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Synchronization of chaos in resistive-capacitive-inductive shunted Josephson junctions

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We present a scheme for chaotic synchronization in two resistive- capacitive-inductive shunted Josephson junctions (RCLSJJs) by using another chaotic RCLSJJ as a driving system. Numerical simulations show that whether the two RCLSJJs are chaotic or not before being driven, they can realize chaotic synchronization with a suitable driving intensity, under which the maximum condition Lyapunov exponent (MCLE) is negative. On the other hand, if the driving system is in different periodic states or chaotic states, the two driven RCLSJJs can be controlled into the periodic states with different period numbers or chaotic states but still maintain the synchronization.

Keywords: chaos synchronization, Josephson junction, maximum condition Lyapunov exponent
PACC: 0545, 7450

1. Introduction

The research activities in chaos and nonlinear dynamics about Josephson junction (JJ) are growing due to its extensive applications, such as superconducting quantum interference devices (SQUIDs) as well as microwave devices,[1−3] and the high critical-current junctions are preferred in many applications.[4] In order to further study the JJs with higher critical currents, the RCLSJJ model was proposed by Whan *et al*[5] based on previous investigations of a resistive-capacitive shunted junction (RCSJ) model.[6−10] Moreover the RCLSJJ model avoids the shortage of the RCSJ model and leads to better agreement of the numerical simulations with the experimentally obtained I-V curves.[5] The RCLSJ model is found to be more used in high-frequency and filter applications,[4,11,12] and its chaotic features have been reported.[13−15] Recently, chaotic synchronization in the RCLSJJs has been an interesting subject of research.[16] The concept of chaotic synchronization was first introduced by Pecora *et al*.[17] And then some synchronization methods, including mutual coupling[18,19] and variable feedback,[20] etc, were proposed, and many experts have successfully achieved the chaotic synchronizations in various systems[21−23] by using these methods. And different chaotic synchronization types, such as completed synchronization[17,24] and generalized synchronization,[25,26] have been also described.

In this paper, we study chaos behaviour in the RCLSJJ by changing external bias currents and give the parameter conditions for chaotic generation. A scheme for chaotic synchronization in two RCLSJJs is presented by using another RCLSJJ as a driving system, and the validity of this scheme is also confirmed numerically.

2. Synchronization scheme

Here, a chaotic synchronization scheme for two RCLSJJs is presented based on a chaotic driving synchronization method,[27] where the output signal from a driving system is used to drive two driven systems, and the synchronization criterion is that when the MCLE is negative, the two driven systems can be synchronized. Our scheme is shown in Fig.1.

Here, systems S_1, S_2 and S_3 are three RCLSJJs, with S_1 taken as the driving system, and S_2 and S_3 regarded as two driven systems which have identical parameter values and different initial conditions; R_{sn} and L_n are the shunt resistance and inductance, respectively; I_n, I_{sn}, and I_{cn} are the external dc bias current, the shunt current flowing through R_{sn}, and

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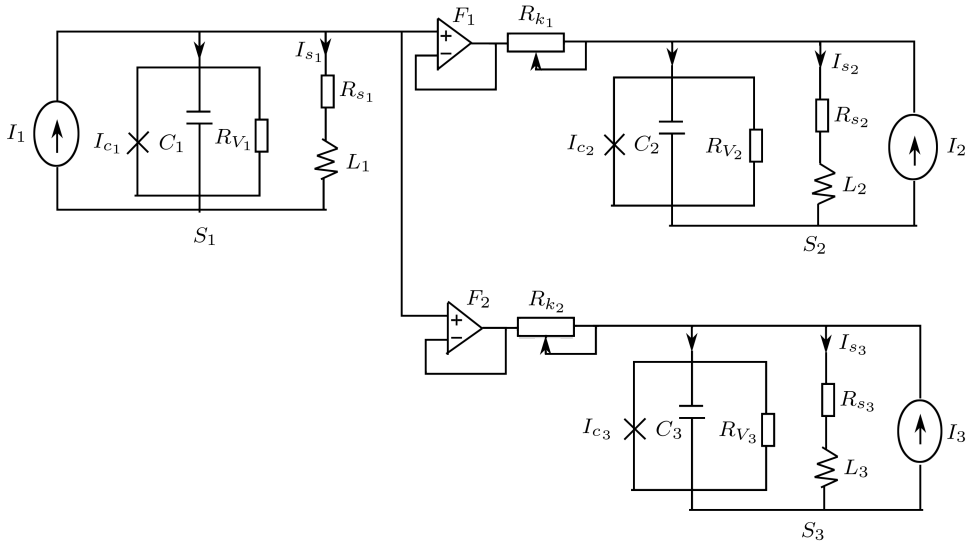


Fig.1. Synchronization scheme.

the junction critical current, respectively; C_n and R_{Vn} are the junction capacitance and the junction resistance, respectively; subscripts $n = 1$, $n = 2$, and $n = 3$ denote systems S_1, S_2, and S_3, respectively. The driving circuits are formed by R_{k1}, R_{k2}, F_1, and F_2.[28] System S_1 is not affected by systems S_2 and S_3, and its dimensionless dynamical equations keep their original forms as follows:[5,13]

$$\begin{aligned}\frac{\mathrm{d}\gamma_1}{\mathrm{d}\tau} &= \upsilon_1,\\ \frac{\mathrm{d}\upsilon_1}{\mathrm{d}\tau} &= \frac{1}{\beta_{c1}}(i_1 - g_1\upsilon_1 - \sin(\gamma_1) - i_{s1}),\\ \frac{\mathrm{d}i_{s1}}{\mathrm{d}\tau} &= \frac{1}{\beta_{L1}}(\upsilon_1 - i_{s1}).\end{aligned} \tag{1}$$

For systems S_2 and S_3, their dynamical behaviours are influenced by system S_1, so their dynamics equations are changed, and their dimensionless equations are shown as follows:

$$\begin{aligned}\frac{\mathrm{d}\gamma_n}{\mathrm{d}\tau} &= \upsilon_n,\\ \frac{\mathrm{d}\upsilon_n}{\mathrm{d}\tau} &= \frac{1}{\beta_{cn}}(i_n - g_n\upsilon_n - \sin(\gamma_n)\\ &\quad -i_{sn} + k_(\upsilon_1 - \upsilon_n)),\\ \frac{\mathrm{d}i_{sn}}{\mathrm{d}\tau} &= \frac{1}{\beta_{Ln}}(\upsilon_n - i_{sn}).\end{aligned} \tag{2}$$

In Eq.(2), subscripts $n = 2$ and $n = 3$ denote systems S_2 and S_3, respectively. For Eqs.(1) and (2), the dimensionless variables and parameters are shown as follows: The γ_n is the difference in phase between the wave functions across the junction. The normalized time: $\tau = \omega t$, with

$$\omega = \omega_n = 2eI_{cn}R_{sn}/\hbar. \tag{3}$$

The normalized current:

$$i_n = I_n/I_{cn}, \quad i_{sn} = I_{sn}/I_{cn}. \tag{4}$$

The normalized junction voltage:

$$\upsilon_n = V_n/I_{cn}R_{sn}. \tag{5}$$

The dimensionless parameters:

$$\beta_{cn} = 2eI_{cn}R_{sn}^2C_n/\hbar, \tag{6}$$

$$\beta_{Ln} = 2eI_{cn}L_n/\hbar, \tag{7}$$

$$g_n = R_{sn}/R_{Vn}, \tag{8}$$

where $n = 1$, $n = 2$, and $n = 3$ represent systems S_1, S_2, and S_3, respectively. And the normalized driving intensity is $k = R_{s_2}/R_{k1} = R_{s_3}/R_{k2}$. We can appropriately adjust R_{k1} and R_{k2}, so as to obtain a driving signal with an appropriate intensity and realize the chaotic synchronization.

3. Chaotic behaviour in the RCLSJJ

The dynamical equation of the RCLSJJ without being driven is the same as Eq.(1) with excluding subscript 1. To show chaotic behaviours in the RCLSJJ, its dynamical equation is solved, with bias current i

changing at the fixed values: $\beta_c = 0.707$, $\beta_L = 2.68$, and $g = 0.0478$. In this work, the differential equations are numerically solved by using the fourth-order Runge- Kutta method with a fixed step $\Delta\tau = 0.01$, and the Lyapunov exponents are computed by using the algorithms given by Wolf *et al.*[29] The voltage bifurcation diagrams are plotted by recording the local maxima in the voltage time series.[30] The parametric values used in calculation are obtained from Refs.[5, 13] at a temperature $T = 7.60\,\mathrm{K}$. The bifurcation diagram and the maximum Lyapunov exponent are presented in Fig.2, where one can find that for $1.090 < i < 1.156$ and $1.164 < i < 1.227$ the RCLSJJ is in chaotic states, which are occasionally interrupted by periodic windows; for other i values in a selected range, this junction is in the periodic state. Thus the state of the RCLSJJ can be changed by adjusting the external bias current.

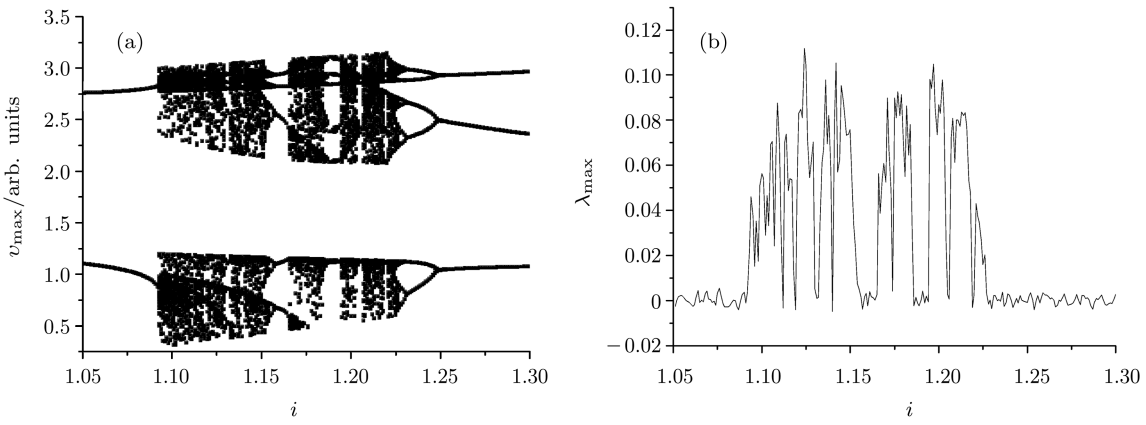


Fig.2. The bifurcation diagram of the RCLSJJ versus i (a) and the maximum Lyapunov exponent $\lambda_{\max}$ versus i (b), with the parameters being: $\beta_c = 0.707, \beta_L = 2.68$, and $g = 0.0478$.

4. Numerical simulations

To illustrate the process of chaotic synchronization, firstly, we consider case 1 in which driving system S_1 is in a chaotic state and systems S_2 and S_3 are in various dynamics states (periodic and chaotic) before they are driven. According to Fig.2, in order to make systems S_1 lie in a chaotic state, we take the parameters as follows: $i_1 = 1.15$, $\beta_{c1} = 0.707$, $g_1 = 0.0478$, and $\beta_{L1} = 2.68$; in order to make original systems S_2 and S_3 stay in different states, we vary their bias currents from $i_2 = i_3 = 1.05$ to $i_2 = i_3 = 1.20$ and maintain $g_2 = g_3 = 0.0478$, $\beta_{c2} = \beta_{c3} = 0.707$, and $\beta_{L2} = \beta_{L3} = 2.68$. The initial values are $\gamma_1^0 = 0.2, \upsilon_1^0 = 2.5$, $i_{s1}^0 = 0.2$, $\gamma_2^0 = 0.1$, $\upsilon_2^0 = 2.0$, $i_{s2}^0 = 0.1$, $\gamma_3^0 = 0.3$, $\upsilon_3^0 = 2.2$, and $i_{s3}^0 = 0.5$. So, numerical results of Eqs.(1) and (2) show whether systems S_2 and S_3 were in chaotic states previously, when they are driven by S_1 with a driving intensity $k > 0.365$, they can enter into the chaotic synchronization states, i.e. $\gamma_2 = \gamma_3$, $\upsilon_2 = \upsilon_3$, and $i_{s2} = i_{s3}$. To show the synchronization phenomenon, now we take $i_2 = i_3 = 1.10$, with other parameter values being the same as those in Fig.2. Then, the MCLE of systems S_2 and S_3 is plotted in Fig.3.

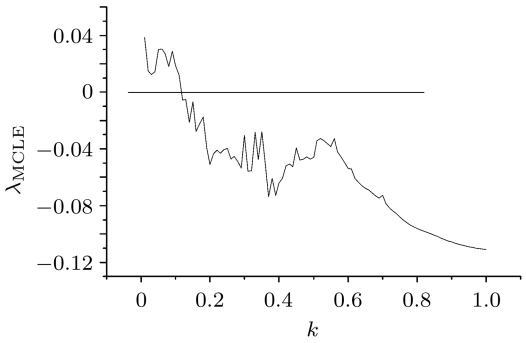


Fig.3. The MCLE of systems S_2 and S_3 versus the driving intensity k with parameters being $g_1 = g_2 = g_3 = 0.0478$, $\beta_{c1} = \beta_{c2} = \beta_{c3} = 0.707$, $\beta_{L1} = \beta_{L2} = \beta_{L3} = 2.68$, $i_1 = 1.15$ and $i_2 = i_3 = 1.10$.

In Fig.3, one can find that for $k > 0.119$ the MCLE is negative, namely, two driven systems S_2 and S_3 can be synchronized, with the value of k selected in this range. The synchronization phenomenon with $k = 0.3$ can be clearly seen in Fig.4, where panels (a), (b), and (d), (e) show that the attractors of driven systems S_2 and S_3 are influenced by driving system

S_1. From Fig.4(c) we cannot see a regular relation between v_2 and v_3; the relationship is complex, so systems S_2 and S_3 are not synchronized before being driven. From Fig.4(f) one can find $v_2 = v_3$, which always holds true after the transient process has died out, other corresponding variables are also the same, thus systems S_2 and S_3 are synchronized after being driven with $k = 0.3$.

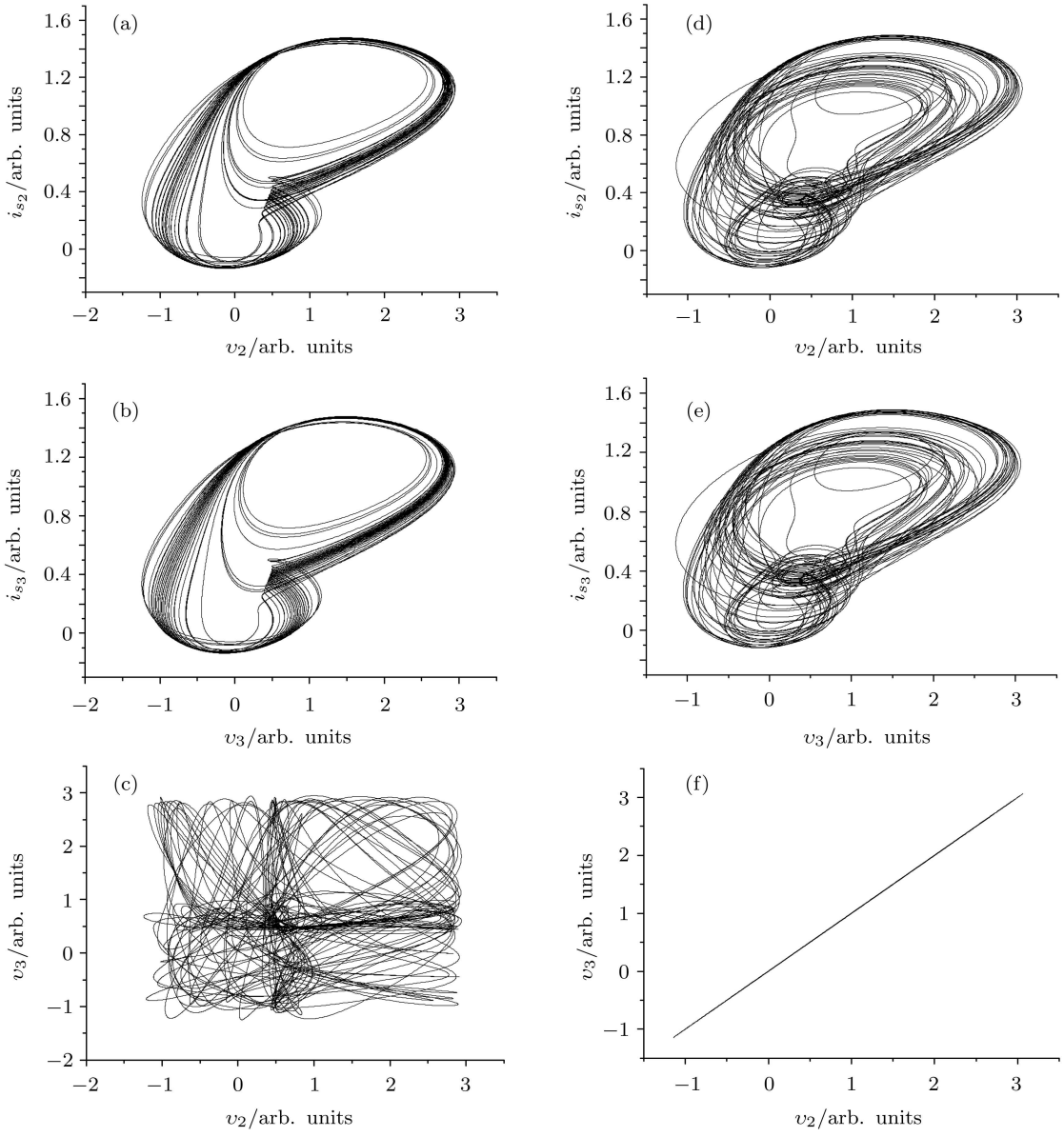


Fig.4. Chaotic synchronization in systems S_2 and S_3 with $k = 0.3$. Panels (a), (b), (d) and (e) show the strange attractors on planes $v_2 - i_{s2}$ and $v_3 - i_{s3}$ before and after being driven, and panels (c) and (f) indicate the relations between v_2 and v_3 before and after being driven. Other parameter values are the same as those in Fig.3.

Secondly, we study the synchronization for case 2 in which systems S_2 and S_3 are originally in chaotic states and system S_1 is in various states. So, we take the parameter values of systems S_2 and S_3 as those of system S_1 in case 1. To make system S_1 stay in different states by adjusting the value of parameter β_{L1}[13] or bias current i_1 (Fig.2), we take its parameter values as follows: $\beta_{c1} = 0.707$, $g_1 = 0.0478$, $i_1 = 1.18$, with the value of β_{L1} varying from 2.50 to 2.70, or $\beta_{L1} = 2.68$, with the value of i_1 changing from 1.15 to 1.18. With these parameters, the numerical results show that the states of driven systems S_2 and S_3 vary with the state of the driving system S_1, and the bigger the value of k is, the more systems S_2 and S_3 are af-

fected. With the diving intensity $k \geq 0.65$, the states of S_2 and S_3 are the same as that of S_1, in the meantime systems S_2 and S_3 can remain in synchronized states. Therefore when systems S_2 and S_3 are synchronized, they can be in different synchronization states (periodic or chaotic) by changing the state of system S_1. The changes of the synchronization state are shown in Fig.5, where one can find that for $2.54 \leq \beta_L \leq 2.65$ and $1.15 \leq i \leq 1.18$, with the increase of β_L or i, the driving system and two driven systems can simultaneously suppress their chaotic states into periodic states through reverse period-doubling bifurcations and then enter into chaotic states again through intermittency routes. Hence, this scheme can also be used for controlling chaos in RCLSJJs. In order to further show the effect of Fig.5, according to Figs.5(a) and 5(b), for instance, $\beta_{L1} = 2.575$ and $\beta_{L1} = 2.650$ are chosen. The corresponding attractors of systems S_1, S_2, and S_3 are presented in Fig.6. Here systems S_2 and S_3 have realized synchronization so they have the same attractors as shown in Figs.6(c) and 6(d). Figures 6(a) and 6(b) show that the driving system is in a periodic-5 (P5) state and a chaotic state, respectively; Figs.6(c) and 6(d) show that the driven systems are also in P5 state and chaotic state, respectively. Similarly, according to Figs.5(c) and 5(d), $i_1 = 1.16$ and $i_1 = 1.18$ are chosen, the corresponding attractors are similar to those in Fig.6.

Based on the results above, we extend chaotic synchronization in two driven RCLSJJs to more driven RCLSJJs. As an example, the case in which there exist one driving system and three driven systems is studied, and their parameters are the same as those above, but the initial conditions are different. The calculation results indicate that these three driven RCLSJJs can also be synchronized if their MCLEs are negative. This is useful in practical applications of RCLSJJs.

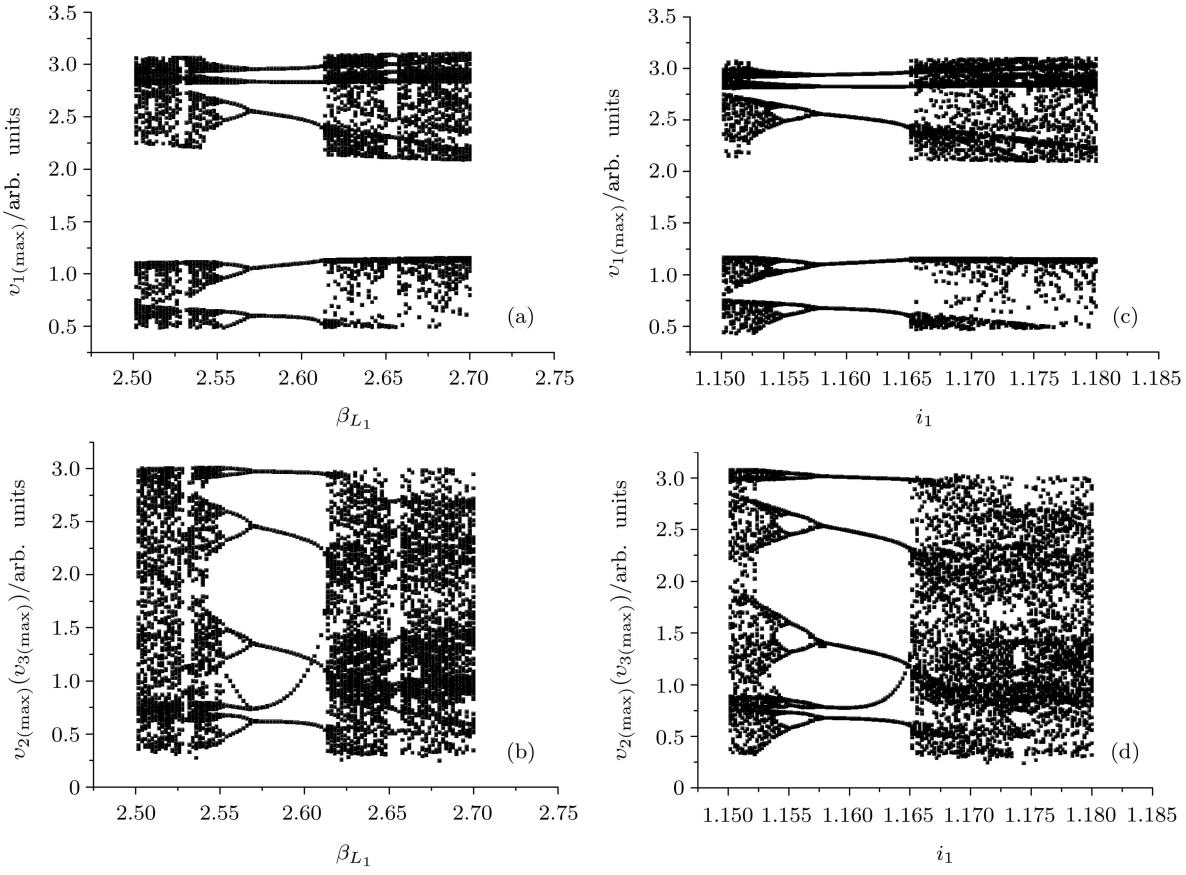


Fig.5. The bifurcation diagram of the driving system and the driven systems versus β_{L1}, at $i_1 = 1.18$ in (a) and (b), and the Bifurcation diagram of the driving system and driven systems versus i_1, at $\beta_{L1} = 2.68$ in (c) and (d), with other parameters fixed at $\beta_{c1} = 0.707$, $g_1 = 0.0478$, $k = 0.65$, $i_2 = i_3 = 1.15$, $\beta_{c2} = \beta_{c3} = 0.707$, $g_2 = g_3 = 0.0478$, and $\beta_{L2} = \beta_{L3} = 2.68$.

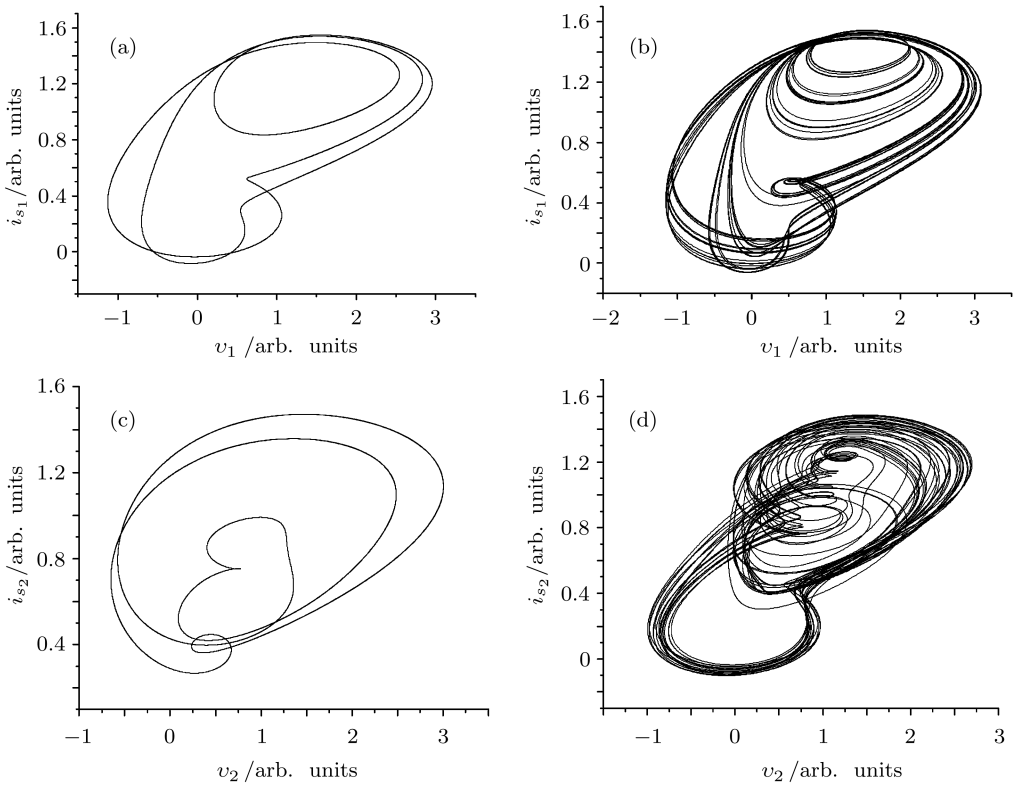


Fig.6. The P5 attractor on planes $v_1 - i_{s1}$ and $v_2 - i_{s2}$, at $\beta_{L1} = 2.575$ in (a) and (c), and the chaotic attractor on planes $v_1 - i_{s1}$ and $v_2 - i_{s2}$, at $\beta_{L1} = 2.650$ in (b) and (d), with a driving intensity $k = 0.65$, and other parametric values being the same as those in Figs.5 (a) and 5(b).

In addition, all synchronization results above belong to the complete synchronization in the driven systems. Moreover, this complete synchronization can occur only if the driving intensity k satisfies the conditions, with which the driving system and driven system can realize a generalized synchronization, namely, the generalized synchronization between the driving system and driven system leads to a complete synchronization between the driven systems.

5. Conclusion

This paper has studied the parameter conditions for chaotic generation in the RCLSJJ by changing external bias currents and presented a scheme for chaotic synchronization in two RCLSJJs based on the chaotic driving synchronization method. Numerical simulations have demonstrated that this scheme can be effectively used for the chaotic synchronization in RCLSJJs by appropriately adjusting the driving intensity. Also, the chaotic and periodic synchronization states in two or several driven RCLSJJs can be obtained by changing the state of the driving system. And the bifurcation diagrams of the driving and driven systems versus the parameter of the driving system are plotted. These are useful for implementing the chaotic synchronization and the chaos control in RCLSJJs.

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