+ + \hat{a}_i^+\hat{b}_i^+\right]|00\rangle_i, \tag{22}$$

where $\eta_i = |\eta_i|\,{\rm e}^{{\rm i}\varphi_i}$, $|00\rangle_i$ is the two-mode vacuum state. The entangled state $|\eta\rangle_i$ is constructed based on the idea of quantum entanglement of Einstein, Podolsky and Rosen.[18] Using $[\hat{a}_i, \hat{a}_i^+] = [\hat{b}_i, \hat{b}_i^+] = 1$, we see that $|\eta\rangle_i$ obeys the eigenvector equations

$$\begin{aligned} & (\hat{a}_i - \hat{b}_i^+)\,|\eta\rangle_i = \eta_i\,|\eta\rangle_i \\ & (\hat{b}_i - \hat{a}_i^+)\,|\eta\rangle_i = -\eta_i^*\,|\eta\rangle_i\,. \end{aligned} \tag{23}$$

The set of $|\eta\rangle_i$ makes up a complete representation

$$\int \frac{{\rm d}^2\eta_i}{\pi}\,|\eta\rangle_{i\,i}\,\langle\eta| = 1. \tag{24}$$

We can construct the bosonic phase operator for Josephson junction[10,15]

$$\begin{aligned} \mathrm{e}^{\mathrm{i}\hat{\Phi}_i} &= \sqrt{\frac{\hat{a}_i - \hat{b}_i^+}{\hat{a}_i^+ - \hat{b}_i}}, \\ \mathrm{e}^{-\mathrm{i}\hat{\Phi}_i} &= \sqrt{\frac{\hat{a}_i^+ - \hat{b}_i}{\hat{a}_i - \hat{b}_i^+}}, \\ \cos\hat{\Phi}_i &= \frac{1}{2}(\mathrm{e}^{\mathrm{i}\hat{\Phi}_i} + \mathrm{e}^{-\mathrm{i}\hat{\Phi}_i}), \end{aligned} \tag{25}$$

because in the $|\eta\rangle_i$ representation, $\mathrm{e}^{\mathrm{i}\hat{\Phi}_i}$ behaves as a phase

$$\mathrm{e}^{\mathrm{i}\hat{\Phi}_i}|\eta\rangle_i = \mathrm{e}^{\mathrm{i}\varphi_i}|\eta\rangle_i, \quad \mathrm{e}^{-\mathrm{i}\hat{\Phi}_i}|\eta\rangle_i = \mathrm{e}^{-\mathrm{i}\varphi_i}|\eta\rangle_i. \tag{26}$$

Noting $[\hat{a}_i^+ - \hat{b}_i, \hat{a}_i - \hat{b}_i^+] = 0$, so we know that they can reside in the same $\sqrt{\ }$. It then follows $\hat{\Phi}_i = \frac{1}{2\mathrm{i}}\ln\frac{\hat{a}_i - \hat{b}_i^+}{\hat{a}_i^+ - \hat{b}_i}$, $\hat{\Phi}_i|\eta\rangle_i = \varphi_i|\eta\rangle_i$. From Eq.(23) we can derive

$$\begin{aligned} \hat{n}_i|\eta\rangle_i \equiv& (\hat{a}_i^+\hat{a} - \hat{b}_i^+\hat{b}_i)|\eta\rangle_i \\ =& [\hat{a}_i^+(\eta_i + \hat{b}_i^+) - \hat{b}_i^+(\hat{a}_i^+ - \eta_i^*)]|\eta\rangle_i \\ =& |\eta_i|(\hat{a}_i^+\mathrm{e}^{\mathrm{i}\varphi_i} + \hat{b}_i^+\mathrm{e}^{-\mathrm{i}\varphi_i})|\eta\rangle_i \\ =& -\mathrm{i}\frac{\partial}{\partial\varphi_i}|\eta\rangle_i. \end{aligned} \tag{27}$$

So

$$\begin{aligned} {}_i\langle\eta|[\hat{\Phi}_i, \hat{n}_i] =& \left[\varphi_i, \mathrm{i}\frac{\partial}{\partial\varphi_i}\right]{}_i\langle\eta| \\ =& -\mathrm{i}_i\langle\eta| \to [\hat{\Phi}_i, \hat{n}_i] = -\mathrm{i}, \\ [\cos\hat{\Phi}_i, \hat{n}_i] =& \mathrm{i}\sin\hat{\Phi}_i, \\ [\sin\hat{\Phi}_i, \hat{n}_i] =& -\mathrm{i}\cos\hat{\Phi}_i, \end{aligned} \tag{28}$$

which embodies number-phase quantization. As a consequence of Eq.(28), the classical Hamiltonian given by Eq.(16) is quantized as

$$\begin{aligned} \hat{H}_i =& E_{\mathrm{c}}^{(i)}(\hat{n}_i - N_{\mathrm{g}i})^2 + E_j(1 - \cos\hat{\Phi}_i) \\ & - 2N_{\mathrm{g}i}^2 e^2/C_{\mathrm{g}i}, \end{aligned} \tag{29}$$

$$\hat{H}_{\mathrm{int}} = E_{\mathrm{c,c}}(\hat{n}_1 - N_{\mathrm{g}1})(\hat{n}_2 - N_{\mathrm{g}2}), \tag{30}$$

where we have quantitized φ_i as $\hat{\Phi}_i$. At the end of this section we mention that in quantum optics theory the phase operator for describing eight-port interferometer was proposed by Hradil.[19]

4. Operator Josephson equations

From the Heisenberg equation and Eqs.(28)–(30) we derive

$$2e\frac{\mathrm{d}}{\mathrm{d}t}\hat{n}_i = \frac{2e}{i\hbar}[\hat{n}_i, \hat{H}_i + \hat{H}_{\mathrm{int}}] = \frac{2eE_j}{\hbar}\sin\hat{\Phi}_i \equiv \hat{I}_i, \tag{31}$$

and using Eq.(15) we deduce

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\hat{\Phi}_1 =& \frac{1}{\hbar}\left[\frac{(2e)^2}{C_{\mathrm{eff}}^{(1)}}(\hat{n}_1 - N_{\mathrm{g}1}) + \frac{(2e)^2}{2C_{\mathrm{c,eff}}}(\hat{n}_2 - N_{\mathrm{g}2})\right] \\ \equiv& \frac{2e}{\hbar}\hat{U}_1. \end{aligned} \tag{32}$$

Comparing Eq.(32) with $\dot{\varphi}_i = 2eu_i/\hbar$ given by Eq.(2), and using Eq.(20) we see that the effective voltage drop across the first junction is

$$\begin{aligned} \hat{U}_1 =& \frac{2e}{C_{\mathrm{eff}}^{(1)}}(\hat{n}_1 - N_{\mathrm{g}1}) + \frac{2e}{2C_{\mathrm{c,eff}}}(\hat{n}_2 - N_{\mathrm{g}2}) \\ =& \frac{2e}{W}\left[C_{\sum 2}(\hat{n}_1 - N_{\mathrm{g}1}) + \frac{C_{\mathrm{c}}}{2}(\hat{n}_2 - N_{\mathrm{g}2})\right]. \end{aligned} \tag{33}$$

So the excess number of Cooper-pairs in the second island affects the effective voltage of the first junction, and this effect is proportional to the coupling capacitance C_{c}/W. It tells us that the two Josephson circuits should be examined in its totality. Similarly, we have

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\hat{\Phi}_2 =& \frac{1}{\hbar}\left[\frac{(2e)^2}{C_{\mathrm{eff}}^{(1)}}(\hat{n}_2 - N_{\mathrm{g}2}) + \frac{(2e)^2}{2C_{\mathrm{c,eff}}}(\hat{n}_1 - N_{\mathrm{g}1})\right] \\ \equiv& \frac{2e}{\hbar}U_2, \end{aligned} \tag{34}$$

and

$$\begin{aligned} \hat{U}_2 =& \frac{2e}{C_{\mathrm{eff}}^{(1)}}(\hat{n}_2 - N_{\mathrm{g}2}) + \frac{2e}{2C_{\mathrm{c,eff}}}(\hat{n}_1 - N_{\mathrm{g}1}) \\ =& \frac{2e}{W}\left[C_{\sum 1}(\hat{n}_2 - N_{\mathrm{g}2}) + \frac{C_{\mathrm{c}}}{2}(\hat{n}_1 - N_{\mathrm{g}1})\right]. \end{aligned} \tag{35}$$

Consequently, the work performed by the Josephson current in a time interval Δt through the first junction is

$$\begin{aligned} \hat{U}_1\hat{I}_1\Delta t =& \Delta t\frac{(2e)^2E_j}{\hbar W}\sin\hat{\Phi}_1\left[C_{\sum 2}(\hat{n}_1 - N_{\mathrm{g}1})\right. \\ & \left. + \frac{C_{\mathrm{c}}}{2}(\hat{n}_2 - N_{\mathrm{g}2})\right]. \end{aligned} \tag{36}$$

In the next section we examine how the light radiation applied to the first Josephson junction affects the second junction.

5. Phase squeezing

When an extra energy (say, light radiation) is applied to the first Josephson junction, in reference to Eq.(36) we can take the form of corresponding Hamiltonian as follows:

$$\hat{H}' = \frac{\lambda}{W}\sin\hat{\Phi}_1\left[C_{\sum 2}(\hat{n}_1 - N_{\mathrm{g}1}) + \frac{C_{\mathrm{c}}}{2}(\hat{n}_2 - N_{\mathrm{g}2})\right], \tag{37}$$

where λ is a coupling constant, and the constant term $\frac{(2e)^2 E_j}{\hbar}$ is absorbed into λ. Taking $\hat{H}'$ in the interaction picture and according to the Heisenberg equation of motion in this picture, we derive

$$\frac{\mathrm{d}}{\mathrm{d}t}\sin\hat{\Phi}_1 = \frac{1}{\mathrm{i}\hbar}[\sin\hat{\Phi}_1, \hat{H}'] = -\frac{\lambda C_{\sum 2}}{2\hbar W}\sin(2\hat{\Phi}_1), \quad (38)$$

and

$$\frac{\mathrm{d}}{\mathrm{d}t}\cos\hat{\Phi}_1 = \frac{1}{\mathrm{i}\hbar}[\cos\hat{\Phi}_1, \hat{H}'] = \frac{\lambda C_{\sum 2}}{\hbar W}\sin^2\hat{\Phi}_1. \quad (39)$$

It then follows

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\tan\frac{\hat{\Phi}_1}{2} =& \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1-\cos\hat{\Phi}_1}{\sin\hat{\Phi}_1}\right)\\ =& -\frac{\lambda C_{\sum 2}}{\hbar W}\tan\frac{\hat{\Phi}_1}{2}, \end{aligned} \quad (40)$$

which has the solution

$$\tan\frac{\hat{\Phi}_1}{2} = \exp\left\{-\frac{\lambda C_{\sum 2}}{\hbar W}t\right\}\tan\frac{\hat{\Phi}_1(0)}{2}. \quad (41)$$

This solution shows that the squeezing of $\tan\frac{\hat{\Phi}_1}{2}$ with a damping factor $\exp\left\{-\frac{\lambda C_{\sum 2}}{\hbar W}t\right\}$. (For a review of squeezed states we refer to Ref.[20]). Since the Josephson current is caused by the phase-difference between two superconductors, and since the two Josephson junctions should be considered in its totality due to the capacitor C_{c}, the squeezing of $\tan\frac{\hat{\Phi}_1}{2}$, of course, accompanies the variation of number of Cooper pairs, both in the first island and in the second island. For the second junction, though it is not directly radiated by light, we still have

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\sin\hat{\Phi}_2 =& \frac{1}{\mathrm{i}\hbar}[\sin\hat{\Phi}_2, \hat{H}']\\ =& -\frac{\lambda C_{\mathrm{c}}}{2\hbar W}\sin\hat{\Phi}_1\cos\hat{\Phi}_2, \end{aligned} \quad (42)$$

so

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\Phi}_2 = -\frac{\lambda}{\hbar W}\frac{C_{\mathrm{c}}}{2}\sin\hat{\Phi}_1, \quad (43)$$

which shows how the phase difference operator $\hat{\Phi}_2$ in the second junction is affected by phase difference operator $\hat{\Phi}_1$ in the first junction.

In summary, via the Hamilton dynamic approach we have constructed a Hamiltonian operator for double-Josephson-junction mesoscopic circuit coupled by a capacitor; then by virtue of the Heisenberg equation we have shown how the capacitance coupling affects the induction voltage across each Josephson junction. The entangled state representation is underlying for proposing Cooper-pair number-phase quantization and the Hamiltonian operator for the whole system. Since the Heisenberg approach is concise, our analysis in this work may be extended to tackle other complicated mesoscopic circuits involving Josephson junctions.

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