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Interference nature of light

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Interference of light has been reinvestigated theoretically by linear superposition of two different state-vector functions, of which each describes the photons from one of two different light sources in both polarization and intensity. By the use of microscopic parameters for a photon, namely probability amplitude and phase, it is again validated that interference of light occurs only between the same photons possessing a set of the selfsame eigenvalues.

Keywords: photon, state-vector function, interference
PACC: 1480A, 4210J, 4225K

1. Introduction

Since Zernike[1] published a classic paper in 1938 in which he introduced a quantitative measure of coherence by the use of Young's interference experiment, the experiment has played a central role in developing the optical coherence theory. Recently, the optical interference effect has reattracted special attention, because Roychowdhury and Wolf[2] first elucidated the role of polarization of light in forming the interference pattern in terms of the cross-spectral density matrix of the electric field. On the other hand, however, just as Feynman[3] in 1965 pointed out that the central mystery of quantum theory was encapsulated in the experiment with two holes, he said that the interference of light was 'a phenomenon which is impossible, absolutely impossible, to explain in any classical way ...' In our recent paper,[4] we have made an attempt to avoid classical concept of the electric field, and successfully explained Young's double-hole experiment in terms of state-vector function (SVF) for a photon, which completely describes all the quantum behaviours of photons in the experiment.

In the present paper, we follow Wolf's idea about polarization of light and more discuss the interference between two light beams in terms of SVF for a photon. The two light beams come from two separated pinholes of Young's double-hole experiment or two independent lasers, and they are different not only in polarization but also in intensity. By the use of linear superposition of two SVFs describing two different light beams in an abstract Hilbert space, we deduce a new formulation, which can describe the interference patterns in an actual physical space by using microscopic parameters for a photon. We believe that the formulation can help to clarify the interference nature of light quantitatively. The interference of light occurs, in principle, only between the same photons possessing a set of the selfsame eigenvalues. In other words, the interference of light occurs only between left spin photons or between right spin photons; it is impossible for the interference of light to occur between left spin photon and right spin photon.

There has been a general expression of one-dimensional (1D) SVF for a photon,[4–6] expressed as

$$|\boldsymbol{\mathcal{A}}(t,z)\rangle = \frac{1}{\sqrt{2}}\left[\sigma_+^{(1)}\mathrm{e}^{\mathrm{i}\alpha}\begin{pmatrix}1\\ \mathrm{i}\end{pmatrix} + \sigma_-^{(1)}\mathrm{e}^{\mathrm{i}\beta}\begin{pmatrix}1\\ -\mathrm{i}\end{pmatrix}\right] \times \mathrm{e}^{-\mathrm{i}(\omega t-\kappa z)}. \quad (1)$$

The photons described with the SVF possess the energy eigenvalue

$$E = \left\langle \boldsymbol{\mathcal{A}}(t,z)\left|\mathrm{i}\hbar\frac{\partial}{\partial t}\right|\boldsymbol{\mathcal{A}}(t,z)\right\rangle = \hbar\omega,$$

the momentum eigenvalue $p_z = \left\langle \boldsymbol{\mathcal{A}}(t,z)\left|-\mathrm{i}\hbar\frac{\partial}{\partial z}\right|\boldsymbol{\mathcal{A}}(t,z)\right\rangle = \hbar\kappa$, and the angular momentum expectation value $\bar{S}_z = \langle\boldsymbol{\mathcal{A}}(t,z)|\,\hat{S}_z\,|\boldsymbol{\mathcal{A}}(t,z)\rangle =$

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$\hbar\left[\left(\sigma_+^{(1)}\right)^2-\left(\sigma_-^{(1)}\right)^2\right]$, where $\hat{S}_z=-\mathrm{i}\hbar\begin{pmatrix}0&1\\-1&0\end{pmatrix}$ is the spin operator for a photon; the real coefficients $\sigma_+^{(1)}$ and $\sigma_-^{(1)}$ are probability amplitudes; and α and β are phases for left and right spin photons, respectively. The SVF satisfies the normalization, giving rise to the relationship $\langle\boldsymbol{\mathcal{A}}(t,z)|\,|\boldsymbol{\mathcal{A}}(t,z)\rangle=\left[\sigma_+^{(1)}\right]^2+\left[\sigma_-^{(1)}\right]^2=1$. Here $\langle\boldsymbol{\mathcal{A}}(t,z)|\,|\boldsymbol{\mathcal{A}}(t,z)\rangle$ is defined as the inner product of SVF $|\boldsymbol{\mathcal{A}}(t,z)\rangle$ and its Hermitian adjoint $|\boldsymbol{\mathcal{A}}(t,z)\rangle^\dagger=\widetilde{|\boldsymbol{\mathcal{A}}(t,z)\rangle}^{\,*}=\langle\boldsymbol{\mathcal{A}}(t,z)|$, in which symbol '†' denotes the Hermitian adjoint, '$*$' refers to complex conjugate, and '$\sim$' means transpose.

The real part of the SVF represents the electromagnetic vector potential for an electromagnetic wave, namely a light wave, of which the general expression is given by[5,6]

$$\begin{aligned}\boldsymbol{\mathcal{A}}(t,z)&=\mathrm{Re}\,|\boldsymbol{\mathcal{A}}(t,z)\rangle\\&=\frac{\sigma_+^{(1)}}{\sqrt{2}}\Big[\boldsymbol{i}\cos(\omega t-\kappa z-\alpha)\\&\quad+\boldsymbol{j}\sin(\omega t-\kappa z-\alpha)\Big]\\&\quad+\frac{\sigma_-^{(1)}}{\sqrt{2}}\Big[\boldsymbol{i}\cos(\omega t-\kappa z-\beta)\\&\quad-\boldsymbol{j}\sin(\omega t-\kappa z-\beta)\Big].\end{aligned}\tag{2}$$

When $\sigma_+^{(1)}=1$ and $\sigma_-^{(1)}=0$, expression (2) represents a left circularly polarized light wave

$$\boldsymbol{\mathcal{A}}_+(t,z)=\frac{1}{\sqrt{2}}\Big[\boldsymbol{i}\cos(\omega t-\kappa z-\alpha)+\boldsymbol{j}\sin(\omega t-\kappa z-\alpha)\Big].\tag{2a}$$

When $\sigma_+^{(1)}=0$ and $\sigma_-^{(1)}=1$, expression (2) represents a right circularly polarized light wave

$$\boldsymbol{\mathcal{A}}_-(t,z)=\frac{1}{\sqrt{2}}\Big[\boldsymbol{i}\cos(\omega t-\kappa z-\beta)-\boldsymbol{j}\sin(\omega t-\kappa z-\beta)\Big].\tag{2b}$$

When $\sigma_+^{(1)}=\sigma_-^{(1)}=\dfrac{\sqrt{2}}{2}$, expression (2) is reduced to

$$\boldsymbol{\mathcal{A}}_0(t,z)=\cos\left(\omega t-\kappa z-\frac{\beta+\alpha}{2}\right)\cdot\left[\boldsymbol{i}\cos\left(\frac{\beta-\alpha}{2}\right)+\boldsymbol{j}\sin\left(\frac{\beta-\alpha}{2}\right)\right].\tag{2c}$$

If the difference $(\beta-\alpha)$ in phase between the left and right spin photons in each pair is a constant, expression (2c) represents a linearly polarized light wave, of which the polarization orientation is $\phi=\left(\dfrac{\beta-\alpha}{2}\right)$; if the distribution of these polarization orientations is symmetrical in their domain $(-\pi,\pi)$ of definition, expression (2c) represents a natural light wave.

$$|\boldsymbol{\mathcal{A}}_1(t,\boldsymbol{r}_1)\rangle=\frac{1}{2}\left[\sigma_{+,1}^{(1)}\mathrm{e}^{\mathrm{i}\alpha_1}\begin{pmatrix}1\\\mathrm{i}\end{pmatrix}+\sigma_{-,1}^{(1)}\mathrm{e}^{\mathrm{i}\beta_1}\begin{pmatrix}1\\-\mathrm{i}\end{pmatrix}\right]\times\mathrm{e}^{-\mathrm{i}(\omega t-\boldsymbol{\kappa}\cdot\boldsymbol{r}_1)},\tag{3a}$$

$$|\boldsymbol{\mathcal{A}}_2(t,\boldsymbol{r}_2)\rangle=\frac{1}{2}\left[\sigma_{+,2}^{(1)}\mathrm{e}^{\mathrm{i}\alpha_2}\begin{pmatrix}1\\\mathrm{i}\end{pmatrix}+\sigma_{-,2}^{(1)}\mathrm{e}^{\mathrm{i}\beta_2}\begin{pmatrix}1\\-\mathrm{i}\end{pmatrix}\right]\times\mathrm{e}^{-\mathrm{i}(\omega t-\boldsymbol{\kappa}\cdot\boldsymbol{r}_2)}.\tag{3b}$$

2. Formulation

The outline of the Young's double-hole experiment is similar to that in Ref.[4] except for two independent light sources as shown in Fig.1.

The 1D SVF for a photon with respect to the origin of its region at hole 1 and that at hole 2 can be written as, respectively,[4]

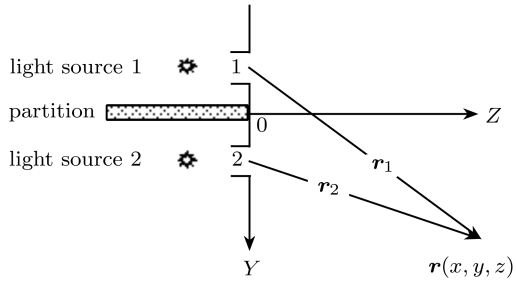


Fig.1. Outline of the Young's double-hole experiment with two independent light sources.

We have supposed that the geometrical symmetry of the double-hole requires the two SVFs each to contribute one half of the total probability, unity, i.e.

$$\begin{aligned}&\langle \boldsymbol{\mathcal{A}}_1(t,\boldsymbol{r}_1)|\,|\boldsymbol{\mathcal{A}}_1(t,\boldsymbol{r}_1)\rangle\\ =\;&\frac{1}{2}\left[\left(\sigma^{(1)}_{+,1}\right)^2+\left(\sigma^{(1)}_{-,1}\right)^2\right]=\frac{1}{2},\end{aligned}\tag{4a}$$

$$\begin{aligned}&\langle \boldsymbol{\mathcal{A}}_2(t,\boldsymbol{r}_2)|\,|\boldsymbol{\mathcal{A}}_2(t,\boldsymbol{r}_2)\rangle\\ =\;&\frac{1}{2}\left[\left(\sigma^{(1)}_{+,2}\right)^2+\left(\sigma^{(1)}_{-,2}\right)^2\right]=\frac{1}{2}.\end{aligned}\tag{4b}$$

The complete SVF that can describe all the quantum behaviours for a photon in the experiment should be linear superposition of the two SVFs, namely

$$|\boldsymbol{\mathcal{A}}_{12}(t,\boldsymbol{r})\rangle=|\boldsymbol{\mathcal{A}}_1(t,\boldsymbol{r}_1)\rangle+|\boldsymbol{\mathcal{A}}_2(t,\boldsymbol{r}_2)\rangle\,.\tag{5}$$

According to the basic rule of quantum mechanics, the probability distribution can be described as $P(\boldsymbol{r})=\langle \boldsymbol{\mathcal{A}}_{12}(t,\boldsymbol{r})|\,|\boldsymbol{\mathcal{A}}_{12}(t,\boldsymbol{r})\rangle$. If $P(\boldsymbol{r})$ is a distribution function of the radius vector $\boldsymbol{r}$, the interference occurs between two quantum states $|\boldsymbol{\mathcal{A}}_1(t,\boldsymbol{r}_1)\rangle$ and $|\boldsymbol{\mathcal{A}}_2(t,\boldsymbol{r}_2)\rangle$, more specifically, between the photons that are described by $|\boldsymbol{\mathcal{A}}_1(t,\boldsymbol{r}_1)\rangle$ and $|\boldsymbol{\mathcal{A}}_2(t,\boldsymbol{r}_2)\rangle$ respectively; otherwise if $P(\boldsymbol{r})=C$ that is a constant, in general, equal to unity, there occurs no interference at all.[4] Because

$$\begin{aligned}&\langle \boldsymbol{\mathcal{A}}_{12}(t,\boldsymbol{r})|\,|\boldsymbol{\mathcal{A}}_{12}(t,\boldsymbol{r})\rangle\\ =\;&(\langle \boldsymbol{\mathcal{A}}_1|+\langle \boldsymbol{\mathcal{A}}_2|)\cdot(|\boldsymbol{\mathcal{A}}_1\rangle+|\boldsymbol{\mathcal{A}}_2\rangle)\\ =\;&\langle \boldsymbol{\mathcal{A}}_1|\,|\boldsymbol{\mathcal{A}}_1\rangle+\langle \boldsymbol{\mathcal{A}}_1|\,|\boldsymbol{\mathcal{A}}_2\rangle+\langle \boldsymbol{\mathcal{A}}_2|\,|\boldsymbol{\mathcal{A}}_1\rangle+\langle \boldsymbol{\mathcal{A}}_2|\,|\boldsymbol{\mathcal{A}}_2\rangle\\ =\;&\frac{1}{2}+\langle \boldsymbol{\mathcal{A}}_1|\,|\boldsymbol{\mathcal{A}}_2\rangle+\langle \boldsymbol{\mathcal{A}}_2|\,|\boldsymbol{\mathcal{A}}_1\rangle+\frac{1}{2}\\ =\;&1+P_{\rm intf}(\boldsymbol{r})\,,\end{aligned}$$

where $P_{\rm intf}(\boldsymbol{r})\equiv\langle \boldsymbol{\mathcal{A}}_1|\,|\boldsymbol{\mathcal{A}}_2\rangle+\langle \boldsymbol{\mathcal{A}}_2|\,|\boldsymbol{\mathcal{A}}_1\rangle$, whether the interference can occur depends on the value of the interference item $P_{\rm intf}(\boldsymbol{r})$, if $P_{\rm intf}(\boldsymbol{r})\neq 0$, the interference makes an appearance, if $P_{\rm intf}(\boldsymbol{r})=0$, no interference comes into being. Before we deduce the general expression, it is appropriate to discuss some particular cases.

Case A The interference between two circularly polarized light beams.

If a pair of SVFs respectively describes left and right circularly polarized light beams, for example $\sigma^{(1)}_{+,1}=1$ and $\sigma^{(1)}_{-,1}=0$ in expression (3a), and $\sigma^{(1)}_{+,2}=0$ and $\sigma^{(1)}_{-,2}=1$ in expression (3b), then

$$|\boldsymbol{\mathcal{A}}_1(t,\boldsymbol{r}_1)\rangle=\frac{1}{2}\mathrm{e}^{\mathrm{i}\alpha_1}\begin{pmatrix}1\\ \mathrm{i}\end{pmatrix}\mathrm{e}^{-\mathrm{i}(\omega t-\boldsymbol{\kappa}\cdot\boldsymbol{r}_1)},\tag{6a}$$

$$|\boldsymbol{\mathcal{A}}_2(t,\boldsymbol{r}_2)\rangle=\frac{1}{2}\mathrm{e}^{\mathrm{i}\beta_2}\begin{pmatrix}1\\ -\mathrm{i}\end{pmatrix}\mathrm{e}^{-\mathrm{i}(\omega t-\boldsymbol{\kappa}\cdot\boldsymbol{r}_2)},\tag{6b}$$

It is apparent that the interference item

$$\begin{aligned}P_{\rm intf}(\boldsymbol{r})=\;&\langle \boldsymbol{\mathcal{A}}_1|\,|\boldsymbol{\mathcal{A}}_2\rangle+\langle \boldsymbol{\mathcal{A}}_2|\,|\boldsymbol{\mathcal{A}}_1\rangle\\ =\;&\frac{1}{2}\mathrm{e}^{-\mathrm{i}\alpha_1}\begin{pmatrix}1&-\mathrm{i}\end{pmatrix}\mathrm{e}^{\mathrm{i}(\omega t-\boldsymbol{\kappa}\cdot\boldsymbol{r}_1)}\cdot\frac{1}{2}\mathrm{e}^{\mathrm{i}\beta_2}\begin{pmatrix}1\\ -\mathrm{i}\end{pmatrix}\mathrm{e}^{-\mathrm{i}(\omega t-\boldsymbol{\kappa}\cdot\boldsymbol{r}_2)}\\ &+\frac{1}{2}\mathrm{e}^{-\mathrm{i}\beta_2}\begin{pmatrix}1&\mathrm{i}\end{pmatrix}\mathrm{e}^{\mathrm{i}(\omega t-\boldsymbol{\kappa}\cdot\boldsymbol{r}_2)}\cdot\frac{1}{2}\mathrm{e}^{\mathrm{i}\alpha_1}\begin{pmatrix}1\\ \mathrm{i}\end{pmatrix}\mathrm{e}^{-\mathrm{i}(\omega t-\boldsymbol{\kappa}\cdot\boldsymbol{r}_1)}\\ =\;&\frac{1}{4}\mathrm{e}^{-\mathrm{i}(\alpha_1-\beta_2)}\left[1+(-\mathrm{i})^2\right]\mathrm{e}^{-\mathrm{i}\boldsymbol{\kappa}\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)}+\frac{1}{4}\mathrm{e}^{\mathrm{i}(\alpha_1-\beta_2)}\left[1+(\mathrm{i})^2\right]\mathrm{e}^{\mathrm{i}\boldsymbol{\kappa}\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)}\\ =\;&\frac{1}{2}\left[1+(-1)\right]\cos\left[(\alpha_1-\beta_2)+\boldsymbol{\kappa}\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)\right]\equiv 0.\end{aligned}$$

So that it is impossible for the interference to occur between the left and right spin photons. If the SVFs of the pair both describe, for example, left circularly polarized light beams, then they are

$$|\boldsymbol{\mathcal{A}}_1(t,\boldsymbol{r}_1)\rangle=\frac{1}{2}\mathrm{e}^{\mathrm{i}\alpha_1}\begin{pmatrix}1\\ \mathrm{i}\end{pmatrix}\mathrm{e}^{-\mathrm{i}(\omega t-\boldsymbol{\kappa}\cdot\boldsymbol{r}_1)},\tag{7a}$$

$$|\boldsymbol{\mathcal{A}}_2(t,\boldsymbol{r}_2)\rangle=\frac{1}{2}\mathrm{e}^{\mathrm{i}\alpha_2}\begin{pmatrix}1\\ \mathrm{i}\end{pmatrix}\mathrm{e}^{-\mathrm{i}(\omega t-\boldsymbol{\kappa}\cdot\boldsymbol{r}_2)},\tag{7b}$$

and

$$\begin{aligned} & P_{\text{intf}}\left(\alpha_1, \alpha_2, \boldsymbol{r}\right) \\ = \ & \cos\left[\left(\alpha_1 - \alpha_2\right) + \boldsymbol{\kappa} \cdot \left(\boldsymbol{r}_1 - \boldsymbol{r}_2\right)\right]. \end{aligned} \tag{8a}$$

The interference can occur between the left spin photons, but it depends on phase difference $\Delta\alpha = \alpha_1 - \alpha_2$ that determines whether the interference pattern can be observed or not macroscopically. If $\Delta\alpha$ is a constant, the interference pattern is displayed despite the fact that its distribution is different from the value of $\Delta\alpha$, especially at $\Delta\alpha = 0$ the distribution is independent of the actual value of $\alpha_1 = \alpha_2$, namely $P_{\text{intf}}\left(\alpha_1 = \alpha_2, \boldsymbol{r}\right) = \cos\left[\boldsymbol{\kappa} \cdot \left(\boldsymbol{r}_1 - \boldsymbol{r}_2\right)\right]$. This is just the result obtained by the use of two selfsame light sources.[4] Otherwise, if $\Delta\alpha$ is symmetrical distribution in a phase region, the average value of its cosine function is equal to zero, namely

$$\overline{\cos\left(\Delta\alpha + \delta\right)} \equiv \frac{1}{4\pi}\int_{-2\pi}^{2\pi} \cos\left(\Delta\alpha + \delta\right) d\left(\Delta\alpha\right) = 0,$$

and then

$$\overline{P_{\text{intf}}\left(\alpha_1, \alpha_2, \boldsymbol{r}\right)} = \overline{\cos\left[\Delta\alpha + \boldsymbol{\kappa} \cdot \left(\boldsymbol{r}_1 - \boldsymbol{r}_2\right)\right]} = 0. \tag{9a}$$

Thus the interference pattern is not observed macroscopically. In a similar fashion, the interference item about the right spin photons, namely the interference pattern between two right circularly polarized light beams, is

$$\begin{aligned} & P_{\text{intf}}\left(\beta_1, \beta_2, \boldsymbol{r}\right) \\ = \ & \cos\left[\left(\beta_1 - \beta_2\right) + \boldsymbol{\kappa} \cdot \left(\boldsymbol{r}_1 - \boldsymbol{r}_2\right)\right], \end{aligned} \tag{8b}$$

when $\Delta\beta = \beta_1 - \beta_2$ is of symmetrical distribution, the average value of the interference item is equal to zero, i.e.

$$\overline{P_{\text{intf}}\left(\beta_1, \beta_2, \boldsymbol{r}\right)} = \overline{\cos\left[\Delta\beta + \boldsymbol{\kappa} \cdot \left(\boldsymbol{r}_1 - \boldsymbol{r}_2\right)\right]} = 0. \tag{9b}$$

Case B The interference between two linearly polarized light beams.

The pair of SVFs is expressed as

$$\begin{aligned} \left|\boldsymbol{\mathcal{A}}_1\left(t, \boldsymbol{r}_1\right)\right\rangle = & \frac{1}{2\sqrt{2}}\left[\mathrm{e}^{\mathrm{i}\alpha_1}\begin{pmatrix}1\\ \mathrm{i}\end{pmatrix} + \mathrm{e}^{\mathrm{i}\beta_1}\begin{pmatrix}1\\ -\mathrm{i}\end{pmatrix}\right] \\ & \times \mathrm{e}^{-\mathrm{i}\left(\omega t - \boldsymbol{\kappa}\cdot\boldsymbol{r}_1\right)}, \end{aligned} \tag{10a}$$

$$\begin{aligned} \left|\boldsymbol{\mathcal{A}}_2\left(t, \boldsymbol{r}_2\right)\right\rangle = & \frac{1}{2\sqrt{2}}\left[\mathrm{e}^{\mathrm{i}\alpha_2}\begin{pmatrix}1\\ \mathrm{i}\end{pmatrix} + \mathrm{e}^{\mathrm{i}\beta_2}\begin{pmatrix}1\\ -\mathrm{i}\end{pmatrix}\right] \\ & \times \mathrm{e}^{-\mathrm{i}\left(\omega t - \boldsymbol{\kappa}\cdot\boldsymbol{r}_2\right)}, \end{aligned} \tag{10b}$$

which are just the general expressions (3a) and (3b) when $\sigma_{+,1}^{(1)} = \sigma_{-,1}^{(1)} = \sigma_{+,2}^{(1)} = \sigma_{-,2}^{(1)} = \dfrac{1}{\sqrt{2}}$, so here exists a pair of corresponding linearly polarized light waves similar to the form of expression (2c)

$$\boldsymbol{\mathcal{A}}_1\left(t, \boldsymbol{r}_1\right) = \mathrm{Re}\left|\boldsymbol{\mathcal{A}}_1\left(t, \boldsymbol{r}_1\right)\right\rangle = \frac{1}{\sqrt{2}}\cos\left(\omega t - \boldsymbol{\kappa}\cdot\boldsymbol{r}_1 - \frac{\beta_1 + \alpha_1}{2}\right)\left[\boldsymbol{i}\cos\left(\frac{\beta_1 - \alpha_1}{2}\right) + \boldsymbol{j}\sin\left(\frac{\beta_1 - \alpha_1}{2}\right)\right],$$

$$\boldsymbol{\mathcal{A}}_2\left(t, \boldsymbol{r}_2\right) = \mathrm{Re}\left|\boldsymbol{\mathcal{A}}_2\left(t, \boldsymbol{r}_2\right)\right\rangle = \frac{1}{\sqrt{2}}\cos\left(\omega t - \boldsymbol{\kappa}\cdot\boldsymbol{r}_2 - \frac{\beta_2 + \alpha_2}{2}\right)\left[\boldsymbol{i}\cos\left(\frac{\beta_2 - \alpha_2}{2}\right) + \boldsymbol{j}\sin\left(\frac{\beta_2 - \alpha_2}{2}\right)\right].$$

Their polarization orientations are $\phi_1 = (\alpha_1 - \beta_1)/2$ and $\phi_2 = (\alpha_2 - \beta_2)/2$, respectively. Owing to

$$\begin{aligned} \left|\boldsymbol{\mathcal{A}}_{12}\left(t, \boldsymbol{r}\right)\right\rangle = & \left|\boldsymbol{\mathcal{A}}_1\left(t, \boldsymbol{r}_1\right)\right\rangle + \left|\boldsymbol{\mathcal{A}}_2\left(t, \boldsymbol{r}_2\right)\right\rangle \\ = & \frac{\mathrm{e}^{-\mathrm{i}\omega t}}{2\sqrt{2}}\left[\mathrm{e}^{\mathrm{i}\left(\alpha_1 + \boldsymbol{\kappa}\cdot\boldsymbol{r}_1\right)}\begin{pmatrix}1\\ \mathrm{i}\end{pmatrix} + \mathrm{e}^{\mathrm{i}\left(\beta_1 + \boldsymbol{\kappa}\cdot\boldsymbol{r}_1\right)}\begin{pmatrix}1\\ -\mathrm{i}\end{pmatrix} + \mathrm{e}^{\mathrm{i}\left(\alpha_2 + \boldsymbol{\kappa}\cdot\boldsymbol{r}_2\right)}\begin{pmatrix}1\\ \mathrm{i}\end{pmatrix} + \mathrm{e}^{\mathrm{i}\left(\beta_2 + \boldsymbol{\kappa}\cdot\boldsymbol{r}_2\right)}\begin{pmatrix}1\\ -\mathrm{i}\end{pmatrix}\right], \end{aligned} \tag{11}$$

by simple arithmetical calculations it is not difficult to validate that

$$\begin{aligned} & P_{\text{intf}}\left(\alpha_1, \alpha_2, \beta_1, \beta_2, \boldsymbol{r}\right) \\ = \ & \frac{1}{2}\left\{\cos\left[\left(\alpha_1 - \alpha_2\right) + \boldsymbol{\kappa}\cdot\left(\boldsymbol{r}_1 - \boldsymbol{r}_2\right)\right] + \cos\left[\left(\beta_1 - \beta_2\right) + \boldsymbol{\kappa}\cdot\left(\boldsymbol{r}_1 - \boldsymbol{r}_2\right)\right]\right\} \end{aligned} \tag{12a}$$

$$= \cos\left[\frac{\left(\alpha_1 - \alpha_2\right) - \left(\beta_1 - \beta_2\right)}{2}\right]\cos\left[\frac{\left(\alpha_1 - \alpha_2\right) + \left(\beta_1 - \beta_2\right)}{2} + \boldsymbol{\kappa}\cdot\left(\boldsymbol{r}_1 - \boldsymbol{r}_2\right)\right] \tag{12b}$$

$$= \cos\left[\left(\frac{\alpha_1 - \beta_1}{2}\right) - \left(\frac{\alpha_2 - \beta_2}{2}\right)\right]\cos\left[\left(\frac{\alpha_1 + \beta_1}{2}\right) - \left(\frac{\alpha_2 + \beta_2}{2}\right) + \boldsymbol{\kappa}\cdot\left(\boldsymbol{r}_1 - \boldsymbol{r}_2\right)\right]. \tag{12c}$$

Let $\Delta\phi = \phi_1 - \phi_2 = \left(\frac{\alpha_1 - \beta_1}{2}\right) - \left(\frac{\alpha_2 - \beta_2}{2}\right)$ denote the crossing angle between polarization orientations of the two linearly polarized light waves, it is clear from expression (12c) that the cosine function $\cos(\Delta\phi)$ completely determines the interference characteristic: if the two polarization orientations are parallel to each other, i.e. $\Delta\phi = 0$, then $\cos(\Delta\phi) = 1$ that is the maximum. Especially at $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$, the interference item is obtained to be $P_{\text{intf}}(\alpha_1 = \alpha_2,\ \beta_1 = \beta_2, \boldsymbol{r}) = \cos[\boldsymbol{\kappa}\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2)]$ from expression (12b), which means that the interference item is independent of the values of those parameters. This is the same result as that obtained by the use of two selfsame light sources.[4] If the two polarization orientations are perpendicular to each other, i.e. $|\Delta\phi| = \pi/2$, then $\cos(\Delta\phi) = 0$, thus the interference pattern disappears macroscopically.

Case C The interference between two natural light beams.

The natural light beam can be regard as the linear superposition of a group of linearly polarized light beams with symmetrically distributed polarization orientations. When two natural light beams are described with the selfsame SVF except for different positions of their grid origins, namely in expressions (10a) and (10b) $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$ except $\boldsymbol{r}_1 \neq \boldsymbol{r}_2$, then by the use of expression (12b), the formula is directly reduced to

$$\begin{aligned} &P_{\text{intf}}(\alpha_1 = \alpha_2,\quad \beta_1 = \beta_2,\quad \boldsymbol{r}) \\ &= \cos[\boldsymbol{\kappa}\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2)]. \end{aligned} \tag{13}$$

This is also the above result that has been obtained.[4] If two independent natural light beams are described with different SVFs, namely there is no relation between the polarization orientations $\phi_1 = (\alpha_1 - \beta_1)/2$ and $\phi_2 = (\alpha_2 - \beta_2)/2$, then they both are of symmetrical distribution, and all the phases α_1, β_1, α_2, and β_2 turn random. Consequently the phase difference $\Delta\alpha = \alpha_1 - \alpha_2$ and $\Delta\beta = \beta_1 - \beta_2$ are of symmetrical distribution, too. According to the above analysis $\overline{\cos(\Delta\alpha + \delta)} = \overline{\cos(\Delta\beta + \delta)} = 0$, it is easy to obtain from expression (12a) the average value of the interference item to be equal to zero, i.e.

$$\begin{aligned} &\overline{P_{\text{intf}}(\alpha_1, \alpha_2, \beta_1, \beta_2, \boldsymbol{r})} \\ &= \frac{1}{2}\left\{\overline{\cos[\Delta\alpha + \boldsymbol{\kappa}\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2)]}\right. \\ &\quad \left. + \overline{\cos[\Delta\beta + \boldsymbol{\kappa}\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2)]}\right\} = 0. \end{aligned} \tag{14}$$

So the interference pattern of two independent natural light beams is not observed macroscopically, either.

Case D The interference between two arbitrarily polarized light beams with the same intensity.

Here the pair of SVFs $|\boldsymbol{\mathcal{A}}_1(t, \boldsymbol{r}_1)\rangle$ and $|\boldsymbol{\mathcal{A}}_2(t, \boldsymbol{r}_2)\rangle$ can be directly expressed by using expressions (3a) and (3b), and it is not difficult to obtain

$$\begin{aligned} &P_{\text{intf}}\left(\sigma^{(1)}_{+,1}, \sigma^{(1)}_{-,1}, \sigma^{(1)}_{+,2}, \sigma^{(1)}_{-,2}, \alpha_1, \alpha_2, \beta_1, \beta_2, \boldsymbol{r}\right) \\ &= \sigma^{(1)}_{+,1}\sigma^{(1)}_{+,2}\cos[(\alpha_1 - \alpha_2) + \boldsymbol{\kappa}\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2)] \\ &\quad + \sigma^{(1)}_{-,1}\sigma^{(1)}_{-,2}\cos[(\beta_1 - \beta_2) + \boldsymbol{\kappa}\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2)]. \end{aligned} \tag{15}$$

When $\sigma^{(1)}_{+,1} = \sigma^{(1)}_{+,2} = 1$ and $\sigma^{(1)}_{-,1} = \sigma^{(1)}_{-,2} = 0$, expression (15) is reduced to expression (8a) or (9a); when $\sigma^{(1)}_{+,1} = \sigma^{(1)}_{+,2} = 0$ and $\sigma^{(1)}_{-,1} = \sigma^{(1)}_{-,2} = 1$, expression (15) is reduced to expression (8b) or (9b); when $\sigma^{(1)}_{+,1} = \sigma^{(1)}_{-,1} = \sigma^{(1)}_{+,2} = \sigma^{(1)}_{-,2} = \sqrt{2}/2$, expression (15) is reduced to expressions (12a), (13) or (14).

Case E The interference between two arbitrarily polarized light beams with different intensities.

Two separated holes irradiated by two independent light sources are equivalent to two independent lasers. The photons that pass through one of two holes, in general, should have a different probability from the photos that pass through another hole, which is dependent on the hole size, so that the pair of SVFs can be expressed as

$$\begin{aligned} |\boldsymbol{\mathcal{A}}_1(t, \boldsymbol{r}_1)\rangle = \frac{a}{\sqrt{2}}&\left[\sigma^{(1)}_{+,1}\mathrm{e}^{\mathrm{i}\alpha_1}\begin{pmatrix}1\\ \mathrm{i}\end{pmatrix} + \sigma^{(1)}_{-,1}\mathrm{e}^{\mathrm{i}\beta_1}\begin{pmatrix}1\\ -\mathrm{i}\end{pmatrix}\right] \\ &\times \mathrm{e}^{-\mathrm{i}(\omega t - \boldsymbol{\kappa}\cdot\boldsymbol{r}_1)}, \end{aligned} \tag{16a}$$

$$\begin{aligned} |\boldsymbol{\mathcal{A}}_2(t, \boldsymbol{r}_2)\rangle = \frac{b}{\sqrt{2}}&\left[\sigma^{(1)}_{+,2}\mathrm{e}^{\mathrm{i}\alpha_2}\begin{pmatrix}1\\ \mathrm{i}\end{pmatrix} + \sigma^{(1)}_{-,2}\mathrm{e}^{\mathrm{i}\beta_2}\begin{pmatrix}1\\ -\mathrm{i}\end{pmatrix}\right] \\ &\times \mathrm{e}^{-\mathrm{i}(\omega t - \boldsymbol{\kappa}\cdot\boldsymbol{r}_2)}. \end{aligned} \tag{16b}$$

Since

$$\begin{aligned} &\langle\boldsymbol{\mathcal{A}}_1(t, \boldsymbol{r}_1)|\,|\boldsymbol{\mathcal{A}}_1(t, \boldsymbol{r}_1)\rangle \\ &= a^2\left\{\left[\sigma^{(1)}_{+,1}\right]^2 + \left[\sigma^{(1)}_{-,1}\right]^2\right\} = a^2 \end{aligned}$$

and

$$\begin{aligned} &\langle\boldsymbol{\mathcal{A}}_2(t, \boldsymbol{r}_2)|\,|\boldsymbol{\mathcal{A}}_2(t, \boldsymbol{r}_2)\rangle \\ &= b^2\left\{\left[\sigma^{(1)}_{+,2}\right]^2 + \left[\sigma^{(1)}_{-,2}\right]^2\right\} = b^2 \end{aligned}$$

represent the respective probabilities, the real numbers $0 \leq a \leq 1$ and $0 \leq b \leq 1$ are just probability amplitudes of the photons that respectively pass through

holes 1 and 2 in the Young's double-hole experiment, and the normalization is represented as

$$a^2 + b^2 = 1. \tag{17}$$

The general expression of the interference item in the Young's double-hole experiment with two independent light sources that possess different polarizations and different intensities is given as

$$\begin{aligned} & P_{\text{intf}}\left(a, b, \sigma_{+,1}^{(1)}, \sigma_{-,1}^{(1)}, \sigma_{+,2}^{(1)}, \sigma_{-,2}^{(1)}, \alpha_1, \alpha_2, \beta_1, \beta_2; \boldsymbol{r}\right) \\ = & \ 2ab\left\{\sigma_{+,1}^{(1)}\sigma_{+,2}^{(1)}\cos\left[(\alpha_1 - \alpha_2) + \boldsymbol{\kappa}\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2)\right] + \sigma_{-,1}^{(1)}\sigma_{-,2}^{(1)}\cos\left[(\beta_1 - \beta_2) + \boldsymbol{\kappa}\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2)\right]\right\}. \end{aligned} \tag{18}$$

It is obvious that whether $a = 0$ or $b = 0$, the interference item must be equal to zero. According to Ref.[4], the spatially averaged interference item can be expressed as

$$\begin{aligned} & \overline{P_{\text{intf}}\left(a, b, \sigma_{+,1}^{(1)}, \sigma_{-,1}^{(1)}, \sigma_{+,2}^{(1)}, \sigma_{-,2}^{(1)}, \alpha_1, \alpha_2, \beta_1, \beta_2; \boldsymbol{r}\right)} \\ \equiv & \lim_{V\to\infty}\frac{1}{V}\int_V P_{\text{intf}}\left(a, b, \sigma_{+,1}^{(1)}, \sigma_{-,1}^{(1)}, \sigma_{+,2}^{(1)}, \sigma_{-,2}^{(1)}, \alpha_1, \alpha_2, \beta_1, \beta_2; \boldsymbol{r}\right)\mathrm{d}V \\ = & \ 2ab\left[\sigma_{+,1}^{(1)}\sigma_{+,2}^{(1)}\cos(\alpha_1 - \alpha_2) + \sigma_{-,1}^{(1)}\sigma_{-,2}^{(1)}\cos(\beta_1 - \beta_2)\right]\frac{\sin\kappa\mathrm{d}}{\kappa\mathrm{d}}, \end{aligned} \tag{19}$$

where $d \equiv |\boldsymbol{r}_1 - \boldsymbol{r}_2|$ is the distance between hole 1 and hole 2. When $a = b = \dfrac{1}{\sqrt{2}}$, $\sigma_{+,1}^{(1)} = \sigma_{+,2}^{(1)} = \sigma_{-,1}^{(1)} = \sigma_{-,2}^{(1)} = \dfrac{1}{\sqrt{2}}$, $\alpha_1 = \alpha_2$, and $\beta_1 = \beta_2$, expression (19) is reduced to

$$\overline{P_{\text{intf}}\left(a = b = \frac{1}{\sqrt{2}}, \sigma_{+,1}^{(1)} = \sigma_{+,2}^{(1)} = \sigma_{-,1}^{(1)} = \sigma_{-,2}^{(1)} = \frac{1}{\sqrt{2}}, \alpha_1 = \alpha_2, \beta_1 = \beta_2; \boldsymbol{r}\right)} = \frac{\sin\kappa\mathrm{d}}{\kappa\mathrm{d}}$$

that is a normalized interference function, and it has been obtained by the use of identical light sources each with an equivalent intensity.[4]

3. Discussion

As early as in 1930, Dirac,[7] in his famous textbook on quantum mechanics, made a basic statement on optical interference: 'Each photon interferes only with itself. Interference between two different photons never occurs.' While this assertion was ceaselessly quoted, there always existed some doubts. It has brought a continual argument to researchers for three quarters of a century long. Since then an impressive progress in the development of both light sources and detectors has been made. Especially in 1963 Magyar and Mendel[8] observed the effect arising from two independent laser beams, and in 1967 Pfleegor and Mendel[9] provided experimental evidence of interference between strongly damped laser beams, pointing out that 'It is found that interference takes place even under conditions in which the light intensities are so low that, with high probability, one photon is absorbed before the next one is emitted by one or the other source.' So that in 1986 Paul[10] asserted: 'Hence, the second part of Dirac's famous statement quoted above has actually been disproved.'

From the point of view of classical optics, it is self-evident that optical interference comes from the redistribution of light intensities. From the point of view of quantum optics, however, optical interference is unnecessarily the redistribution of light intensities, and it is even not a redistribution of photons themselves, but it is the redistribution of probability amplitudes of photons. It is just as expression (18) shows that this redistribution occurs only between the same photons that possess a set of the selfsame eigenvalues, besides the same energy eigenvalue $E = \hbar\omega$ and the same momentum eigenvalue $p_z = \hbar\kappa$, particularly the same angular momentum eigenvalue $\hat{s}_{z+} = +1\hbar$ or $\hat{s}_{z-} = -1\hbar$. On the right-hand side of expression (18), the first item represents the interference between left spin photons that pass through hole 1 or hole 2; the

second item represents the interference between right spin photons that also pass through hole 2 or hole 1. Furthermore, as mentioned above, although the phase itself does not affect the interference between two single photons microscopically, the differences in phase between two groups of photons that possess a set of selfsame eigenvalues but different phases, determines whether the interference pattern can be observed macroscopically.

Finally, we try to complete Dirac's famous statement quoted above as follows: every photon interferes only with itself that possesses a set of the selfsame eigenvalues. Interference between two different photons that possess different eigenvalues never occurs.

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