Vol 17 No 2, February 2008
1674-1056/2008/17(02)/0585-07

Chinese Physics B

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# The mass effect of the quark phase transition in supernova core*

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(Received 3 May 2007; revised manuscript received 7 September 2007)

Constituent quark mass model is adopted as a tentative one to study the phase transition between two-flavour quark matter and more stable three-flavour quark matter in the core of supernovae. The result shows that the transition has a significant influence on the increasing of the core temperature, the neutrino abundance and the neutrino energies, which contributes to the enhancement of the successful probability of supernova explosion. However, the equilibrium values of these parameters (except the temperature) from the constituent quark mass model in this work are slightly bigger than those obtained from the other model. And we find that the constituent quark mass model is also applicable to describing the transition in the supernova core.

**Keywords:** quark transitions, constituent quark mass, supernova, reaction rates
**PACC:** 2165, 1480D, 9760

## 1. Introduction

On the basis of the theory of Itoh[1] and Bodmer,[2] Witten proposed a famous conjecture that the matter, which consists of comparable numbers of $u$-, $d$- and $s$-quarks, is more stable than hadronic matter.[3] It greatly enlightens the theoretic prediction of the existence of strange quark stars. And somewhat later Farhi and Jaff performed their investigations according to the Massachusetts Institute of Technology (MIT) bag model in a wide range of quantum chromodynamics (QCD) parameters to support this idea.[4] Strange quark stars, which are believed to be more stable than neutron stars, are true matters existing in nature. On the basis of this conjecture and its deductions, many authors (see references in Ref.[5]) have dedicated much effort to the interesting and intriguing discussion on properties of strange quark stars, such as mass, radius, crust, formation, existence form, cooling, surface radiation, and so on. The significance has surpassed Witten's conjecture itself.

The existence and formation of strange quark stars has been intensively investigated. One of the formations of strange quark star is expected to come true during the collapse of the core of a massive star after the supernova explosion.[6] The phase transition in that process intensively affects the supernova explosion.[7] Generally, electron capture[8−10] induces core collapse till the central density exceeds the nuclear matter density, and then the shock wave is formed at the edge of proto-neutron star (PNS) and moves outwards quickly. Unfortunately, a number of numerical simulations indicate that the shock cannot rush out of the iron core due to photodisintegration. Neutrino delay explosion mechanism is regarded as a most possible one to solve this problem, because convective energy transition of strong neutrino from PNS can heat the stalled shock.[11] Although the multi-dimensional models based on a neutrino-driven scenario have been elaborately studied, the problem about triggering a successful supernova explosion is still open.[12]

Gentile *et al*[13] and Dai *et al*[14] investigated the effects of the phase transition of nuclear matter to two-flavour quark matter and the phase transition of the two-flavour quark matter to three-flavour quark matter on supernova explosion respectively. Anand *et al*[15] also investigated the conversions in supernovas and neutron stars under the strong interaction between quarks at a finite temperature. Their results

*Project supported by the National Natural Science Foundation of China (Grant No 10347008), the Scientific Research and Fund of Sichuan Provincial Education Department (Grant No.2006A079) and the Science and Technological Foundation of China West Normal University.

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all show that the energy of the revived shock wave can be enhanced, and then the successful probability of a supernova explosion is increased. And the production and emission of neutrinos caused by the conversion have a great influence on the cooling process of supernovas and pulsars.[6,16]

Both the u-quark mass and d-quark mass are neglected by Dai *et al*[14] and Anand *et al*[15] in their calculations except s-quark mass (200MeV), and a set of quark masses they chose is close to the range of current quark mass. Constituent quark mass has successfully been used to describe the fundamental properties of hadrons in a low-lying regime including the mass of the meson.[17] Besides this, it can also be chosen to study the hadron-quark phase transition in neutron stars[18] and some properties of the neutron star as an important input parameter.[19] Following the illumination and accounting for the uncertainties of the quark mass (e.g. the quark mass was chosen artificially in Ref.[20]), constituent quark mass model is adopted as a tentative one to discuss its influence on the quark conversion in a core-collapse supernova in this paper. And we suppose that the phase transition from nuclear matter to two-flavour quark matter has already taken place in supernova inner core, then the process from two-flavour quark matter to strange matter subsequently occurs, which will be taken into account in the following sections.

With regard to the phase transition process in supernova, the following treatment should be reasonable.[14−16] Firstly, neutrinos, quarks (u, d, s) and electrons are treated as strong relativistic and degenerate Fermi gases. Secondly, the strange quark matter in the inner core is approximately treated as a homogeneous matter. The calculations by Gentile *et al* show that the strange quark matter in the inner core is less than $0.3M_{\odot}$. And the quark matter density is not assumed to vary during the conversion. Thirdly, all particles are characterized with the same temperature due to strong interactions.

## 2. Phase transition reaction rates with constituent quark mass

In the supernova core, the most important process is the producing of strange quark by the following weak interaction:

$$\mathrm{u}_{(1)} + \mathrm{d} \leftrightarrow \mathrm{u}_{(2)} + \mathrm{s}, \tag{1}$$

of which the concrete roles are indicated below.[5] First, it is a channel to detect strangelets from relativistic ion collision in an earth laboratory. Second, it determines the rate of neutron stars burning into strange stars. Third, it plays an important role in forming and evaporating quark nuggets in the universe. Fourth, it determines the bulk viscosity of quark matter, and further determines the stability properties and maximum rotation velocity of neutron stars or strange stars. Besides reaction (1) other two weak interactions are needed to achieve an equilibrium state in the strange quark matter,

$$\mathrm{u} + \mathrm{e}^- \leftrightarrow \mathrm{d} + \mathrm{v_e}, \tag{2}$$

$$\mathrm{u} + \mathrm{e}^- \leftrightarrow \mathrm{s} + \mathrm{v_e}. \tag{3}$$

Since neutrinos are highly degenerate in the core of supernova, the reactions in which antineutrinos take part are neglected, and the strong corrections are not considered either. Strange quark matter will be in an equilibrium state because of these weak interactions, and the chemical equilibrium between fermions means that

$$\mu_{\mathrm{d}} + \mu_{\mathrm{v_e}} = \mu_{\mathrm{s}} + \mu_{\mathrm{v_e}} = \mu_{\mathrm{u}} + \mu_{\mathrm{e}}, \tag{4}$$

where the effect of neutrino is not neglected.

Let $k = \hbar = c = 1$, the number of reactions per unit time and per unit volume for reaction (1) can be obtained from the Weinberg–Salam theory as follows:

$$\Gamma^{+}_{(\mathrm{ud}\to\mathrm{us})} = \frac{2\times 36}{(2\pi)^8} G_{\mathrm{F}}^2 \cos^2\theta_{\mathrm{c}} \sin^2\theta_{\mathrm{c}} \int \left[\prod_{i=1}^{4} \mathrm{d}\boldsymbol{p}_i^3\right] \delta^{(4)}(P_1 + P_2 - P_3 - P_4) S (P_1\cdot P_2)(P_3\cdot P_4), \tag{5}$$

where $S = f(E_1)f(E_2)[1-f(E_3)][1-f(E_4)]$ with $f(E_i) = \left[1+\exp\left(\dfrac{E_i-\mu_i}{T}\right)\right]^{-1}$, and $i = 1,\cdots,4$ corresponding to particles of reaction (1) successively, $\mu_i$, $E_i$, $P_i = (E_i, \boldsymbol{p}_i)$ representing chemical potential,

energy and four-momentum of particle $i$ respectively, $T$ being temperature, $f(E_i)$ being Fermi-Dirac distribution function; $G_{\rm F}$ is the Fermi constant ($1.435\times10^{-49}{\rm erg\cdot cm^{-3}}$) (1erg=$10^{-7}$J); $\theta_{\rm c}$ is the Cabibbo angle ($\cos^2\theta_{\rm c}=0.974$); coefficient 36 comes from spin and colour degrees of quarks. When quark mass is considered, the relation between energy and momentum becomes $E_q=\sqrt{p_q^2+m_q^2}$. For simplicity, we let

$$a_q=\frac{E_q}{p_q},\quad b_q=\frac{{\rm d}E_q}{{\rm d}p_q},\quad q={\rm u,d,s}. \tag{6}$$

Inserting expression (6) into expression (5), we can obtain the forward reaction rate in reaction (1)

$$\Gamma^{+}_{({\rm ud}\to{\rm us})}=\frac{18}{\pi^6}G_{\rm F}^2\cos^2\theta_{\rm c}\sin^2\theta_{\rm c}\frac{p_{\rm F}(u)^2p_{\rm F}(d)}{b_{\rm u}^2b_{\rm d}}\times T_{11}^5A_1M(\xi_1){\rm s}^{-1}, \tag{7}$$

where

$$\begin{aligned}A_1=&\frac{1}{p_{\rm F}(4)}\int_0^\infty\frac{1}{x^2}{\rm d}x\left\{\prod_{i=1}^4\sin p_{\rm F}(i)x+\ c_1c_2\prod_{i=1}^4(\cos p_{\rm F}(i)x-\frac{\sin p_{\rm F}(i)x}{p_{\rm F}(i)x})\right.\\&+c_1(\cos p_{\rm F}(1)x-\frac{\sin p_{\rm F}(1)x}{p_{\rm F}(1)x})(\cos p_{\rm F}(2)x-\frac{\sin p_{\rm F}(2)x}{p_{\rm F}(2)x})\sin p_{\rm F}(3)x\sin p_{\rm F}(4)x\\&\left.+\ c_2(\cos p_{\rm F}(3)x-\frac{\sin p_{\rm F}(3)x}{p_{\rm F}(3)x})(\cos p_{\rm F}(4)x-\frac{\sin p_{\rm F}(4)x}{p_{\rm F}(4)x})\sin p_{\rm F}(1)x\sin p_{\rm F}(2)x\right\};\end{aligned} \tag{8}$$

$$M(\xi_1)=\int_{-\infty}^{(\mu_{\rm s}-m_{\rm s})/T}(\mu_{\rm s}/T-x)\sqrt{(\mu_{\rm s}/T-x)^2-m_{\rm s}^2/T^2}\frac{(x+\xi_1)^2+\pi^2}{(1+{\rm e}^x)(1+{\rm e}^{-x-\xi_1})}{\rm d}x; \tag{9}$$

$p_{\rm F}(i)$ is the Fermi momentum of particle $i$; $T_{11}$ means the temperature in units of $10^{11}$K; $\xi_1=(\mu_{\rm d}-\mu_{\rm s})/T$, as can be seen from Eq.(4), it just characterizes the deviation from chemical equilibrium; and $A_1$ represents the momentum conservation, with $c_1=1/(a_{\rm u}a_{\rm d})$ and $c_2=1/(a_{\rm u}a_{\rm s})$.[14] Define the baryon number density as

$$n_{\rm b}=\frac{n_{\rm u}+n_{\rm d}+n_{\rm s}}{3}. \tag{10}$$

According to the former assumption that the quark matter density is constant, baryon and lepton abundances can be defined as

$$Y_i=\frac{n_i}{n_{\rm b}},\quad i={\rm u,d,s,e,\nu_e}. \tag{11}$$

Substituting $M(\xi_1)$ by $M(-\xi_1)$ in expression (7), we can gain its reverse reaction rate. Therefore the net reaction rate per nuclei of reaction (1) is

$$\begin{aligned}\Gamma_1=&\frac{\Gamma_{({\rm ud}\leftrightarrow{\rm us})}}{n_{\rm b}}=45.9T_{11}^5\frac{(Y_{\rm u}^2Y_{\rm d})^{1/3}}{b_{\rm u}^2b_{\rm d}}\\&\times A_1[M(\xi_1)-M(-\xi_1)]{\rm s}^{-1},\end{aligned} \tag{12}$$

The net rate per baryon for reaction (2) is given by

$$\begin{aligned}\Gamma_2=&\frac{\Gamma_{({\rm ue}\leftrightarrow{\rm d\nu_e})}}{n_{\rm b}}\\=&6.64\times10^5T_{11}^3\left(\frac{n_{\rm b}}{n_0}\right)^{2/3}\frac{(Y_{\rm u}Y_{\rm e}Y_{\rm d}Y_{\rm \nu_e}^2)^{1/3}}{b_{\rm u}b_{\rm e}b_{\rm d}}\\&\times A_2\xi_2(\xi_2^2+4\pi^2){\rm s}^{-1},\end{aligned} \tag{13}$$

where $n_0$ is the saturation density ($\approx0.155{\rm fm}^{-3}$). Likewise, substituting $c_1$ and $c_2$ in $A_1$ by $1/a_{\rm u}$ and $1/a_{\rm d}$ respectively, one can obtain $A_2$. $\xi_2=(\mu_{\rm u}+\mu_{\rm e}-\mu_{\rm d}-\mu_{\rm \nu_e})/T$. The neutrino energy production rate per baryon can also be obtained to be

$$\begin{aligned}\varepsilon_2(\xi_2)=&1.17\times10^{-2}T_{11}^6\frac{(Y_{\rm u}Y_{\rm e}Y_{\rm d})^{1/3}}{b_{\rm u}b_{\rm e}b_{\rm d}}\\&\times A_2[I(\xi_2)-I(-\xi_2)](\text{erg / s}),\end{aligned} \tag{14}$$

where

$$I(\xi_2)=\int_{-\infty}^{\mu_{\rm \nu_e}/T}\frac{(\mu_{\rm \nu_e}/T-x)^3[(x+\xi_2)^2+\pi^2]}{(1+{\rm e}^x)(1+{\rm e}^{-x-\xi_2})}{\rm d}x. \tag{15}$$

The net rate per baryon for reaction (3) is

$$\begin{aligned}\Gamma_3=&\frac{\Gamma_{({\rm ue}\leftrightarrow{\rm s\nu_e})}}{n_{\rm b}}\\=&1.77\times10^4T_{11}^3\left(\frac{n_{\rm b}}{n_0}\right)^{2/3}\frac{(Y_{\rm u}Y_{\rm e}Y_{\rm s}Y_{\rm \nu_e}^2)^{1/3}}{b_{\rm u}b_{\rm e}b_{\rm s}}\\&\times A_3\xi_3(\xi_3^2+4\pi^2){\rm s}^{-1}.\end{aligned} \tag{16}$$

Similarly, $A_3$ can be obtained by replacing $c_1$ and $c_2$ in $A_1$ with $1/a_{\rm u}$ and $1/a_{\rm s}$ respectively. $\xi_3=(\mu_{\rm u}+\mu_{\rm e}-\mu_{\rm s}-\mu_{\rm \nu_e})/T$. The neutrino energy production rate is

$$\begin{aligned}\varepsilon_3(\xi_3)=&3.12\times10^{-4}T_{11}^6\frac{(Y_{\rm u}Y_{\rm e}Y_{\rm s})^{1/3}}{b_{\rm u}b_{\rm e}b_{\rm s}}\\&\times A_3[I(\xi_3)-I(-\xi_3)](\text{erg / s}).\end{aligned} \tag{17}$$

$I(\xi_3)$ can be obtained by substituting $\xi_2$ with $\xi_3$ in expression (15), and the total neutrino energy production rate is given by

$$\varepsilon_{\nu_e} = \varepsilon_2(\xi_2) + \varepsilon_3(\xi_3). \tag{18}$$

So we can acquire the total neutrino energy due to its being trapped.

## 3. Conversion of quark matter into strange matter

Reaction rate formulas derived from the above are applied to the conversion in the inner core of supernova, and an equation group is constructed as follows:

$$\frac{\mathrm{d}Y_\mathrm{d}}{\mathrm{d}t} = -\Gamma_1 + \Gamma_2, \tag{19}$$

$$\frac{\mathrm{d}Y_\mathrm{u}}{\mathrm{d}t} = -\Gamma_2 - \Gamma_3. \tag{20}$$

Because of baryon number conservation and charge neutrality, one can have

$$Y_\mathrm{s} = 3 - Y_\mathrm{u} - Y_\mathrm{d}, \tag{21}$$

$$2Y_\mathrm{u} = Y_\mathrm{d} + Y_\mathrm{s} + 3Y_\mathrm{e}. \tag{22}$$

Conversions (2) and (3) do not change lepton abundance because neutrinos are trapped, namely

$$Y_\mathrm{e} + Y_{\nu_e} = Y_L \tag{23}$$

is a constant during the transition. The chemical potential of particle $i$ is written as

$$\mu_i = \sqrt{p_\mathrm{F}(i)^2 + m_i^2}, \quad i = \mathrm{u, d, s, e, \nu_e}, \tag{24}$$

where

$$p_\mathrm{F}(i) = 417.5 \left(\frac{n_\mathrm{b}}{n_0} \times \frac{Y_i}{g_i}\right)^{1/3} \mathrm{MeV}, \tag{25}$$

with $g_\mathrm{u} = g_\mathrm{d} = g_\mathrm{s} = 6$, $g_\mathrm{e} = 2$ and $g_{\nu_e} = 1$. The temperature as a function of time can be obtained from the first law of thermodynamics,[14,15]

$$\frac{\mathrm{d}T_{11}}{\mathrm{d}t} = -\left(732.9 T_{11} \sum_i \frac{Y_i}{p_\mathrm{F}(i)}\right)^{-1} \times \left[(\mu_\mathrm{u} + \mu_\mathrm{e} - \mu_\mathrm{s} - \mu_v)\frac{\mathrm{d}Y_\mathrm{u}}{\mathrm{d}t} + (\mu_\mathrm{d} - \mu_\mathrm{s})\frac{\mathrm{d}Y_\mathrm{d}}{\mathrm{d}t}\right]. \tag{26}$$

Using the initial conditions, the influence of the quark mass on the conversion can be gained. Other initial conditions, except for quark mass, are chosen to be those given by Dai *et al*[14] and those cited in his paper: 1) the two values for baryon number density in the inner core are adopted to be $1.5n_0$ and $2.5n_0$, corresponding to two masses of strange quark matter in the inner core: $0.245M_\odot$ and $0.307M_\odot$ respectively; 2) the initial temperatures are chosen to be 10, 20, and 30MeV; 3) the values of lepton abundance $Y_\mathrm{L}$ are taken as 0.32, 0.36 and 0.40. Neutrino abundance then can be given by[21]

$$Y_\nu = 0.38Y_\mathrm{L}^2 + 0.1Y_\mathrm{L} - 0.0145. \tag{27}$$

The constituent quark masses are chosen as $m_\mathrm{u} = m_\mathrm{d} = 309.15\mathrm{MeV}$ and $m_\mathrm{s} = 354\mathrm{MeV}$,[22] which is called quark mass model I. A set of quark masses cited from Refs.[14,15], $m_\mathrm{u} = m_\mathrm{d} = 0$ and $m_\mathrm{s} = 200\mathrm{MeV}$, is called quark mass model II. For a given initial abundance $Y_i$ of particle $i$, initial temperature $T_{11}$, baryon number density and quark mass, both the variations of the particle abundance $Y_i$, temperature $T_{11}$ and the total neutrino energy production rate $\varepsilon_{\nu_e}$ as functions of time and their equilibrium values are obtained by integrating Eqs.(19), (20) and (26).

**Table 1.** The initial and final values of parameters in the conversion for the constituent quark mass with the baryon number density $n_\mathrm{b} = 1.5n_0$. Subscripts 'i' and 'f' refer to the initial and final respectively. $T$ is in units of MeV, and $E$ in units of $10^{52}$erg.

| Mass Model | $Y_{\nu_e\mathrm{i}}$ | $Y_{\nu_e\mathrm{f}}$ | $T_\mathrm{i}$ | $T_\mathrm{f}$ | $Y_\mathrm{ui}$ | $Y_\mathrm{uf}$ | $Y_\mathrm{di}$ | $Y_\mathrm{df}$ | $Y_\mathrm{si}$ | $Y_\mathrm{sf}$ | $Y_\mathrm{ei}$ | $Y_\mathrm{ef}$ | $E_{\nu_e\mathrm{i}}$ | $E_{\nu_e\mathrm{f}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| I | 0.0564 | 0.1072 | 10 | 21.9558 | 1.2636 | 1.2128 | 1.7364 | 1.2012 | 0 | 0.5860 | 0.2636 | 0.2128 | 1.10 | 1.7855 |
| II | | 0.1010 | | 22.1955 | | 1.2190 | | 1.3005 | | 0.4805 | | 0.2190 | | 1.6631 |
| I | 0.0707 | 0.1226 | 20 | 27.6749 | 1.2893 | 1.2374 | 1.7107 | 1.1875 | 0 | 0.5752 | 0.2893 | 0.2374 | 1.69 | 2.4612 |
| II | | 0.1167 | | 27.8424 | | 1.2433 | | 1.2864 | | 0.4704 | | 0.2433 | | 2.3208 |
| I | 0.0863 | 0.1383 | 30 | 35.3585 | 1.3137 | 1.2617 | 1.6863 | 1.1738 | 0 | 0.5645 | 0.3137 | 0.2617 | 2.59 | 3.4666 |
| II | | 0.1329 | | 35.4751 | | 1.2671 | | 1.2725 | | 0.4604 | | 0.2671 | | 3.3095 |

**Table 2.** The initial and final values of parameters in the conversion for the constituent quark mass with the baryon number density $n_{\rm b} = 2.5n_0$. Subscripts 'i' and 'f' refer to the initial and final respectively. $T$ is in units of MeV, and $E$ in units of $10^{52}$erg.

| Mass Model | $Y_{\nu_e i}$ | $Y_{\nu_e f}$ | $T_{\rm i}$ | $T_{\rm f}$ | $Y_{\rm ui}$ | $Y_{\rm uf}$ | $Y_{\rm di}$ | $Y_{\rm df}$ | $Y_{\rm si}$ | $Y_{\rm sf}$ | $Y_{\rm ei}$ | $Y_{\rm ef}$ | $E_{\nu_e i}$ | $E_{\nu_e f}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| I | 0.0564 | 0.1117 | 10 | 29.3949 | 1.2636 | 1.2083 | 1.7364 | 1.1159 | 0 | 0.6758 | 0.2636 | 0.2083 | 1.10 | 2.3061 |
| II | | 0.1083 | | 30.8147 | | 1.2117 | | 1.1888 | | 0.5996 | | 0.2117 | | 2.2413 |
| I | 0.0707 | 0.1277 | 20 | 33.6947 | 1.2893 | 1.2323 | 1.7107 | 1.1029 | 0 | 0.6648 | 0.2893 | 0.2323 | 1.69 | 3.0425 |
| II | | 0.1247 | | 34.8919 | | 1.2353 | | 1.1756 | | 0.5891 | | 0.2353 | | 2.9801 |
| I | 0.0863 | 0.1439 | 30 | 40.0945 | 1.3137 | 1.2561 | 1.6863 | 1.0900 | 0 | 0.6539 | 0.3137 | 0.2561 | 2.59 | 4.0622 |
| II | | 0.1415 | | 41.0693 | | 1.2585 | | 1.1627 | | 0.5788 | | 0.2585 | | 4.0092 |

# 4. Numerical results and discussion

All the equilibrium values (including those of pre-conversion) are given in Tables 1 and 2. After the conversion, the values of the quantities obtained from the constituent quark mass (model I), such as $T$, $Y_{\rm d}$, $Y_{\rm u}$ and $Y_{\rm e}$, decrease to a certain extent compared with the corresponding results from the quark mass model II. However, the result is opposite for the other quantities $Y_{\nu_e}$, $Y_{\rm s}$ and $E_{\nu_e}$ etc. The larger the constituent quark mass, the smaller the parameter $b_q$ in expression (6). It leads to bigger reaction rates in Eqs.(12), (13) and (16). This means that more transitions have occurred in supernova core. So the decreasing of $Y_{\rm u}$ and $Y_{\rm e}$ and the increasing of $Y_{\nu_e}$, $Y_{\rm s}$ and $E_{\nu_e}$ can easily be explained by the qualitative analysis; the explanation of the decreasing of $Y_{\rm d}$ needs further analysis, which is given in the following paragraph. With the temperature $T$ decreasing after the conversion due to larger constituent quark mass, the u- and d-quarks must have larger kinetic energies (mainly coming from thermal energy) in order to be able to create s-quarks.

The deviation of the results with quark mass model I from the ones with quark mass model II can be obtained from Tables 1 and 2. For the baryon number density $n_{\rm b} = 1.5n_0$, the maximum decrement of the temperature $T$ is 1%, and the maximum increments of the neutrino abundance $Y_{\nu_e}$, s-quark abundance $Y_{\rm s}$ and total neutrino energy in the core $E_{\nu_e}$ are 6%, 23% and 7% respectively. For the baryon number density $n_{\rm b} = 2.5n_0$, the maximum decrement of $T$ is about 5%, the maximum increments of $Y_{\nu_e}$, $Y_{\rm s}$ and $E_{\nu_e}$ are 3%, 13% and 3% respectively. One can see that the deviations are not significant except for the case for s-quark abundance $Y_{\rm s}$. The reason is that the reaction rate $\Gamma_1$ is directly proportional to $T_{11}^5$ which is two orders of magnitude bigger than both $\Gamma_2$ and $\Gamma_3$. And by further studying $M(\xi_1)$ numerically, we find that $\Gamma_1$ is bigger than both $\Gamma_2$ and $\Gamma_3$ at any moment. This means that reaction (1) is dominant to make a contribution to the bigger deviations of $Y_{\rm s}$ and $Y_{\rm d}$, which can also explain the decreasing of $Y_{\rm d}$. The temperature, neutrino abundance and total neutrino energies in supernova core are displayed as a function of time in Figs.1, 2 and 3 based on the constituent quark mass(model I), which are parallel closely to the results of Ref.[14] obtained by using the quark model II. Therefore, according to the results we can conclude that the constituent quark mass can also be adopted to study the quark transition in supernova core successfully.

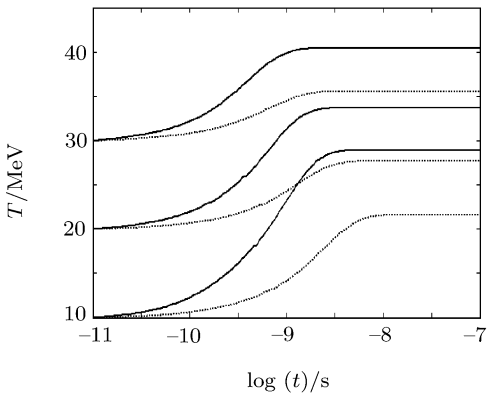



**Fig.1.** The temperature vs time for lepton abundance $Y_{\rm L} = 0.36$ (Mass Model I). The solid and dot lines are corresponding to $n_{\rm b} = 2.5n_0$ and $n_{\rm b} = 1.5n_0$, each series of lines from the bottom to the top are at initial temperature $T = 10, 20$ and 30MeV.

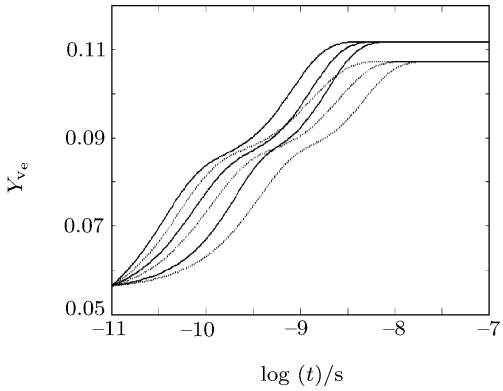



**Fig.2.** The neutrino abundance vs time for lepton abundance $Y_{\rm L} = 0.32$ (Mass Model I). The solid and dot lines are corresponding to $n_{\rm b} = 2.5n_0$ and $n_{\rm b} = 1.5n_0$, each series of lines from the top to the bottom are at initial temperature $T = 30, 20$ and 10MeV.

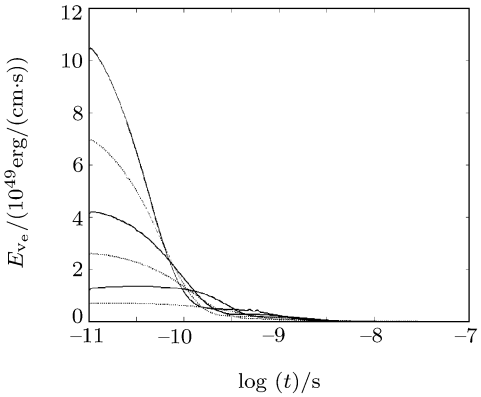



**Fig.3.** The neutrino production rate vs time for lepton abundance $Y_{\rm L} = 0.32$ (Mass Model I). The solid and dot lines are corresponding to $n_{\rm b} = 2.5n_0$ and $n_{\rm b} = 1.5n_0$, each series of lines from the bottom to the top are at initial temperature $T = 10, 20$ and 30MeV.

According to Tables 1 and 2, one can find that the bigger the baryon number density, the more reactions occur with the same initial condition for one type of quark mass model. The results are also displayed in Figs.1, 2 and 3. Because both $\Gamma_2$ and $\Gamma_3$ are directly proportional to $n_{\rm b}^{2/3}$, in other words, the larger the baryon number density $n_{\rm b}$, the larger numbers of u- and d-quarks are converted into s-quarks according to the assumption of the baryon number density conservation in this paper.

The following discussion is based on the constituent quark mass (model I). One can also find from Tables 1 and 2 that after the conversion the maximum increments of the temperature are 120% and 194% for the baryon number density $n_{\rm b} = 1.5n_0$ and $2.5n_0$ respectively. Figure 3 indicates that the production process of neutrinos ends off almost within the first half time. At the beginning, the production of neutrinos is raised slightly (the trend is obvious with the decreasing of initial temperature). We further study the rates of reactions (2) and (3) numerically, and find that at the beginning of the conversion, reaction (2) is dominant, of which the reaction rate $\Gamma_2$ is an order higher than $\Gamma_3$. Hence the production of the neutrinos is determined mainly by reaction (2) at the beginning. However, the rate of reaction (3) rises with the rate of reaction (2) dropping. Thus the production rate of neutrinos increases slightly again in the vicinity of $10^{-9.5}$s in Fig.3, which corresponds to the rapid increasing of the neutrino abundance again after $10^{-9}$s in Fig.2. The numbers of neutrinos produced by reactions (2) and (3), as a consequence of a competition between them, will be of the same order of magnitude. In Fig.3 the energy production rate of neutrinos in supernova slightly increases with time twice, which is not different from the situation in neutron stars discussed in Ref.[23]. The reason is that neutrinos in supernova are trapped, but in neutron star the neutrino can escape freely as soon as it is produced. The trends of neutrino abundance varying with time are alike for different sets of initial parameters. And the neutrino abundances of the final state have the same values for different initial temperatures, with the same baryon number density and lepton abundance. Because the Fermi momentum of particle, which is only dependent on the baryon number density, is independent of temperature. And this leads to the same difference in chemical potential between two sides of reactions (1), (2) and (3) no matter what their initial temperatures are. After the conversion, the maximum increments of neutrino abundances are 90% and 98% when $n_{\rm b} = 1.5n_0$ and $2.5n_0$, respectively.

The increases of temperature, neutrino abundance and total neutrino energies, play important roles in supernova explosion from the angle of the delayed time neutrino-heating mechanism. This is because in the stage of the shock wave forming and moving outwards, the deleptonization and the heating of a PNS depend on temperature and neutrino number density.[24] According to the chiral hadronic model[25] or the finite temperature Brueckner–Hartree–Fock theory with three-body forces, the temperature behaviour of nuclear matter can be studied to further ascertain the properties of the PNS born following supernova explosion.[26−28] Thus the result based on constituent quark mass will also further influence the dynamics of the supernova evolution, and greatly increase the successful probability of a supernova explosion.

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