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Dynamical behaviour of Liu system with time delayed feedback*

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This paper investigates the dynamical behaviour of the Liu system with time delayed feedback. Two typical situations are considered and the effect of time-delay parameter on the dynamics of the system is discussed. It is shown that the Liu system with time delayed feedback may exhibit interesting and extremely rich dynamical behaviour. The evolution of the dynamics is shown to be complex with varying time-delay parameter. Moreover, the strange attractor like 'wormhole' is detected via numerical simulations.

Keywords: Liu system, time delayed feedback, dynamical behaviour, strange attractor
PACC: 0547

1. Introduction

The bifurcations and chaotic dynamics of nonlinear systems have been studied widely in the last decades. Hence, various three-dimensional (3D) chaotic systems were found and investigated, showing some interesting, and sometimes unexpected results. As reported by Smale,[1] there are eighteen challenging mathematical problems for the twenty-first century, in which the fourteenth problem is about the Lorenz attractor. Thus, the Lorenz and 3D Lorenz-like systems are of great practical importance. In the last two decades, various Lorenz-like systems were discovered and studied.[2−9] The main topic in this field is the analysis of bifurcations, chaos and its compound structures, controlling chaos, chaos synchronization, and so on.[10−20] Recently, Liu *et al*[9] developed a new 3D chaotic system and have shown some interesting results. In this study, the Liu system is further studied. However, a time delayed feedback signal is added to the equations of Liu system. Due to the applying of time delayed feedbacks, the modified Liu system can display much richer dynamical behaviour, as described below.

2. Liu system with time delayed feedbacks and some interesting results

In Ref.[9], the autonomy differential equations that describe the Liu system are

$$\dot{x} = a(y - x), \quad \dot{y} = bx - kxz, \quad \dot{z} = -cz + hx^2, \tag{1}$$

where a, b, c, k, and h are real constants, x, y and z are the state variables. As shown in Ref.[9], when (a, b, c, k, h) = (10, 40, 2.5, 1, 4), the Liu system acts as a chaotic attractor. The chaotic figure for system (1) is shown in Fig.1.

Now, we consider the Liu delay system denoted as system (2), which is described by the following equations:

$$\dot{x} = a(y - x),$$

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$$\dot{y} = bx - kxz - K_2(y(t-\tau) - y(t)),$$
$$\dot{z} = -cz + hx^2, \qquad (2)$$

in which K_2 is feedback gain and τ is delay time.

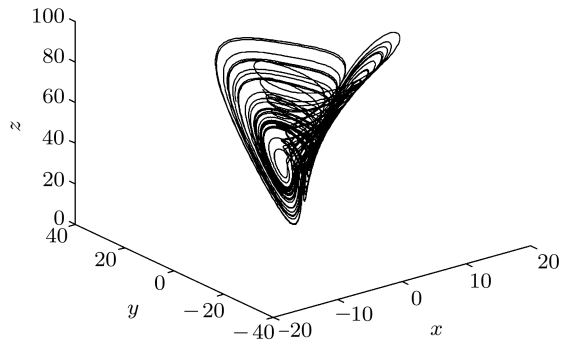


Fig.1. Liu chaotic attractor.

Solutions of Eq.(2) were obtained by using a Runge-Kutta integration algorithm, with a step size of 0.0025. For the current study, the system parameters are specified as (a, b, c, k, h) = (10, 40, 2.5, 1, 4), throughout the paper. Fix $K_2 = 5$ and let τ vary. Then system (2) is numerically calculated by using $\tau \in [0,\ 0.6]$, with the increment of τ equal 0.005. The bifurcation diagram of y against τ is presented in Fig.2. It is found that there exist complex bifurcation branches in this parameter region. As shown in this

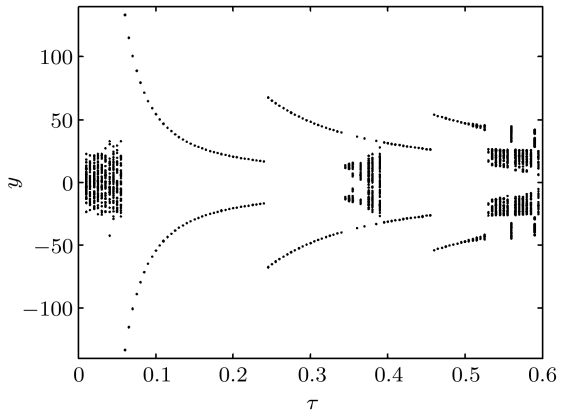


Fig.2. Bifurcation diagram for system (2) with the variable parameter of τ.

figure, periodic, chaotic or quasi-periodic motions may occur. However, the quasi-periodic and chaotic motions can not be distinguished from each other in the bifurcation diagrams. Hence, more extensive calculations have produced the phase portraits and power spectrum diagrams with various values of τ. In Fig.3, it is observed that system (2) may display various motions, as mentioned in the foregoing. Figs.3(b) and 3(e) show periodic orbits; 3(a) and 3(d) show chaotic motions; 3(c) and 3(f) show quasi-periodic motions, which can be demonstrated via construction of power spectrum diagrams. It is noted that periodic and quasi-periodic orbits result in power spectrum function that will have limited frequency bandwidths. For chaotic motions, however, the power spectrum function will have significant values for broad frequency bandwidths, which can be clearly seen in Fig.4. Moreover, it ought to be mentioned that, the two new attractors represented in Figs.3(c) and 3(f) were not found in the Liu system. The attractor shown in Fig.3(f) is very strange like wormhole.

In what follows, we consider another modified Liu system with time delay feedbacks denoted as system (3), which is described by the following equations:

$$\dot{x} = a(y - x) - K_1(x(t-\tau) - x(t)),$$
$$\dot{y} = bx - kxz - K_2(y(t-\tau) - y(t)),$$
$$\dot{z} = -cz + hx^2 - K_3(z(t-\tau) - z(t)), \qquad (3)$$

in which K_1, K_2, K_3 are feedback gains and τ is delay time.

Similarly, we fix $K_1 = K_2 = K_3 = 5$ and let τ vary. Sample results are given in Figs.5–7. In this case, system (3) is chaotic when τ is small. However, a period-1 motion occur at $\tau \approx 0.0378$, which finally evolves to another new strange attractor, as shown in Fig.6(b). This attractor is quasi-periodic as demonstrated in Fig.7.

3. Conclusion

Thus, in this study, the Liu system with time delayed feedbacks has been studied and some new dynamical behaviours are represented for two typical modified Liu systems. It is shown that these two modified Liu systems can display very rich dynamics when time delayed feedbacks are added to the equations of Liu system. Particularly, the strange attractor like wormhole is detected. In addition, this study shows that it is easy to switch from a chaotic trajectory to a regular periodic orbit when τ is varied in a large range (see Figs.2 and 5). This implies that the chaotic orbits of the Liu system can be controlled with time delayed feedback.

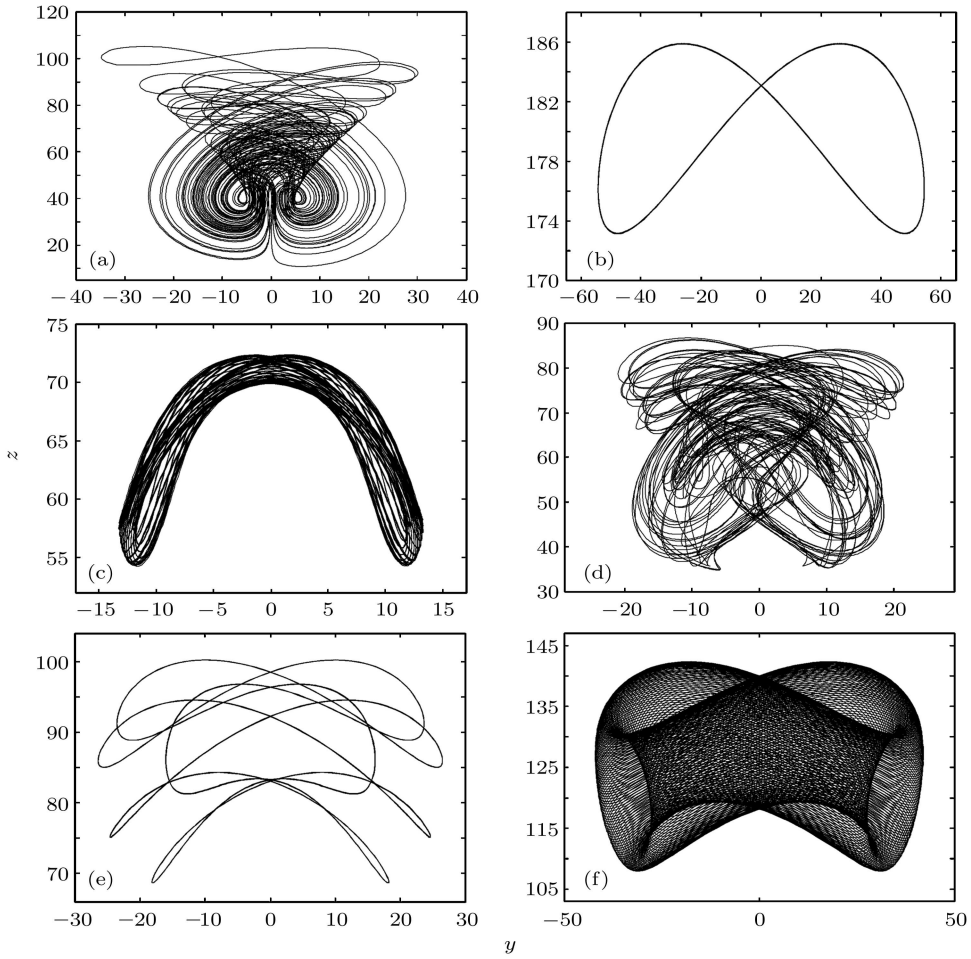


Fig.3. Phase portraits on $y-z$ plane for system (2) with various τ: (a) $\tau = 0.03$; (b) $\tau = 0.1$; (c) $\tau = 0.345$; (d) $\tau = 0.38$; (e) $\tau = 0.53$; (f) $\tau = 0.59$.

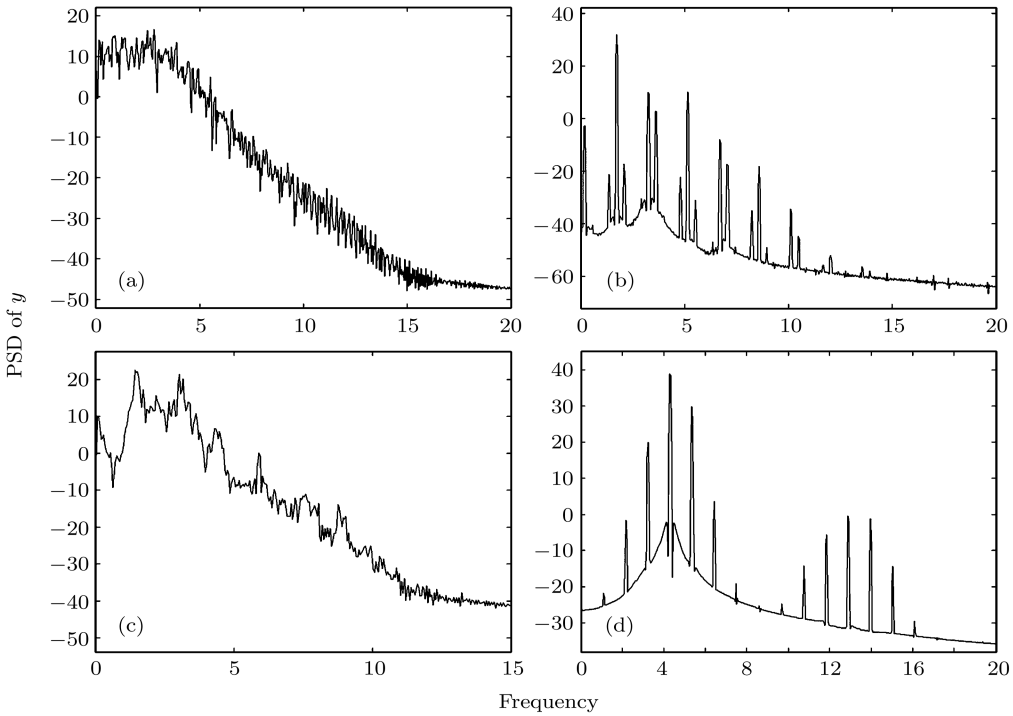


Fig.4. Power spectrum of y for system (2) with various τ: (a) $\tau = 0.03$; (b) $\tau = 0.345$; (c) $\tau = 0.38$; (d) $\tau = 0.59$.

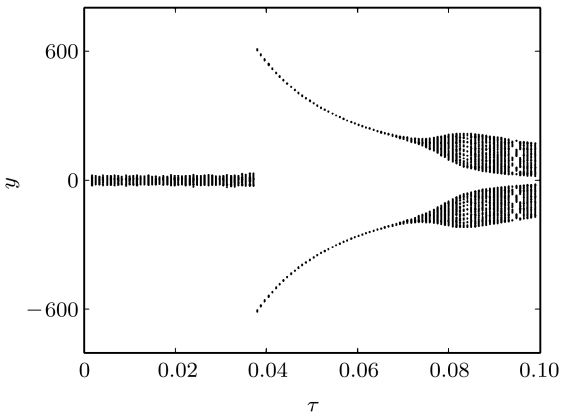


Fig.5. Bifurcation diagram for system (3) with the variable parameter of τ.

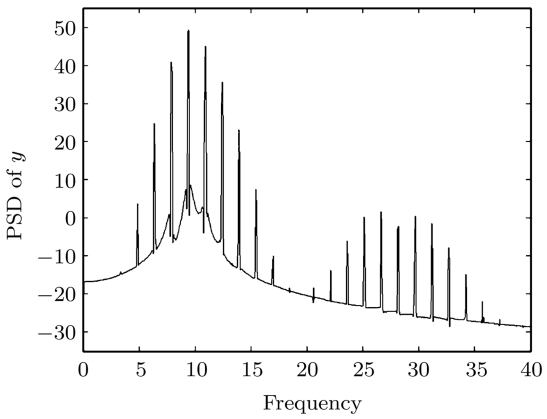


Fig.7. Power spectrum of quasi-periodic behaviour for system (3), and $\tau = 0.08$.

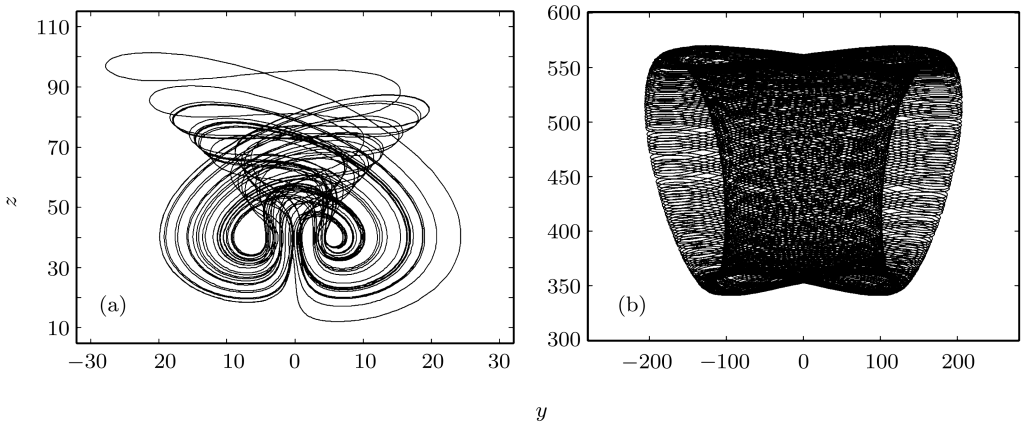


Fig.6. Phase portraits on $y - z$ plane for system (3) with various τ: (a) $\tau = 0.02$; (b) $\tau = 0.08$.

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