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Chaotic motion of the dynamical system under both additive and multiplicative noise excitations*

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With both additive and multiplicative noise excitations, the effect on the chaotic behaviour of the dynamical system is investigated in this paper. The random Melnikov theorem with the mean-square criterion that applies to a type of dynamical systems is analysed in order to obtain the conditions for the possible occurrence of chaos. As an example, for the Duffing system, we deduce its concrete expression for the threshold of multiplicative noise amplitude for the rising of chaos, and by combining figures, we discuss the influences of the amplitude, intensity and frequency of both bounded noises on the dynamical behaviour of the Duffing system separately. Finally, numerical simulations are illustrated to verify the theoretical analysis according to the largest Lyapunov exponent and Poincaré map.

Keywords: Melnikov theory, bounded noise, Lyapunov exponent, Poincaré map

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1. Introduction

Chaotic phenomena in nonlinear dynamical systems have been studied in the last two decades due to their great potential applications. It is known that chaotic behaviour is complex and unpredictable and one prominent feature is extremely sensitive to initial conditions and variation of parameters. Now, there exist a considerable number of articles about this topic in the literature.[1−5] Jing and Huang[6] considered a three-well Duffing system with two external periodic forces, and gave the criteria of existence of chaos under periodic and quasi-periodic perturbations by using the Melnikov's method, while Ref.[7] discussed the chaotic behaviour of a nonlinear damped three-well Φ^6-Van der Pol oscillator under external and parametric excitations in detail by using the Melnikov analysis.

Noises exist inevitably in actual circumstances, so some people switched to the discussion on the influence of noise on chaotic motion of the dynamical system.[8−13] Gan[14] studied the additive noise-induced chaos and discussed the effect of noise on erosion of safe basin in the softening Duffing oscillator. The random Melnikov approach was also adopted to obtain the necessary conditions for the rising of chaos. Simulations were given to verify the feasibility of the theory. The effect of multiplicative bounded noise on the chaotic behaviour of the Duffing oscillator was studied by Liu *et al*,[15] where the mean-square criterion of the random Melnikov process was employed to detect the chaotic dynamics in the system. Under additive bounded noise, the influence on the chaotic motion of a prototypical bistable system was studied analytically and numerically by Sun *et al*.[16]

But up to now, less effort has been devoted to studying the influence on the system behaviour under both additive and multiplicative noise excitations. So in this paper, we investigate the effect on the chaotic motion of the dynamical system subjected to both additive and multiplicative noise excitations.

This paper is organized as follows. In Section 2, the random Melnikov theorem with the mean-square criterion is analysed under both additive and multiplicative noise excitations; meanwhile bounded noise is introduced briefly. In Section 3, we apply the random Melnikov theorem to the Duffing system to deduce the concrete expression that is the necessary conditions of chaos occurring. While Section 4 gives numerical simulations by means of the Lyapunov exponents and Poincaré map. Section 5 presents some conclusions drawn from the present study.

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2. General description

The Melnikov method is an analytical tool to provide the criterion for the occurrence of chaos.[14−16] The Melnikov technique is a perturbation method of proving the existence of transverse homoclinic or heteroclinic orbits. It was first applied by Holmes[17] to the study of the chaotic attractor of a periodically driven Duffing oscillator with negative linear stiffness. Wiggins[18,19] introduced a generalized version of the Melnikov's function for a system subjected to an excitation with multiple frequencies. Frey and Simiu[20] presented a generalized random Melnikov technique to study the effect of noise on near-integrable second-order dynamical system. Xie[21] studied the effect of the chaotic behaviour of a buckled column under parametric excitation using a modified Melnikov method. It was not until 1996 that the mean-square criterion for the Melnikov process was used to study the periodically forced Duffing system with additive random perturbation by Lin and Yim.[22]

In this section, the stochastic Melnikov theorem with the mean-square criterion that applies to a type of dynamical systems is analysed to deduce the necessary conditions of the occurrence of chaotic phenomenon. We can see that the forms of Melnikov theorem in Refs.[14–16] are all particular cases of this method.

Consider a one-degree-of-freedom Hamiltonian system with light damping under both additive and multiplicative noise excitations. The equation of motion of the system is

$$\begin{aligned}\dot{Q} =&\frac{\partial H}{\partial P},\\ \dot{P} =&-\frac{\partial H}{\partial Q}-\varepsilon c(P,Q)\frac{\partial H}{\partial P}+\varepsilon\gamma\cos\Omega t\\ &+\varepsilon f(P,Q)\xi_1(t)+\varepsilon\xi_2(t),\end{aligned}\tag{1}$$

where ε is the small positive parameter; $c(P,Q)$ denotes the coefficient of quasi-linear damping; γ and Ω are the amplitude and frequency of the external harmonic force; $f(Q,P)$ represents the amplitude of excitation; $\xi_1(t)$ and $\xi_2(t)$ are independent bounded noises which are of stationary random process with zero mean.

A harmonic function with constant amplitude and random frequency and phase is called bounded noise. So, $\xi_1(t)$ and $\xi_2(t)$ are expressed as

$$\begin{aligned}&\xi_i(t)=\mu_i\sin(\Omega_i t+\Phi_i),\\ &\Phi_i=\sigma_i B_i(t)+\Gamma_i,\qquad i=1,2,\end{aligned}\tag{2}$$

where μ_i,Ω_i,σ_i are positive constants; $B_1(t)$ and $B_2(t)$ are unit Wiener processes; Γ_1 and Γ_2 are random variables uniformly distributed in $[0,2\pi)$. Accordingly their covariance function and spectral densities can be obtained (see Ref.[16]).

It is assumed that Eq.(1) possesses a hyperbolic fixed point connected to itself by homoclinic or heteroclinic orbits $(q_0(t),p_0(t))$. So, the Melnikov function for Eq.(1) can be derived by using the formula given by Wiggins[19]

$$\begin{aligned}M(t_0)=\int_{-\infty}^{\infty}\frac{\partial H}{\partial P}\Big[&-c(P,Q)\frac{\partial H}{\partial P}+\gamma\cos\Omega(t+t_0)\\ &+f(P,Q)\xi_1(t+t_0)+\xi_2(t+t_0)\Big]\mathrm{d}t,\end{aligned}\tag{3}$$

where $Q(t)$ and $P(t)$ in expression (3) are taken as $q_0(t)$ and $p_0(t)$ of homoclinic (heteroclinic) orbit.

For positive damping, the expectation of $M(t_0)$ is negative, that is,

$$E[M(t_0)]=-\int_{-\infty}^{\infty}c(P,Q)\left(\frac{\partial H}{\partial P}\right)^2\mathrm{d}t<0,$$

which shows that system (1) cannot be chaotic in the sense of average.

Therefore, the mean-square criterion for the stochastic Melnikov process[20] is employed

$$\begin{aligned}\Lambda_1^2&=E\left[\left(\int_{-\infty}^{\infty}c(P,Q)\left(\frac{\partial H}{\partial P}\right)^2\mathrm{d}t\right)^2\right],\\ \Lambda_2^2&=E\left[\left(\int_{-\infty}^{\infty}\frac{\partial H}{\partial P}\gamma\cos\Omega(t+t_0)\mathrm{d}t\right)^2\right],\\ \tilde{\Lambda}_1^2&=E\left[\left\{\int_{-\infty}^{\infty}\frac{\partial H}{\partial P}f(P,Q)\xi_1(t+t_0)\mathrm{d}t\right\}^2\right],\\ \tilde{\Lambda}_2^2&=E\left[\left\{\int_{-\infty}^{\infty}\frac{\partial H}{\partial P}\xi_2(t+t_0)\mathrm{d}t\right\}^2\right],\end{aligned}\tag{4}$$

where $\tilde{\Lambda}_k^2$ $(k=1,2)$ can be obtained from the following expression (their detailed explanation can be seen in Ref.[16]):

$$\tilde{\Lambda}_k^2=\int_{-\infty}^{\infty}|H_k(\omega)|^2S_{\xi_k}(\omega)\mathrm{d}\omega,\quad k=1,2,\tag{5}$$

where $S_{\xi_k}(\omega)$ $(k=1,2)$ is the spectral density of ξ_k $(k=1,2)$ and $H_k(\omega)$ $(k=1,2)$ is the frequency response function of the system, which is the Fourier transform of the impulse response function, i.e.

$$\begin{aligned}H_1(\omega)&=\int_{-\infty}^{\infty}\frac{\partial H}{\partial P}f(P,Q)\mathrm{e}^{-\mathrm{j}\,\omega t}\mathrm{d}t,\\ H_2(\omega)&=\int_{-\infty}^{\infty}\frac{\partial H}{\partial P}\mathrm{e}^{-\mathrm{j}\,\omega t}\mathrm{d}t,\quad k=1,2.\end{aligned}\tag{6}$$

The random Melnikov process has a simple zero in mean-square sense when

$$\Lambda_1^2 = \Lambda_2^2 + \tilde{\Lambda}_1^2 + \tilde{\Lambda}_2^2. \tag{7}$$

3. Analysis of Duffing system

The Duffing equation is a well-known vibration equation, which can describe electrical circuits and mechanical problems like buckled beams when only one mode of vibration is considered. Consider the effect of noise, the Duffing equation can be rewritten as follows:

$$\begin{aligned}\dot{Q} =& P,\\ \dot{P} =& Q - \alpha Q^3 - \varepsilon\beta P + \varepsilon\gamma\cos\Omega t\\ &+ \varepsilon Q\xi_1(t) + \varepsilon\xi_2(t).\end{aligned} \tag{8}$$

Equation (8) can be called the perturbed Duffing equation with stochastic noise, which is much more complex than its deterministic counterpart. The equation of the unperturbed Duffing system is expression (8) with ε=0, and it has a hyperbolic saddle point (0,0) and two homoclinic orbits that are

$$\begin{aligned}p_0(t) &= \pm\sqrt{\frac{2}{\alpha}}\operatorname{sech} t,\\ q_0(t) &= \mp\sqrt{\frac{2}{\alpha}}\operatorname{sech} t\tanh t.\end{aligned} \tag{9}$$

From formula (3), the random Melnikov function of Eq.(8) is deduced as follows:

$$\begin{aligned}M(t_0) =& \int_{-\infty}^{+\infty}[-\beta p_0^2(t) + \gamma p_0(t)\cos\Omega(t+t_0)\\ &+ p_0(t)q_0(t)\xi_1(t+t_0) + p_0(t)\xi_2(t+t_0)]\mathrm{d}t.\end{aligned}$$

The mean-square criterion is adopted here in order to obtain the simple zero of Melnikov function $M(t_0)$. The expressions for Λ_1^2 and Λ_2^2 are

$$\begin{aligned}\Lambda_1^2 =& E\left[\left\{\beta\int_{-\infty}^{+\infty} p_0^2(t)\mathrm{d}t\right\}^2\right]\\ =& \left\langle\frac{4\beta}{3\alpha}\right\rangle^2,\end{aligned} \tag{10}$$

$$\begin{aligned}\Lambda_2^2 =& E\left[\left\{\int_{-\infty}^{\infty}\gamma p_0(t)\cos\Omega(t+t_0)\mathrm{d}t\right\}^2\right]\\ =& \left\langle\pm\sqrt{2}\pi\Omega\gamma\sin\Omega t_0\operatorname{csch}^2\frac{\pi\Omega}{2}\right\rangle^2,\end{aligned} \tag{11}$$

and the expressions of $\tilde{\Lambda}_1^2$ and $\tilde{\Lambda}_2^2$ can be obtained as follows:

$$\tilde{\Lambda}_1^2 = \int_{-\infty}^{\infty}|H_1(\omega)|^2 S_{\xi_1}(\omega)\mathrm{d}\omega,$$

and

$$\tilde{\Lambda}_2^2 = \int_{-\infty}^{\infty}|H_2(\omega)|^2 S_{\xi_2}(\omega)\mathrm{d}\omega.$$

From expressions (6) and (9), the frequency response functions of the system are obtained to be

$$\begin{aligned}H_1(\omega) =& \int_{-\infty}^{\infty} p_0(t)q_0(t)\mathrm{e}^{-\mathrm{j}\,\omega t}\mathrm{d}t\\ =& j\left(\frac{\pi\omega^2}{\alpha}\right)\operatorname{csch}\left(\frac{\pi\omega}{2}\right),\\ H_2(\omega) =& \int_{-\infty}^{\infty} p_0(t)\mathrm{e}^{-\mathrm{j}\,\omega t}\mathrm{d}t\\ =& \pm\sqrt{\frac{2}{\alpha}}\operatorname{sech}\left(\frac{\pi\omega}{2}\right).\end{aligned}$$

Therefore, by using the spectral density that is given in Ref.[16], we can obtain

$$\begin{aligned}\tilde{\Lambda}_1^2 =& \frac{\pi^2\mu_1^2\sigma_1^2}{4\pi\alpha^2}\int_{-\infty}^{\infty}\omega^4\operatorname{csch}^2\left(\frac{\pi\omega}{2}\right)\\ &\times\frac{\omega^2+\Omega_1^2+\sigma_1^4/4}{(\omega^2-\Omega_1^2-\sigma_1^4/4)^2+\sigma_1^4\omega^2}\mathrm{d}\omega,\end{aligned} \tag{12}$$

$$\begin{aligned}\tilde{\Lambda}_2^2 =& \frac{2\mu_2^2\pi^2\sigma_2^2}{4\pi\alpha}\int_{-\infty}^{\infty}\operatorname{sech}^2\left(\frac{\pi\omega}{2}\right)\\ &\times\frac{\omega^2+\Omega_2^2+\sigma_2^4/4}{(\omega^2-\Omega_2^2-\sigma_2^4/4)^2+\sigma_2^4\omega^2}\mathrm{d}\omega.\end{aligned} \tag{13}$$

The criterion for possible chaotic phenomenon based on the mean-square method is

$$E[M^2(t_0)] = -\Lambda_1^2 + \Lambda_2^2 + \tilde{\Lambda}_1^2 + \tilde{\Lambda}_2^2 = 0.$$

Let

$$\begin{aligned}I_1 =& \frac{\pi^2\sigma_1^2}{4\pi\alpha^2}\int_{-\infty}^{\infty}\omega^4\operatorname{csch}^2\left(\frac{\pi\omega}{2}\right)\\ &\times\frac{\omega^2+\Omega_1^2+\sigma_1^4/4}{(\omega^2-\Omega_1^2-\sigma_1^4/4)^2+\sigma_1^4\omega^2}\mathrm{d}\omega,\\ I_2 =& \frac{2\pi^2\sigma_2^2}{4\pi\alpha}\int_{-\infty}^{\infty}\operatorname{sech}^2\left(\frac{\pi\omega}{2}\right)\\ &\times\frac{\omega^2+\Omega_2^2+\sigma_2^4/4}{(\omega^2-\Omega_2^2-\sigma_2^4/4)^2+\sigma_2^4\omega^2}\mathrm{d}\omega,\end{aligned}$$

thus, for the homoclinic orbits ($p_0(t)$, $q_0(t)$), the random Melnikov function can be simplified into

$$\begin{aligned}\left\langle\frac{4\beta}{3\alpha}\right\rangle^2 =& \left\langle\pm\sqrt{2}\pi\Omega\gamma\sin\Omega t_0\operatorname{csch}^2\frac{\pi\Omega}{2}\right\rangle^2\\ &+\mu_1^2 I_1 + \mu_2^2 I_2.\end{aligned}$$

Then the expression of the possible occurrence of chaotic phenomena is

$$\left\langle\frac{4\beta}{3\alpha}\right\rangle^2 \le 2\gamma^2\left\langle\pi\Omega\operatorname{csch}^2\frac{\pi\Omega}{2}\right\rangle^2 + \mu_1^2 I_1 + \mu_2^2 I_2. \tag{14}$$

With the help of inequality (14), for different parameters of bounded noise, we give the threshold of multiplicative bounded noise amplitude μ_1 versus its intensity σ_1 for the onset of chaos, which is shown in Figs.1, 2 and 3.

From Fig.1, for μ_2=0.1, σ_2=1.0 and Ω_2=1.0, it is found that for small values of frequency Ω_1 and small values of intensity σ_1 of the multiplicative bounded noise, as Ω_1 increases, the threshold of noise amplitude μ_1 for the onset of chaos decreases, which can be seen in Fig.1(a); while for slightly larger values of frequency Ω_1 and smaller values of intensity σ_1, the threshold of noise amplitude μ_1 for the occurrence of chaos increases as frequency Ω_1 increases which can be seen in Fig.1(b).

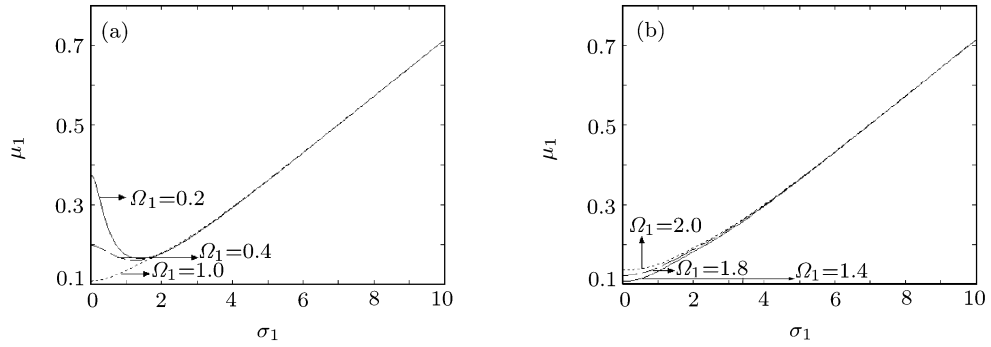


Fig.1. The threshold of multiplicative bounded noise amplitude μ_1 versus its intensity σ_1 for the onset of chaos with μ_2=0.1, σ_2=1.0 and Ω_2=1.0 for the case where Ω_1 and σ_1 are both small (a) and for the case where Ω_1 is slightly larger and σ_1 is smaller (b).

Now, the effect of the additive noise on the threshold of noise amplitude μ_1 for the onset of chaos can be illustrated in Fig.2. It is seen from Fig.2(a) that as the noise amplitude μ_2 increases, the threshold of noise amplitude μ_1 for the occurrence of chaos falls remarkably, that is to say, the phenomenon of chaos is much easier to occur, while for larger values of noise intensity σ_2, from Fig. 2(b), we can see that as the intensity σ_2 increases, the threshold of noise amplitude μ_1 for the rising of chaos increases.

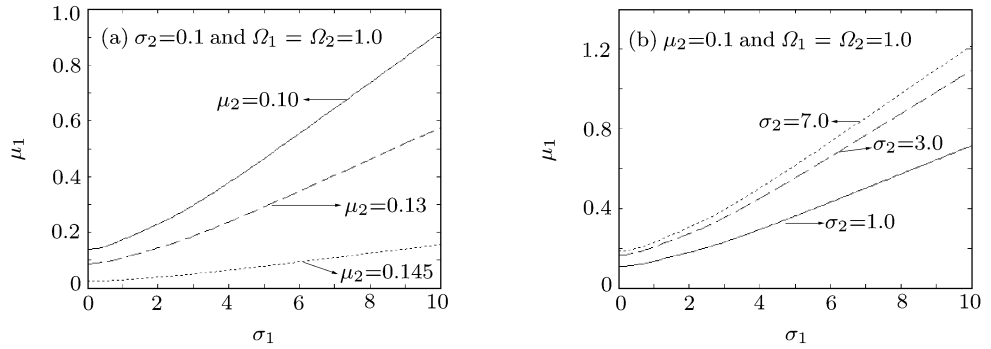


Fig.2. The threshold of multiplicative bounded noise amplitude μ_1 versus its intensity σ_1 for the onset of chaos with σ_2=0.1 and Ω_1 = Ω_2=1.0 at different values of μ_2 (a) and with μ_2=0.1 and Ω_1 = Ω_2=1.0 at different values of σ_2 (b).

Now, we discuss that the influence of the frequency of the additive bounded noise on the threshold of noise amplitude μ_1 for the onset of chaos, which can be seen in Fig.3. For μ_2=0.1 and Ω_1=1.0, it is known from Fig.3(a) that, for smaller values of intensity σ_2 of the additive noise (such as σ_2=0.5), as the frequency Ω_2 increases, the threshold of noise amplitude μ_1 for the occurrence of chaos increases, On the contrary, when intensity σ_2 of the additive noise is larger (such as σ_2=3.0), for different values of frequency Ω_2, we can obtain nearly the same threshold of noise amplitude μ_1 for the occurrence of chaos that can be seen in Fig.3(b).

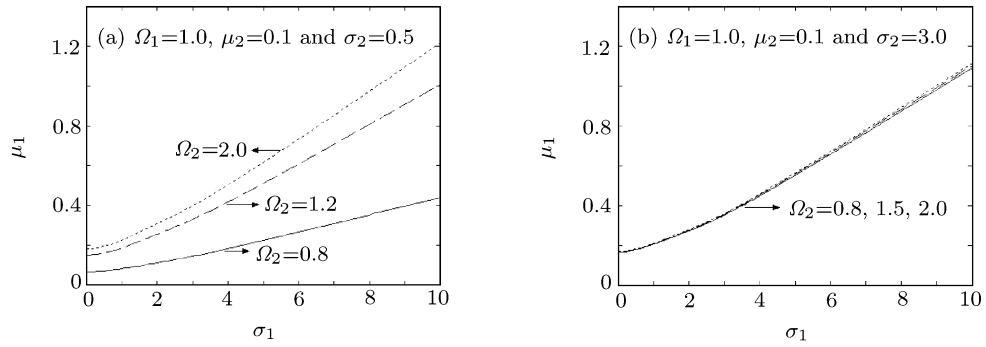


Fig.3. The threshold of multiplicative bounded noise amplitude μ_1 versus its intensity σ_1 for the onset of chaos with Ω_1=1.0, μ_2=0.1 and σ_2=0.5 at different values of Ω_2 (a) and with Ω_1=1.0, μ_2=0.1 and σ_2=3.0 at different values of Ω_2 (b).

4. Numerical simulations

In this section, numerical simulations are given to support the theoretical results obtained in the previous sections. For the Duffing system with bounded noise, the fourth-order Runge–Kutta scheme[23,24] is adopted, while the simulations of bounded noise are the same as those in Ref.[23], where Box-瑒uller method[25] is employed, with parameter values fixed at α=1.0, β=0.2, γ=0.10 and Ω=1.0.

4.1. The largest Lyapunov exponent

The Lyapunov exponent has been widely accepted as a quantitative index to the stability and chaotic motion of the system. It is an average exponential rate of growth or decay of solutions to differential equations. A dynamical system with a positive largest Lyapunov exponent is usually a chaotic system. Thus, the condition for the rising of chaos in dynamical system can also be determined by numerically calculating the largest Lyapunov exponent. Using the Wolf's algorithm, the largest Lyapunov exponent is computed for different parameters of bounded noise, and the curves for the largest Lyapunov exponent λ versus the threshold of multiplicative bounded noise amplitude μ_1 for the onset of chaos are shown in Figs.4, 5, 6 and 7.

All figures indicate that for smaller values of noise amplitude μ_1, the largest Lyapunov exponent λ is negative. As μ_1 increases, the largest Lyapunov exponent changes from negative values to positive values, which means the presence of chaotic phenomenon.

Firstly, the effect of the frequency Ω_1 of the multiplicative bounded noise on the threshold of noise amplitude μ_1 for the onset of chaos will be simulated, and the results are shown in Fig.4. In this simulation, we fix the parameters μ_2=0.1, σ_2=1.0 and Ω_2=1.0, and for the smaller intensity σ_1, we choose it to be 0.5. It is found from Figs.4(a), 4(b), 4(c) and 4(d) that, for smaller values of frequency Ω_1, as Ω_1 increases, the threshold of noise amplitude μ_1 for the onset of chaos decreases accordingly; while, from Figs.4(d), 4(e) and 4(f), we can see that, for slightly larger values of frequency Ω_1, the threshold of noise amplitude μ_1 for the occurrence of chaos increases as frequency Ω_1 increases. All these results are consistent with the analyses of Figs.1(a) and 1(b).

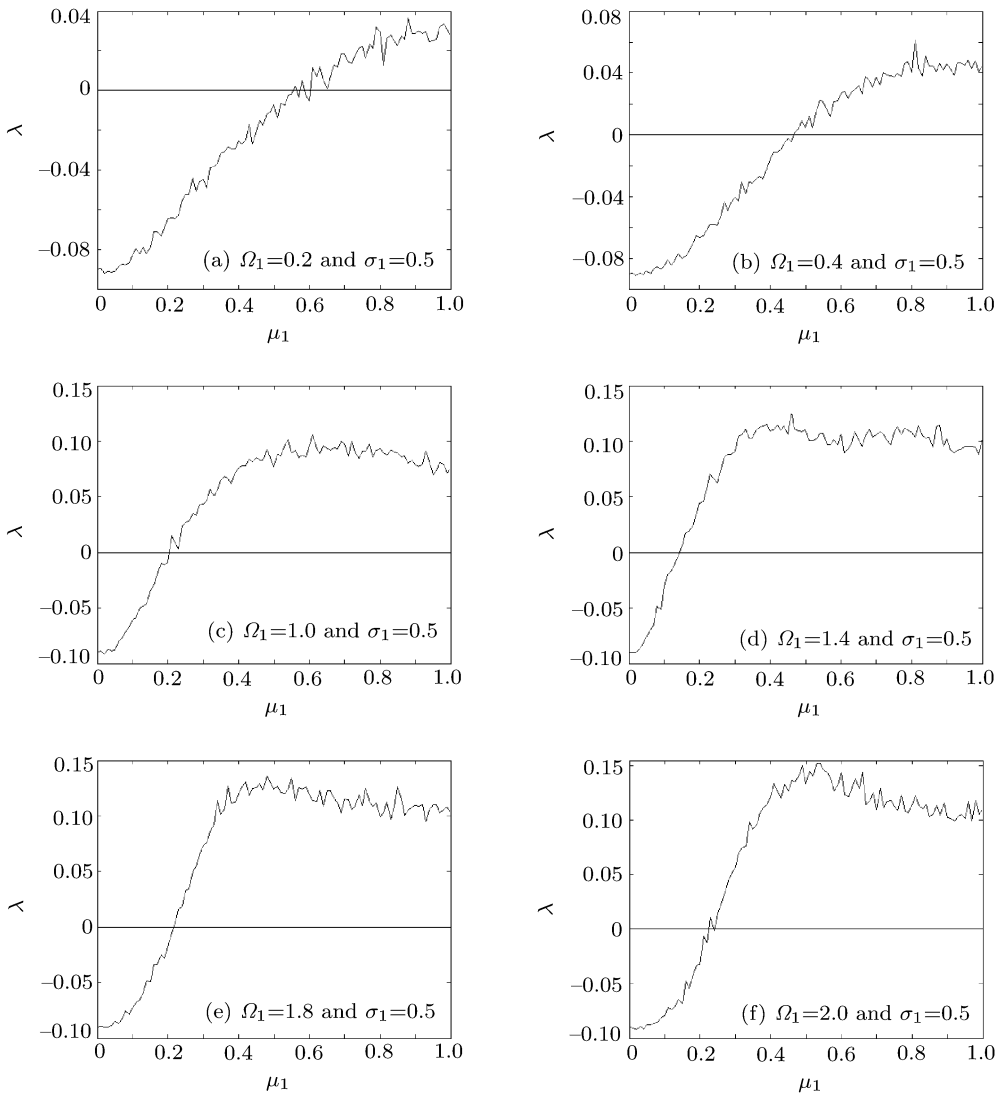


Fig.4. The largest Lyapunov exponent λ versus the threshold of multiplicative bounded noise amplitude μ_1 for the onset of chaos with parameters μ_2 =0.1, σ_2 =1.0 and Ω_2=1.0 at the values of Ω_1 and σ_1: 0.2 and 0.5 (a); 0.4 and 0.5 (b); 1.0 and 0.5 (c); 1.4 and 0.5 (d); 1.8 and 0.5 (e); 2.0 and 0.5 (f).

Secondly, we choose the parameters as σ_1=1.0, Ω_1=1.0 and Ω_2=1.0, and the results of numerical simulation are given in Fig.5. From Figs.5(a), 5(b), 5(c) and 5(d), it follows that, for smaller or larger values of intensity σ_2 of additive noise, as the noise amplitude μ_2 increases, the threshold of noise amplitude μ_1 for the occurrence of chaos decreases. Now, we fix the parameter μ_2=0.145, we can see that for larger values of noise intensity σ_2, as the intensity σ_2 increases, the threshold of noise amplitude μ_1 for the onset of chaos increases which can be found in Figs.5(d), 5(e) and 5(f). All these results are also consistent with the conclusions that are drawn from Figs.2(a) and (b).

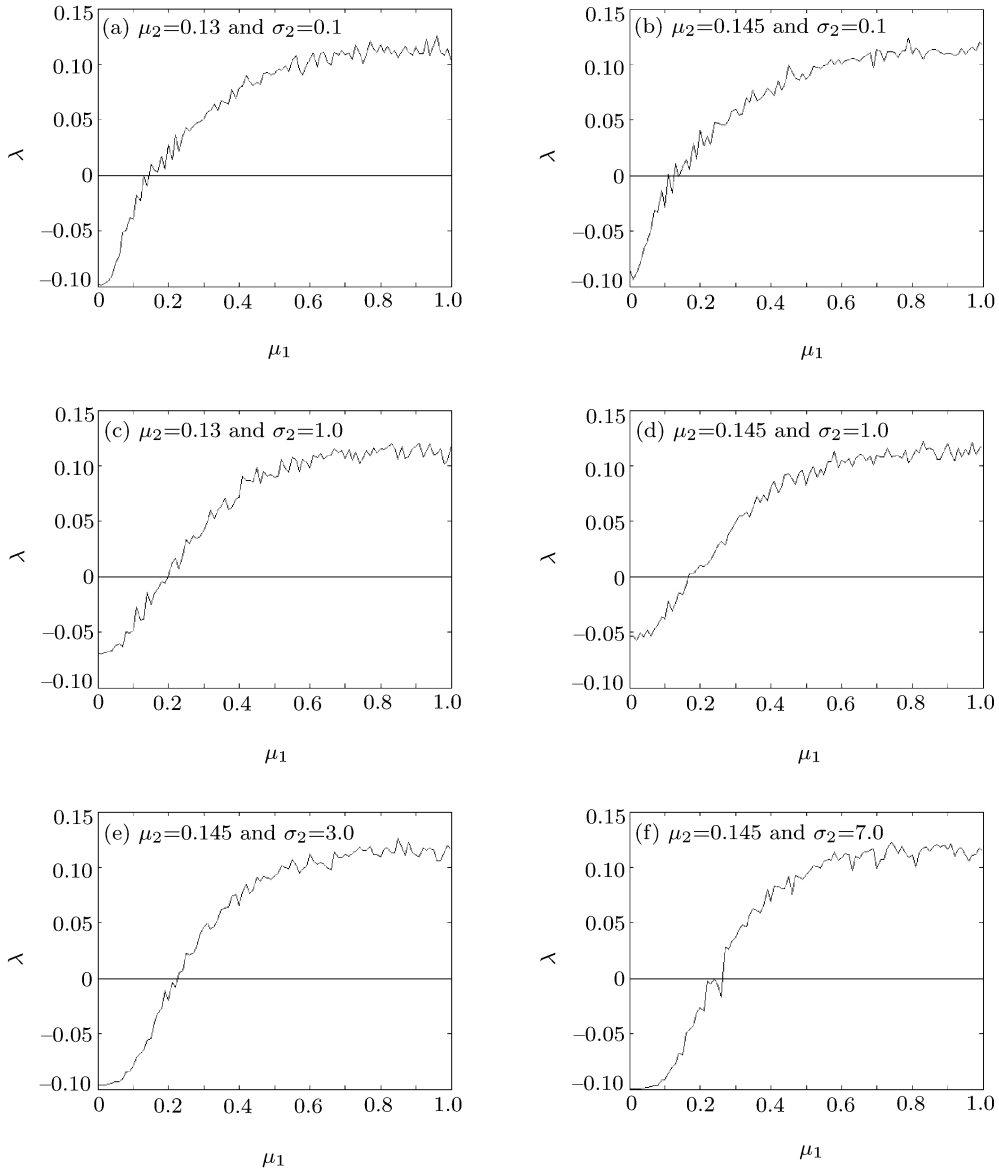


Fig.5. The largest Lyapunov exponent λ versus the threshold of multiplicative bounded noise amplitude μ_1 for the onset of chaos with parameters σ_1=1.0, Ω_1=1.0 and Ω_2=1.0 at the values of μ_2 and σ_2: 0.13 and 0.1 (a); 0.145 and 0.1 (b); 0.13 and 1.0 (c); 0.145 and 1.0 (d); 0.145 and 3.0 (e); 0.145 and 7.0 (f).

Thirdly, the influence of the frequency of the additive bounded noise on the threshold of noise amplitude μ_1 for the onset of chaos is simulated and the results are as follows. It is adopted in simulation that the parameters σ_1=4.0, Ω_1=1.0 and μ_2=0.1. For a smaller intensity σ_2=0.5, it is found in Figs.6(a), 6(b) and 6(c) that as the frequency Ω_2 increases, the threshold of noise amplitude μ_1 for the occurrence of chaos increases slightly. Though it is not apparent, compared with Fig.3(a), the conclusion is still consistent with the theoretic analysis. For a larger intensity σ_2=3.0, we can see from Figs.6(d), 6(e) and 6(f) that for different values of frequency Ω_2, the thresholds of noise amplitude μ_1 for the occurrence of chaos are nearly the same and these thresholds of noise amplitude μ_1 for the occurrence of chaos is about 0.50, which is identical to the analysis of the mean-square criterion for stochastic Melnikov process.

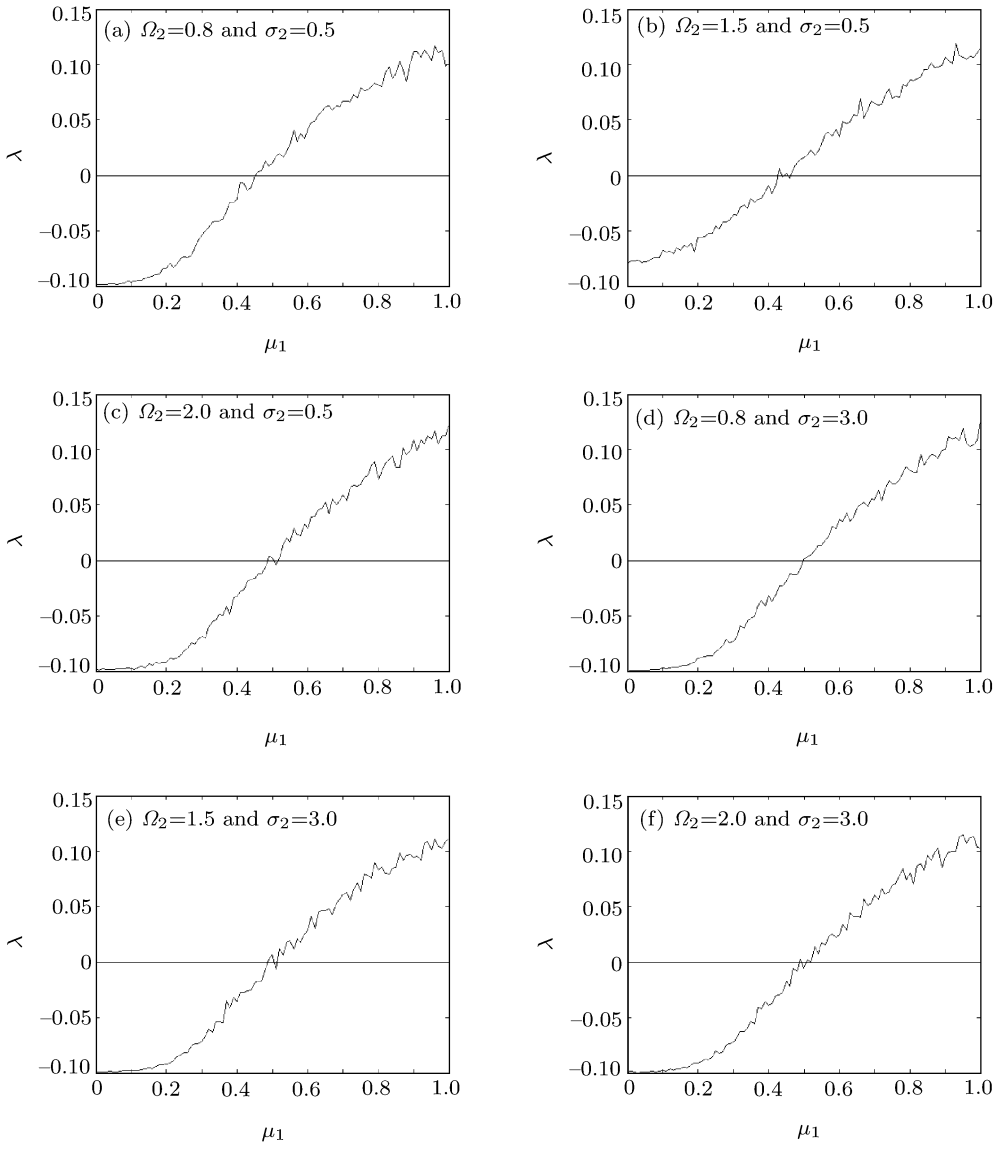


Fig.6. The largest Lyapunov exponent λ versus the threshold of multiplicative bounded noise amplitude μ_1 for the onset of chaos with parameters σ_1=4.0, Ω_1=1.0 and μ_2 =1.0 at the values of Ω_2 and σ_2: 0.8 and 0.5 (a); 1.5 and 0.5 (b); 2.0 and 0.5 (c); 0.8 and 3.0 (d); 1.5 and 3.0 (e); 2.0 and 3.0 (f).

Lastly, for different values of noise intensity σ_1, the threshold of noise amplitude μ_1 for the onset of chaos is investigated without additive noise. It is seen from Figs.7(a) and 7(b) that for smaller values of noise intensity, as σ_1 increases, the threshold of noise amplitude μ_1 for the onset of chaos decreases; while for larger values of noise intensity, from Figs.7(c) and 7(d), it follows that the threshold of noise amplitude μ_1 for the occurrence of chaos increases as noise intensity σ_1 increases, which implies that the diffusion of frequency reduces the effect of bound noise on triggering chaos in the system. These simulation results are the same as the existing conclusions.[15]

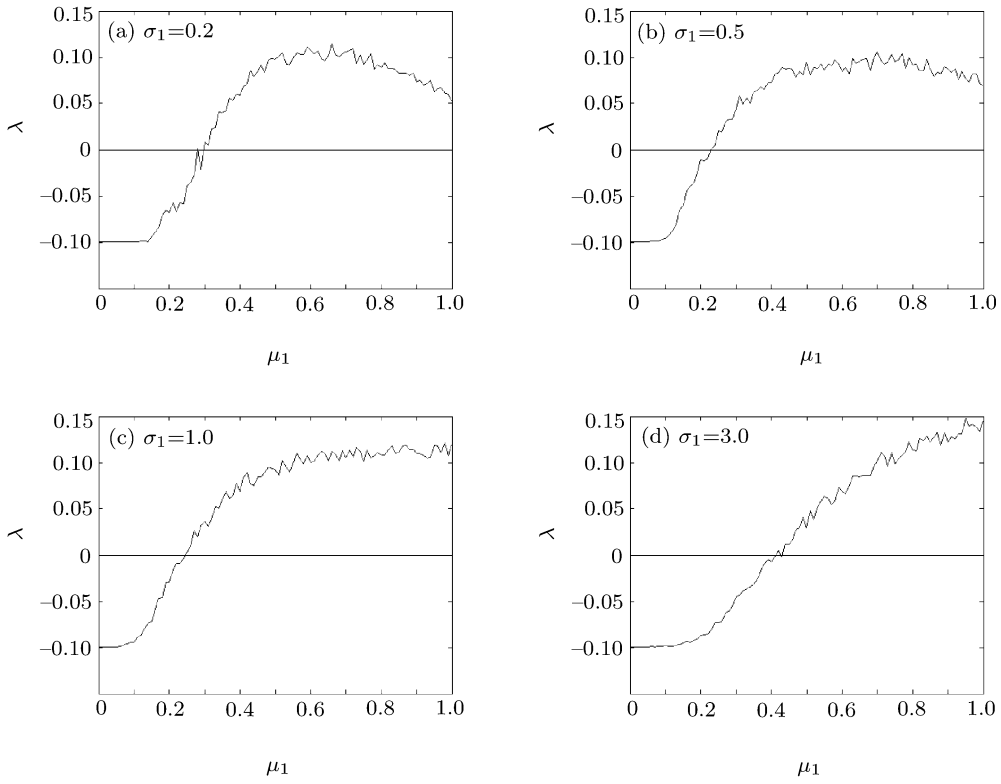


Fig.7. The largest Lyapunov exponent λ versus the threshold of multiplicative bounded noise amplitude μ_1 for the onset of chaos at the values of σ_1: 0.2(a); 0.5(b); 1.0(c); 3.0(d).

4.2. Poincaré maps

Poincaré map [23,26,27] is another powerful tool in analysing the behaviour of nonlinear system, for it can provide a vivid qualitative description of the behaviour of the system.

For a perturbed Duffing system, the successive iteration of Poincaré map is defined as

$$P : \Sigma \to \Sigma,$$
$$\Sigma = \{(Q, P)|t = 0, 2\pi/\Omega, 4\pi/\Omega, \cdots\} \in R^2.$$

One hundred initial points are randomly chosen in the phase plane. With each initial condition, the differential equation is solved and the solution is plotted for each $T = 2\pi/\Omega$; for each initial point, after deleting the first 1000 transients, the succeeding 9000 iteration points are plotted.

By combining Fig.7, without the additive noise, from Fig.8(a), we see that, for μ_1=0.1, σ_1=0.2 and Ω_1=1.0, the Poincaré map shows two smeared rings which indicate the occurrence of the bounded noise and cause system (8) to move nearby period orbit of the original system without noise; and with the increase of the multiplicative noise amplitude μ_1, the area of two smeared rings becomes larger as can be seen in Fig.8(b). It is not until the occurrence of two strange attractors that system (8) can be found to be randomly chaotic (see Fig. 8(c)). While we fix the parameters μ_1=0.35 and σ_1=0.2, it is seen from Figs.8(c) and 8(d) that when Ω_1 changes from 1.0 to 0.2, system (8) changes from randomly chaotic to nearly periodic. At the same time it is found that a different noise intensity σ_1 has a different effect on the behaviour of system (8), which can be seen in Figs.8(e) and 8(f).

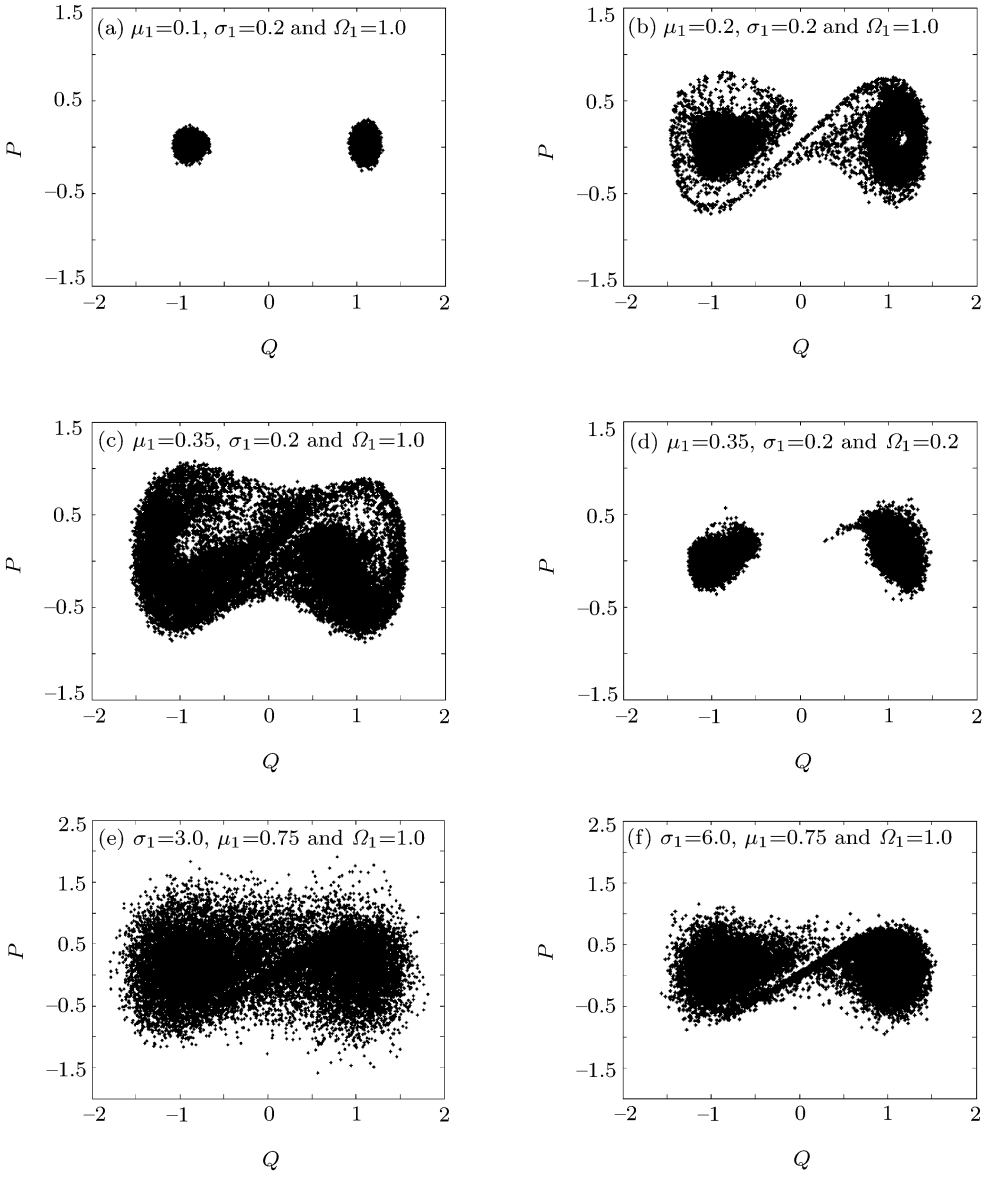


Fig.8. Poincaré maps with parameters μ_1, σ_1 and Ω_1 at values of 0.1, 0.2 and 1.0(a); 0.2, 0.2 and 1.0(b); 0.35, 0.2 and 1.0(c); 0.35, 0.2 and 0.2(d); 0.75, 3.0 and 1.0(e); 0.75, 6.0 and 1.0(f).

According to Fig.5, with the parameters fixed at μ_1=0.18, σ_1=1.0, Ω_1=1.0 and Ω_2=1.0, we can obtain from Figs.9(a) and 9(b) that a different noise amplitude μ_2 has a different influence on the behaviour of chaotic system (8); and it is chosen that the parameters μ_1=0.20, σ_1=1.0, Ω_1=1.0, and Ω_2=1.0, we can see from Figs.9(c) and 9(d) that when the noise intensity σ_2 increases, the system changes from randomly chaotic to nearly periodic, which means that for larger values of noise intensity σ_2, as the intensity σ_2 increases, the threshold of noise amplitude μ_1 for the onset of chaos increases accordingly. Figures 9(e) and 9(f) show the effect of the noise frequency Ω_2 on the chaotic behaviour of the dynamical system with the parameters fixed at μ_1=0.50,σ_1=4.0 and Ω_1=1.0.

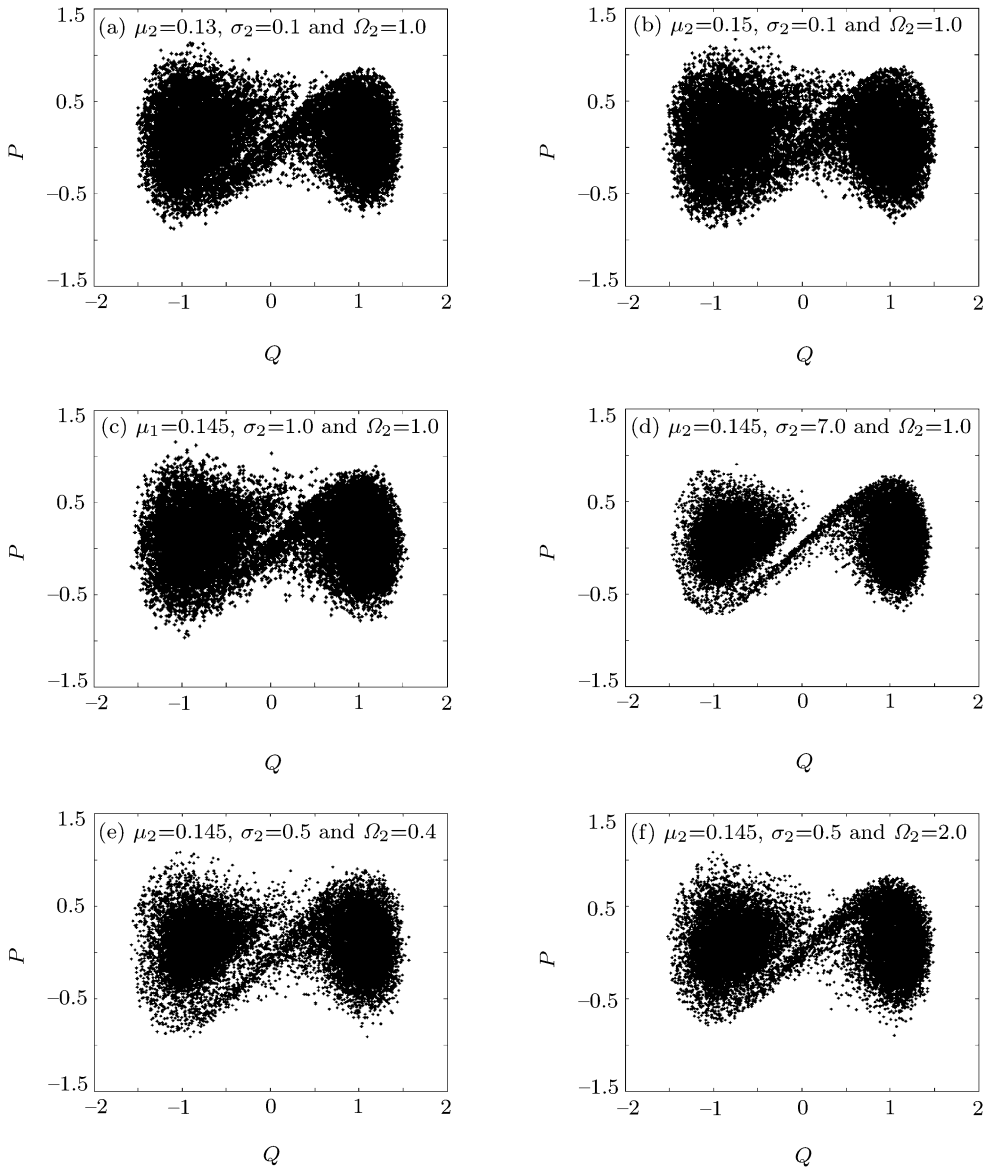


Fig.9. Poincaré maps with parameters μ_1=0.18, σ_1=1.0 and Ω_1=1.0 at the values of μ_2, σ_2 and Ω_2: 0.13, 0.1 and 1.0 (a); 0.15, 0.1 and 1.0 (b) and with parameters μ_1=0.20, σ_1=1.0 and Ω_1=1.0 at the values of μ_2, σ_2 and Ω_2: 0.145, 1.0 and 1.0 (c); 0.145, 7.0 and 1.0 (d) and with parameters μ_1=0.50, σ_1=4.0 and Ω_1=1.0 at the values of μ_2, σ_2 and Ω_2: 0.145, 0.5 and 0.4 (e); 0.145, 0.5 and 2.0 (f).

5. Conclusions

Owing to the randomicity and unpredictability of the noise, argumentation on the perturbed dynamical system has been a hot topic in recent years. So in this paper, the effect on the chaotic behaviour of dynamical system is investigated under both additive and multiplicative noise excitations. Firstly, the random Melnikov theorem with the mean-square criterion is adopted in order to obtain the conditions for the occurrence of chaos. Secondly, we apply this theory to the Duffing system subjected to both additive and multiplicative noise excitations and give the concrete expression of the threshold of noise amplitude for the occurrence of chaos. Finally, numerical simulations are illustrated for the Duffing system to verify the theoretical analysis. According to Lyapunov exponent and Poincaré map, it is further shown that it is reasonable that we use the random Melnikov theorem with the mean-square criterion to obtain the conditions for the possible occurrence of chaos phenomenon for one type of dynamical systems that are discussed in this paper.

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