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The entropic squeezing of superposition of two arbitrary coherent states*

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In this paper the superpositions of two arbitrary coherent states $|\psi\rangle = a|\beta\rangle + b\mathrm{e}^{\mathrm{i}\varphi}|m\beta\mathrm{e}^{\mathrm{i}\delta}\rangle$ are constructed by using the superposition principle of quantum mechanics. The entropic squeezing effects of the quantum states are studied. The numerical results indicate that the amplitudes, the ratio between the amplitudes of two coherent states, the phase difference between the two components and the relative phase of the two coefficients play important roles in the squeezing effects of the position entropy and momentum entropy.

Keywords: coherent state, superposition state, entropy squeezing
PACC: 4250, 0365

1. Introduction

Since coherent state was introduced by Glauber[1] in 1963, the theory of coherent states has been applied to solve mathematical problems in quantum electrodynamics and quantum optics. Currently the coherent state theory and its application have become an important research field of physics.[2] Meanwhile linear and nonlinear superpositions of coherent states have become an important means for preparing quantum states with different quantum properties.[3−7] There are many superposition ways of coherent states. The most basic form is the superposition of two single-mode coherent states. Odd and even coherent states are particular superpositions of coherent states. Odd coherent state has an anti-bunching effect, while even coherent state has a squeezing effect.[8] Schleich *et al*[9] studied the superposition of two coherent states with the different phases but same mean photon number, the obtained superposition state also exhibits squeezing effect. The squeezing effect has practical applications in the fields of high-precision optical measurement, ultra low noise communications, weak signal detection and quantum communication.[10,11] Therefore the squeezing effect of the field has attracted much attention. In this study of the optical squeezing effect, the standard deviation of one quadrature is generally used to measure the quantum fluctuation in the optical field. If the fluctuation in the quadrature is below the vacuum limit there is squeezing effect.[12] Since the standard deviation of the optical density matrix includes only second order statistical matrix but not the high-order statistical matrix message, it is thus unable to accurately measure the quantum fluctuations. However, the information entropy includes the high-order statistical matrix of the density matrix,[13,14] and is therefore better used to measure the quantum fluctuations.

In this paper,the superpositions of two arbitrary coherent states $|\psi\rangle = a|\beta\rangle + b\mathrm{e}^{\mathrm{i}\varphi}|m\beta\mathrm{e}^{\mathrm{i}\delta}\rangle$ are constructed, where m indicates the ratio between the amplitudes of two coherent states, δ is the phase difference of two coherent states, and φ is the relative phase of the two coefficients. Odd coherent states,even coherent states, and the superposition state $N(|\alpha\mathrm{e}^{\mathrm{i}\varphi}\rangle + |\alpha\mathrm{e}^{-\mathrm{i}\varphi}\rangle$ that Schleich studied are particular forms of the superpositions studied in the present paper. The superpositions studied in the present paper can be extended to different places with different phase, different mean photon number, and different superposition coefficients and so on, so it has important universal significance. The entropic squeezing effects of the quantum states are studied. The numerical results show that the amplitudes of the coherent states, the phase difference between the two components and the relative phase of the two coefficients play important roles in the squeezing effects of the position entropy and momentum entropy.

This paper is organized as follows. In Section 2, the superpositions of two arbitrary coherent states are

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constructed. The corresponding position entropy and momentum entropy is obtained. In Section 3, the influences of various parameters on the squeezing effects of position entropy and momentum entropy are discussed. Section 4 is devoted to a short summary of the main results of this paper.

2. The position entropy and momentum entropy of the superposition states

According to the superposition principle of quantum mechanics, the superposition of two arbitrary coherent states can be described by the state vector

$$|\psi\rangle = a\,|\beta\rangle + b\mathrm{e}^{\mathrm{i}\varphi}\left|m\beta\mathrm{e}^{\mathrm{i}\delta}\right\rangle. \tag{1}$$

In Eq.(1), a and b satisfy normalization condition

$$a^2 + b^2 + 2ab\exp\left(|\beta|^2\left(m\cos\delta - \frac{m^2+1}{2}\right)\right) \times \cos(\varphi + m\,|\beta|^2\sin\delta) = 1. \tag{2}$$

In order to satisfy the normalization condition we set

$$b = -r + \sqrt{r^2 + 1 - a^2},$$

where $r = a\exp(t)\cos(\varphi + m\,|\beta|^2\sin\delta)$, $t = (m\cos\delta - (1+m^2)/2)\,|\beta|^2$.

In the Fock representation, the position eigen state $|x\rangle$ and the momentum eigen state $|p\rangle$ can be written as

$$\begin{aligned}|x\rangle &= \pi^{-1/4}\exp[-x^2/2 + \sqrt{2}x\hat{a}^+ - \hat{a}^{+2}/2]\,|0\rangle,\\ |p\rangle &= \pi^{-1/4}\exp[-p^2/2 + \sqrt{2}\mathrm{i}p\hat{a}^+ - \hat{a}^{+2}/2]\,|0\rangle,\end{aligned} \tag{3}$$

where $\hat{a}$ and $\hat{a}^+$ are the annihilation and creation operators respectively.

In the position and momentum representations, the superposition states can be written as

$$\begin{aligned}\langle x|\beta\rangle &= \pi^{-1/4}\exp\left[-\frac{1}{2}(x-x_0)^2\right]\exp\left(-\frac{\mathrm{i}}{2}p_0x_0 + \mathrm{i}p_0x\right),\\ \langle p|\beta\rangle &= \pi^{-1/4}\exp\left[-\frac{1}{2}(p-p_0)^2\right]\exp\left(\frac{\mathrm{i}}{2}p_0x_0 - \mathrm{i}x_0p\right),\\ \left\langle x|m\beta\mathrm{e}^{\mathrm{i}\delta}\right\rangle &= \pi^{-1/4}\exp\left[-\frac{1}{2}(x-x_1)^2\right]\exp\left(-\frac{\mathrm{i}}{2}p_1x_1 + \mathrm{i}p_1x\right),\\ \left\langle p|m\beta\mathrm{e}^{\mathrm{i}\delta}\right\rangle &= \pi^{-1/4}\exp\left[-\frac{1}{2}(p-p_1)^2\right]\exp\left(-\frac{\mathrm{i}}{2}p_1x_1 + \mathrm{i}x_1p\right).\end{aligned} \tag{4}$$

The density operator of superposition state is $\rho = |\psi\rangle\langle\psi|$. The diagonal elements of the density matrix in the position representation and momentum representation are

$$\begin{aligned}\langle x|\,\rho\,|x\rangle &= a^2\pi^{-1/2}\exp[-(x-x_0)^2] + b^2\pi^{-1/2}\exp[-(x-x_1)^2]\\ &\quad + 2ab\pi^{-1/2}\exp\left[-\frac{1}{2}((x-x_0)^2 + (x-x_1)^2)\right]\cos\left[(p_0-p_1)x - \frac{1}{2}(p_0x_0 - p_1x_1) - \varphi\right],\\ \langle p|\,\rho\,|p\rangle &= a^2\pi^{-1/2}\exp[-(p-p_0)^2] + b^2\pi^{-1/2}\exp[-(p-p_1)^2]\\ &\quad + 2ab\pi^{-1/2}\exp\left[-\frac{1}{2}((p-p_0)^2 + (p-p_1)^2)\right]\cos\left[(x_0-x_1)p - \frac{1}{2}(p_0x_0 - p_1x_1) + \varphi\right].\end{aligned} \tag{5}$$

Where

$$\begin{aligned}x_0 &= \sqrt{2}m\,|\beta|\cos\alpha,\\ p_0 &= \sqrt{2}m\,|\beta|\sin\alpha\\ x_1 &= \sqrt{2}m\,|\beta|\cos(\alpha+\delta),\\ p_1 &= \sqrt{2}m\,|\beta|\sin(\alpha+\delta).\end{aligned}$$

The parameters $|\beta|$ and α are given by $\beta = |\beta|\,\mathrm{e}^{\mathrm{i}\alpha}$. According to the definition of the position entropy and momentum entropy in optical field,

$$\begin{aligned}s_x &= -\int_{-\infty}^{\infty}\langle x\,|\rho|\,x\rangle\ln\langle x\,|\rho|\,x\rangle\mathrm{d}x,\\ s_p &= -\int_{-\infty}^{\infty}\langle p\,|\rho|\,p\rangle\ln\langle p\,|\rho|\,p\rangle\mathrm{d}p.\end{aligned} \tag{6}$$

Applying Eq.(5) to Eq.(6), we should be able to cal-

culate the position entropy and momentum entropy of the superposition states.

3. The squeezing effect of position and momentum entropies in the superposition states

An alternative mathematical formulation of the uncertainty principle is provided by the inequality

$$\delta\hat{x}\delta\hat{p} \geq \pi e \quad (\hbar = 1). \tag{7}$$

Where

$$\delta\hat{x} = \exp(s_x), \quad \delta\hat{p} = \exp(s_p). \tag{8}$$

The position operator $\hat{x}$ and momentum operator $\hat{p}$ can be written as

$$\hat{x} = \frac{1}{\sqrt{2}}(\hat{a} + \hat{a}^+), \quad \hat{p} = \frac{1}{\sqrt{2}\mathrm{i}}(\hat{a} - \hat{a}^+). \tag{9}$$

Defining,

$$\delta X = \delta\hat{x} - \sqrt{\pi e}, \quad \delta P = \delta\hat{p} - \sqrt{\pi e}. \tag{10}$$

If $\delta X < 0$ ($\delta P < 0$), then the position (momentum) entropy is squeezed. In order to further study entropic squeezing effects of the superposition states, the entropic squeezing degree is defined as

$$X = \frac{\delta X}{\sqrt{\pi e}}, \quad P = \frac{\delta P}{\sqrt{\pi e}}. \tag{11}$$

The analytical solution of both the position entropy and momentum entropy of the superposition states are not easily solved. To overcome the problem we have to use numerical computations to analyse the variations of the entropic squeezing degree with the changes of $|\beta|$, m, δ and φ.

3.1. The influence of the ratio between the amplitudes of two coherent states on the entropic squeezing effect

In order to study the influence of the ratio between the amplitudes of two coherent states on the entropic squeezing effect, set $|\beta| = 0.5$ and 1.0 respectively, with $\varphi = 0, \delta = \pi/2, a = 0.7$, and m is either 0.6, 1.2, 1.8 or 2.4. The position entropy and momentum entropy as the functions of α are shown in Figs.1 and 2 below. Curves a, b, c and d in the figures represent the results when $m = 0.6$, 1.2, 1.8, 2.4, respectively. The numerical results show: (1) the squeezing ranges of both the position entropy and momentum entropy decrease when the ratio m increases; (2) the obtainable maximal squeezing may strengthen or weaken as m increases.

3.2. The influence of the amplitude $|\beta|$ on the entropic squeezing effect

In order to study the influence of the amplitude $|\beta|$ on the entropic squeezing effect, let $\delta = \pi/2$, $\varphi = 0$, $a = 0.7$, and $m = 0.6$ and 1.2 respectively, $|\beta|$ is either 0.5, 1.0, $\sqrt{2}$ or 2.0. The position entropy and momentum entropy as functions of α are shown in Figs.3 and 4 below. Curves a, b, c and d in the figures represent the results respectively when $|\beta| = 0.5$, 1.0, $\sqrt{2}$, 2.0. The numerical results show: (1) the squeezing ranges of both the position entropy and the momentum entropy decrease as the amplitude $|\beta|$ increases; (2) the squeezing depths can either strengthen or weaken as the amplitude $|\beta|$ increases.

3.3. The influence of phase difference δ of two coherent states on the entropic squeezing effect

The influences of the phase difference of two coherent states on the entropic squeezing effect are shown in Fig.5. The position entropy and momentum entropy as the functions of α are shown in the figure, where $m = 1.2$, $\varphi = 0$, $a = 0.7$, $|\beta| = 1.0$, and $\delta = \pi/6$, $\pi/3$, $\pi/2$, $2\pi/3$, π respectively. The numeric results indicate that there is a nonlinear relationship between the phase difference δ, the position entropy and momentum entropy. Both the position entropy and momentum entropy don't change with δ monotonically.

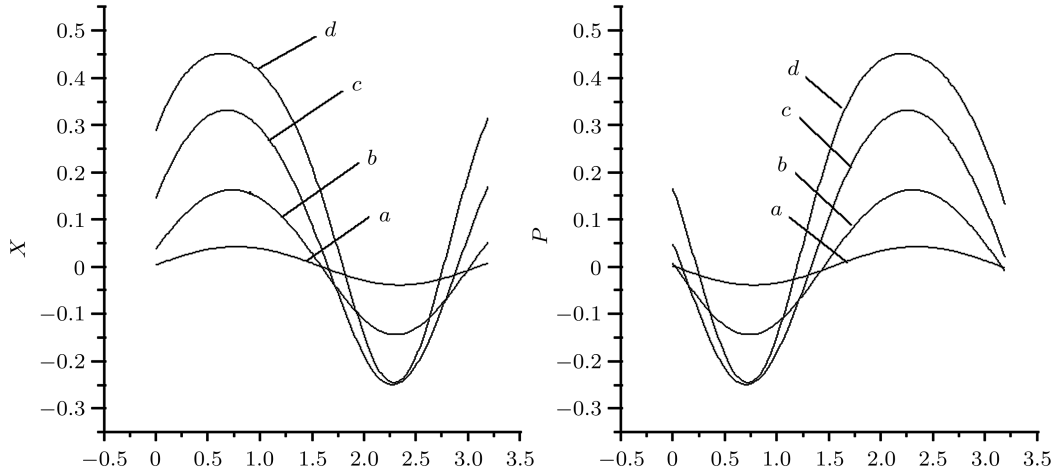


Fig.1. The entropic squeezing parameters X and P as the functions of α where. $|\beta| = 0.5,\ \varphi = 0,\ \delta = \pi/2,\ a = 0.7$ and $m = 0.6$ for Curve a; $m = 1.2$ for Curve b; $m = 1.8$ for Curve c; $m = 2.4$ for Curve d.

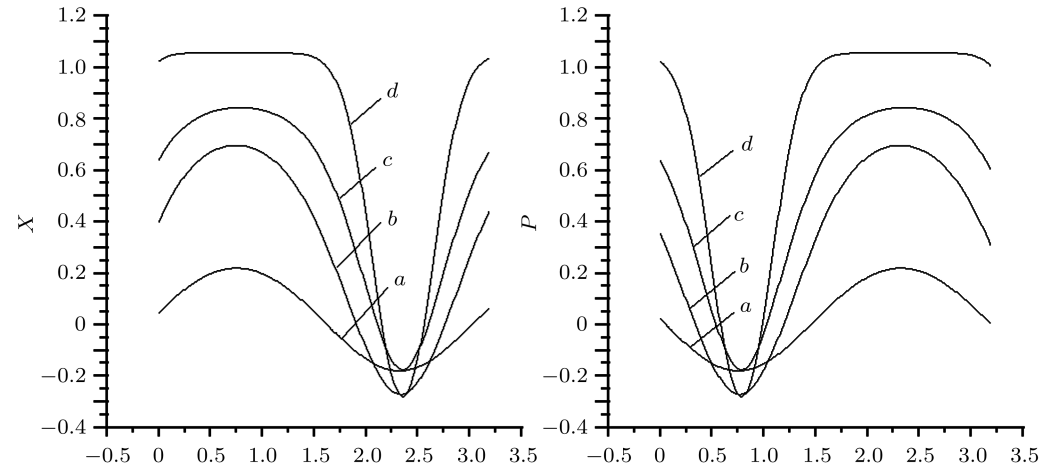


Fig.2. The entropic squeezing parameters X and P as the functions of α where. $|\beta| = 1.0,\ \varphi = 0,\ \delta = \pi/2,\ a = 0.7$ and $m = 0.6$ for Curve a; $m = 1.2$ for Curve b; $m = 1.8$ for Curve c; $m = 2.4$ for Curve d.

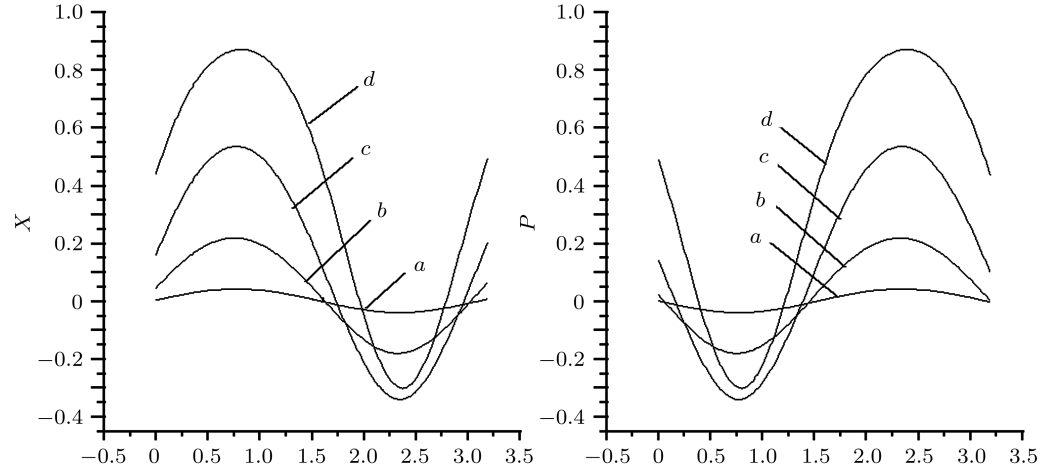


Fig.3. The entropic squeezing parameters X and P as the functions of α where. $m = 0.6,\ \varphi = 0,\ \delta = \pi/2,\ a = 0.7$ and $|\beta| = 0.5$ for Curve a; $|\beta| = 1.0$ for Curve b; $|\beta| = \sqrt{2}$ for Curve c; $|\beta| = 2.0$ for Curve d.

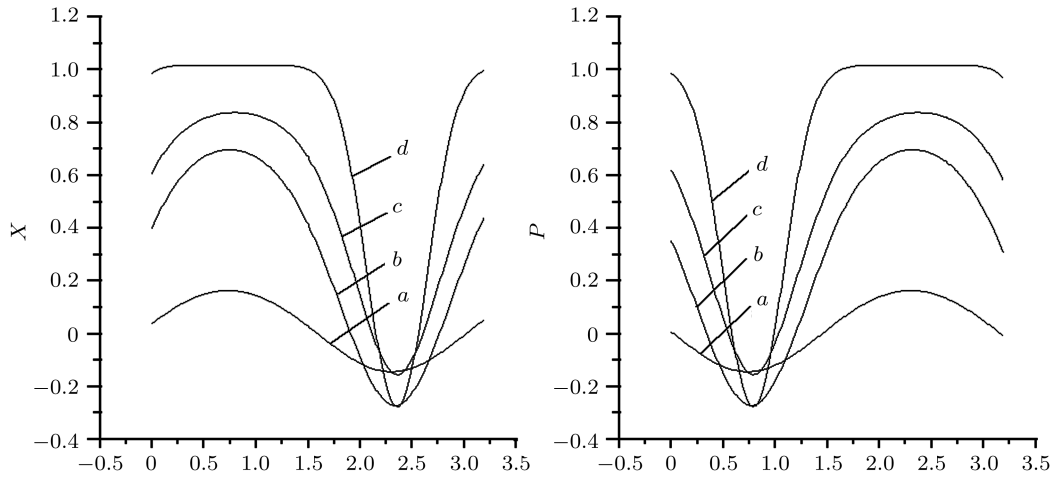


Fig.4. The entropic squeezing parameters X and P as the functions of α where. $m = 1.2$, $\varphi = 0$, $\delta = \pi/2$, $a = 0.7$ and $|\beta| = 0.5$ for Curve a; $|\beta| = 1.0$ for Curve b; $|\beta| = \sqrt{2}$ for Curve c; $|\beta| = 2.0$ for Curve d.

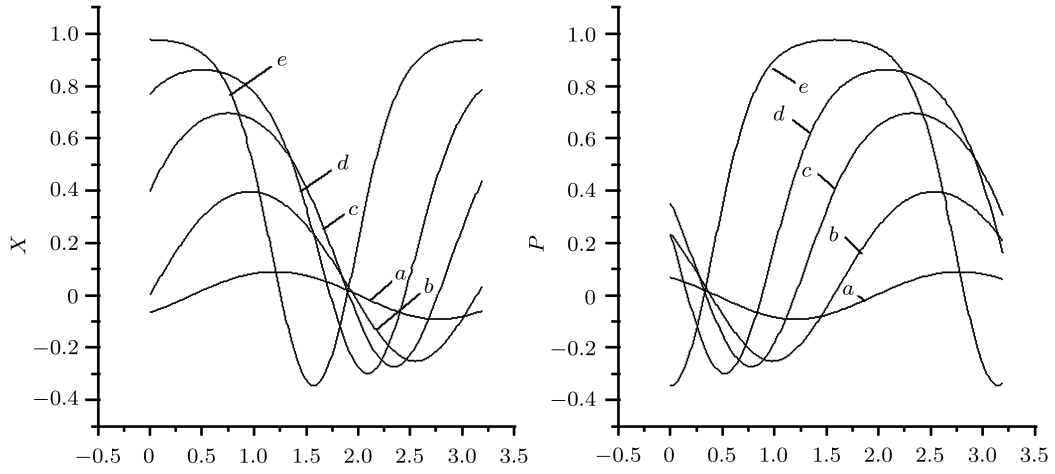


Fig.5. The entropic squeezing parameters X and P as the functions of α where. $m = 1.2$, $\varphi = 0$, $|\beta| = 1.0$, $a = 0.7$ and $\delta = \pi/6$ for Curve a; $\delta = \pi/3$ for Curve b; $\delta = \pi/2$ for Curve c; $\delta = 2\pi/3$ for Curve d; $\delta = \pi$ for Curve e.

3.4. The influence of relative phase of two coefficients on the entropic squeezing effect

In order to study the influence of relative phase φ of two coefficients on the entropic squeezing effect, let $m = 1.2$, $a = 0.7$, $|\beta| = 1.0$, $\delta = \pi/2$ and $\varphi = 0$, $\pi/6$, $\pi/3$, $\pi/2$ respectively. The position entropy and the momentum entropy as the functions of α are shown in Fig.6. The numerical results show when φ increases from 0 to $\pi/2$, the squeezing ranges of both the position entropy and the momentum entropy decrease, and the squeezing depths weaken. It indicates that the squeezing effect weakens as φ increases in the first quadrant.

4. Conclusion

In this paper, the superpositions of two arbitrary coherent states are constructed by using the superposition principle of quantum mechanics. The entropic squeezing effects of the superposition states are studied. The numerical results indicate: (1) the squeezing ranges of both the position entropy and momentum entropy decrease as the amplitudes of the coherent states or the ratio between the amplitudes of two coherent states increases, meanwhile the squeezing depth can either strengthen or weaken; (2) there is a nonlinear relationship between entropic squeezing and the phase difference δ of two coherent states; (3) the squeezing effect weakens as the relative phase of the coefficients increases in the first quadrant.

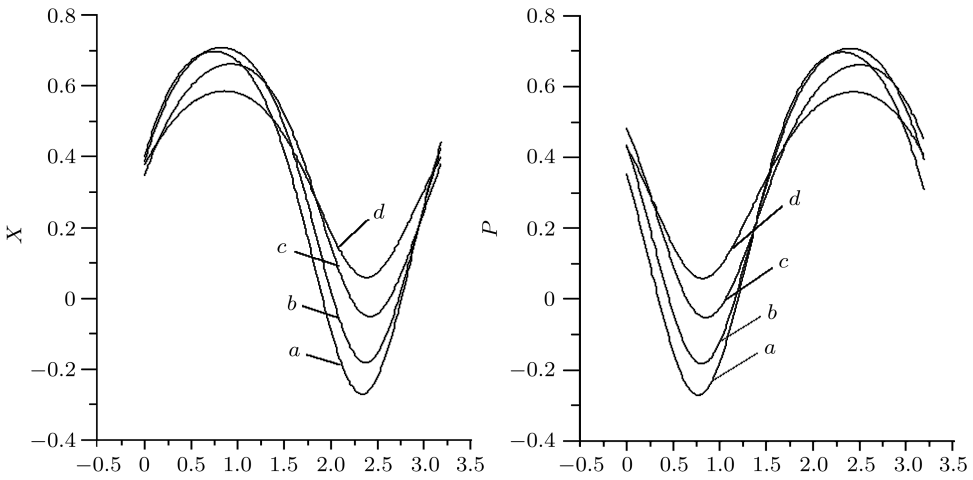


Fig.6. The entropic squeezing parameters X and P as the functions of α where. $m = 1.2$, $\delta = \pi/2$, $|\beta| = 1.0$, $a = 0.7$ and $\varphi = 0$ for Curve a; $\varphi = \pi/6$ for Curve b; $\varphi = \pi/3$ for Curve c; $\varphi = \pi/2$ for Curve d.

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