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Anticontrol of chaos for discrete-time fuzzy hyperbolic model with uncertain parameters*

Zhao Yan(赵 琰)[a)†], Zhang Hua-Guang(张化光)[a)b)], and Zheng Cheng-De(郑成德)[a)c)]

[a)] *School of Information Science and Engineering, Northeastern University, Shenyang* 110004, *China*
[b)] *Key Laboratory of Process Industry Automation of Ministry of Education, Northeastern University, Shenyang* 110004, *China*
[c)] *Department of Mathematics, Dalian Jiaotong University, Dalian* 116028, *China*

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This paper proposes a new method to chaotify the discrete-time fuzzy hyperbolic model (DFHM) with uncertain parameters. A simple nonlinear state feedback controller is designed for this purpose. By revised Marotto theorem, it is proven that the chaos generated by this controller satisfies the Li–Yorke definition. An example is presented to demonstrate the effectiveness of the approach.

Keywords: discrete-time fuzzy hyperbolic model, anticontrol of chaos, parametric uncertainty, nonlinear state feedback

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1. Introduction

Recently, chaotic systems and chaotic phenomena have stimulated the interest of more and more scientists and engineers for their particularity.[1−10] As a reverse process of suppressing or eliminating chaotic behaviours in order to reduce the complexity of an individual system or a coupled system, anticontrol of chaos aims at creating or enhancing the system complexity for some special applications. More precisely, chaotification is to generate some chaotic behaviours from a given system, which is non-chaotic or even is stable originally.[11] It is well-known that chaotification of dynamical systems has many potential applications in electronic, mechanical, biological and medical systems, etc.[12−14] In recent years, Many conventional control methods and special techniques are applied to the anticontrol of chaos for discrete-time dynamical systems or continuous-time dynamical systems, such as linear feedback control,[12] time-delay feedback control,[15] nonlinear feedback control,[16−17] tracking control,[18] adaptive inverse optimal control,[19] impulsive control,[20] saturating control,[21] switching control,[22] etc. However, many results of them are based on the previous Marotto theorem,[23] for which it is proven that there are some inaccuracies in it. In 2003, Li and Chen[24,25] proposed an improved version of Marotto theorem, named Li–Chen–Marotto theorem. But Marotto himself thought that the results given by them were too conservative though they were correct. Then he gave a more relaxed result[26] in 2005. In this paper, the revised Marotto theorem[26] will be applied to prove that a controlled system is chaotic in the sense of Li–Yorke. Moreover, The parametric uncertainty of the system which will be chaotified is considered. To the best knowledge of the authors, problems about chaotifying an uncertain system based on the revised Marotto theorem are seldom investigated in the literature.

Fuzzy hyperbolic model (FHM) is a nonlinear model in nature which is very suitable for expressing nonlinear dynamic properties, and it is easier to design stable and optimal controller based on FHM than other models such as fuzzy dynamic model, T-S model, etc. Moreover, FHM is proved to be an universal approximator, i.e., it can approximate arbitrary nonlinear function on a compact set with arbitrary accuracy.[27−29] This provides a theoretical foundation for using FHM to represent complex nonlinear systems approximately. Therefore, the DFHM can be regarded as an intermedium to chaotify an unknown discrete-time dynamical system. In this paper, a simple nonlinear state feedback controller is designed to chaotify the DFHM with uncertain parameters. There are

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†E-mail: zy_neu@yahoo.com.cn

some relevant results about chaotification of discrete-time T-S fuzzy model in Refs.[30, 31]. But it is a pity that there are some mistakes in the process of proving that there exists a snap-back repeller in the controlled T-S fuzzy system, which can be seen in Remark 3.

The rest of this paper is organized as follows: in Section 2, some definitions and concepts about DFHM and chaos in the sense of Li–Yorke are presented. In Section 3, a nonlinear state feedback controller is designed to chaotify the uncertain DFHM. It is proven that the chaos generated by the controller satisfies the Li–Yorke definition. In Section 4, some simulation results are demonstrated to verify the effectiveness of theoretical analysis. Some conclusions are finally drawn in Section 5.

2. Preliminaries

2.1. Discrete-time fuzzy hyperbolic model (DFHM)

At first we define the fuzzy rule base for inferring DFHM.

Definition 1[28] Given a plant with n state variables $\boldsymbol{x} = (x_1(t), \ldots, x_n(t))^{\mathrm{T}}$, we call the fuzzy rule base as hyperbolic type fuzzy rule base if it satisfies the following conditions:

(1) Every fuzzy rule adopts the following form:

R^l: IF $x_1(k)$ is F_{x_1} and $x_2(k)$ is F_{x_2} and $\ldots$ and $x_n(k)$ is F_{x_n},

THEN $x_i(k+1) = \pm c_{x_1} \pm c_{x_2} \pm \cdots \pm c_{x_n}$, $(l = 1, 2, \ldots, 2^n)$,

where $F_{x_i}(i = 1, 2, \ldots, n)$ are fuzzy sets of $x_i(k)$, which include P (Positive) and N (Negative) two linguistic values. $c_{x_i}(i = 1, 2, \ldots, n)$ is a positive constant corresponding to F_{x_i}, and '$\pm$' stands for either the plus or the minus sign. The actual signs in the THEN-part are determined in the following manner: if the term characterizing F_{x_i} in the IF-part is P, then c_{x_i} in the THEN-part appears with a plus sign; otherwise c_{x_i} appears with a minus sign.

(2) The constant term c_{x_i} in the THEN-part corresponds to F_{x_i} in the IF-part, that is, if there is F_{x_i} term in the IF-part, c_{x_i} must appear in the THEN-part, otherwise c_{x_i} does not appear in the THEN-part.

(3) There are 2^n fuzzy rules in the rule base, that is, it includes all the possible P and N combination of state variables in the IF-part, and all the sign combination of constants in the THEN-part.

Theorem 1[28] Given a hyperbolic type rule base, if we define the membership function of P_{x_i} and N_{x_i} as:

$$\mu_{P_{x_i}}(x_i(k)) = \mathrm{e}^{-1/2\left(x_i(k) - s_{x_i}\right)^2},$$
$$\mu_{N_{x_i}}(x_i(k)) = \mathrm{e}^{-1/2\left(x_i(k) + s_{x_i}\right)^2}, \tag{1}$$

where $s_{x_i} > 0$, then we can derive the following model:

$$\boldsymbol{x}(k+1) = \boldsymbol{A}\tanh(\boldsymbol{S}\boldsymbol{x}(k)), \tag{2}$$

where $\boldsymbol{S} = \mathrm{diag}(s_{x_1}, \ldots, s_{x_n})$, $\tanh(\boldsymbol{S}\boldsymbol{x}(k)) = (\tanh(s_{x_1}x_1(k)), \ldots, \tanh(s_{x_n}x_n(k)))^{\mathrm{T}}$. This model is named discrete-time fuzzy hyperbolic model (DFHM). In the sequel, s_{x_i} is denoted by s_i for simplicity.

2.2. Chaos in the sense of Li–Yorke and a criterion of chaos

Consider a general n-dimensional discrete-time autonomous system with the form

$$\boldsymbol{x}(k+1) = \boldsymbol{g}(\boldsymbol{x}(k)), \tag{3}$$

where $\boldsymbol{g}$ is a C^1 nonlinear map. Let $\boldsymbol{g}^t$ denote the t times of compositions of $\boldsymbol{g}$ with itself. A point, $\boldsymbol{x}^*$, is said to be a p-periodic point of $\boldsymbol{g}$ if $\boldsymbol{g}^p(\boldsymbol{x}^*) = \boldsymbol{x}^*$ but $\boldsymbol{g}^t(\boldsymbol{x}^*) \neq \boldsymbol{x}^*$ for $p \geq t \geq 1$. If $p = 1$, i.e., $\boldsymbol{g}(\boldsymbol{x}^*) = \boldsymbol{x}^*$, $\boldsymbol{x}^*$ is called a fixed point. Let $\boldsymbol{g}'(\boldsymbol{x})$ and $\det(\boldsymbol{g}'(\boldsymbol{x}))$ be the Jacobian of $\boldsymbol{g}$ at the point $\boldsymbol{x}$ and its determinant respectively.

Definition 2[26] Suppose $\boldsymbol{x}^*$ is a fixed point of $\boldsymbol{g}$ with all eigenvalues of $\boldsymbol{g}'(\boldsymbol{x}^*)$ exceeding 1 in magnitude, and suppose that there exists a point $\boldsymbol{x}^0 \neq \boldsymbol{x}^*$ in a repelling neighbourhood of $\boldsymbol{x}^*$, such that for some positive integer m, $\boldsymbol{g}^m(\boldsymbol{x}^0) = \boldsymbol{x}^*$ and $\det((\boldsymbol{g}^m)'(\boldsymbol{x}^0)) \neq 0$. Then $\boldsymbol{x}^*$ is said to be a snap-back repeller of $\boldsymbol{g}$.

Lemma 1[26] If system (3) has a snap-back repeller then the system is chaotic in the sense of Li–Yorke, namely,

(1) there exists a positive integer n such that for every integer $p \geq n$, system (3) has p periodic points;

(2) there exists a scrambled set (an uncountable invariant set S containing no periodic points) such that

(a) $\boldsymbol{g}(S) \subset S$,

(b) for every $\boldsymbol{y} \in S$ and any periodic point $\boldsymbol{x}$ of Eq.(3)

$$\limsup_{k\to\infty} \left\|\boldsymbol{g}^k(\boldsymbol{x}) - \boldsymbol{g}^k(\boldsymbol{y})\right\| > 0,$$

(c) for every $\boldsymbol{x}, \boldsymbol{y} \in S$ with $\boldsymbol{x} \neq \boldsymbol{y}$

$$\lim_{k\to\infty} \sup \left\|\boldsymbol{g}^k(\boldsymbol{x}) - \boldsymbol{g}^k(\boldsymbol{y})\right\| > 0;$$

(3) there exists an uncountable subset $S_0 \subset S$ such that for any $\boldsymbol{x}, \boldsymbol{y} \in S_0$,

$$\lim_{k\to\infty} \inf \left\|\boldsymbol{g}^k(\boldsymbol{x}) - \boldsymbol{g}^k(\boldsymbol{y})\right\| = 0.$$

In Ref.[26], Marotto pointed out that identifying a repelling neighbourhood of $\boldsymbol{x}^*$ is not in general a difficult task. In fact, every closed ball $\boldsymbol{B}(\boldsymbol{x}^*, r)$ defined by Euclidean norm must be one for sufficiently small $r > 0$.

3. Anticontrol of chaos for an uncertain DFHM via a nonlinear state feedback controller

Now, consider the following uncertain DFHM:

$$\boldsymbol{x}(k+1) = (\boldsymbol{A} + \Delta\boldsymbol{A}) \tanh(\boldsymbol{S}\boldsymbol{x}(k)), \tag{4}$$

where $\Delta\boldsymbol{A}$ represents the uncertain parameters.

To chaotify the above model, some assumptions and lemmas are given below.

Assumption 1 $\Delta\boldsymbol{A}$ and $\boldsymbol{S}$ are bounded in the sense of infinite norm, i.e., there exist two positive constants γ and η, such that $\|\Delta\boldsymbol{A}\|_\infty \leq \gamma$, $\|\boldsymbol{S}\|_\infty \leq \eta$. Consequently, it can be inferred that $\|\boldsymbol{A} + \Delta\boldsymbol{A}\|_\infty \leq \rho$, where ρ is another positive constant.

Lemma 2[32]**(Gershgorin disc theorem)** Let $\boldsymbol{A}$ be a complex $n \times n$ matrix whose i, j entry is a_{ij}. For every $i = 1, 2, \ldots, n$, let $r_i \equiv \sum\limits_{j=1, j\neq i}^{n} |a_{ij}|$. Let D_i be a disc with centre a_{ii} and radius r_i. Then, every eigenvalue of $\boldsymbol{A}$ lies in the union of D_i for $i = 1, 2, \ldots, n$, i.e., $\lambda_i \in \bigcup\limits_{j=1}^{n} D_j$ for $i = 1, 2, \ldots, n$.

3.1. The nonlinear state feedback controller design

For the uncertain DFHM, the following simple nonlinear state feedback controller is designed to make it chaotic.

$$\boldsymbol{u}(k) = \delta \sin\left(\frac{\beta\pi}{\delta}\boldsymbol{x}(k)\right) \equiv \left(\delta \sin\left(\frac{\beta\pi}{\delta}\boldsymbol{x}_1(k)\right), \ \delta \sin\left(\frac{\beta\pi}{\delta}\boldsymbol{x}_2(k)\right), \ldots, \ \delta \sin\left(\frac{\beta\pi}{\delta}\boldsymbol{x}_n(k)\right)\right)^{\mathrm{T}}. \tag{5}$$

Lemma 3 (Boundedness) The state vector of the controlled system, under the controller (5), is uniformly bounded by a constant $\rho + \delta$.

Proof The equation of closed-loop system can be written as

$$\boldsymbol{x}(k+1) = (\boldsymbol{A} + \Delta\boldsymbol{A}) \tanh(\boldsymbol{S}\boldsymbol{x}(k)) + \delta \sin\left(\frac{\beta\pi}{\delta}\boldsymbol{x}(k)\right). \tag{6}$$

Note that $\tanh(\cdot)$ and $\sin(\cdot)$ are both bounded functions. Moreover, $\|\boldsymbol{A} + \Delta\boldsymbol{A}\| \leq \rho$, $\|\boldsymbol{u}_k\| \leq \delta$, one has

$$\begin{aligned}
\lim_{k\to\infty} \|\boldsymbol{x}(k)\| &= \lim_{k\to\infty} \left\|(\boldsymbol{A} + \Delta\boldsymbol{A}) \tanh(\boldsymbol{S}\boldsymbol{x}(k-1)) + \delta \sin\left(\frac{\beta\pi}{\delta}\boldsymbol{x}(k-1)\right)\right\| \\
&\leq \lim_{k\to\infty} \|(\boldsymbol{A} + \Delta\boldsymbol{A}) \tanh(\boldsymbol{S}\boldsymbol{x}(k-1))\| + \lim_{k\to\infty} \left\|\delta \sin\left(\frac{\beta\pi}{\delta}\boldsymbol{x}(k-1)\right)\right\| \\
&\leq \|\boldsymbol{A} + \Delta\boldsymbol{A}\| + \delta \\
&\leq \rho + \delta.
\end{aligned}$$

This means that the uncertain DFHM is bounded under the action of the sinusoidal function (5), which is necessary to prove that the controlled system is chaotic.

3.2. Verification of chaos in the controlled uncertain DFHM

In the subsection, the revised Marotto theorem is applied to prove that the controlled system is chaotic

in the sense of Li–Yorke. The relevant result is given as follows.

Theorem 2 There exists a sufficiently large constant β such that the controlled uncertain DFHM is chaotic in the sense of Li–Yorke.

Proof Define

$$\begin{aligned}&\boldsymbol{x}(k+1)\\&=(\boldsymbol{A}+\Delta\boldsymbol{A})\tanh(\boldsymbol{S}\boldsymbol{x}(k))+\delta\sin\left(\frac{\beta\pi}{\delta}\boldsymbol{x}(k)\right)\\&\equiv\boldsymbol{g}(\boldsymbol{x}(k)).\end{aligned}\tag{7}$$

Obviously, $\boldsymbol{x}^*=0$ is the fixed point of $\boldsymbol{g}$, which will be proven to be a snap-back repeller later. Define

$$\begin{aligned}&\boldsymbol{\Phi}(\boldsymbol{S}\boldsymbol{x})\equiv\operatorname{diag}\left(\frac{1}{\cosh^2(s_1x_1)},\dots,\frac{1}{\cosh^2(s_nx_n)}\right),\\&\cos\left(\frac{\beta\pi}{\delta}\boldsymbol{x}\right)\\&\equiv\operatorname{diag}\left(\cos\left(\frac{\beta\pi}{\delta}x_1\right),\dots,\cos\left(\frac{\beta\pi}{\delta}x_n\right)\right).\end{aligned}$$

Note that $\boldsymbol{\Phi}(\boldsymbol{S}\boldsymbol{x}^*)=\boldsymbol{\Phi}(\boldsymbol{0})=\boldsymbol{I}$, $\cos\left(\frac{\beta\pi}{\delta}\boldsymbol{x}^*\right)=\cos(\boldsymbol{0})=\boldsymbol{I}$, where $\boldsymbol{I}$ denotes the identity matrix with appropriate dimensions. Differentiating Eq.(7) at the fixed point $\boldsymbol{x}^*=\boldsymbol{0}$ yields

$$\boldsymbol{g}'(\boldsymbol{x}^*)=(\boldsymbol{A}+\Delta\boldsymbol{A})\cdot\boldsymbol{S}+\pi\beta\boldsymbol{I}.\tag{8}$$

Based on Lemma 2, one can assert that all $\lambda_i(\boldsymbol{g}'(\boldsymbol{x}^*))$ $(i=1,2,\dots,n)$ lie in the union of all D_i of radius $\sum\limits_{j=1,j\neq i}^{n}|(a_{ij}+\Delta a_{ij})s_j|$ centred at $(a_{ii}+\Delta a_{ii})s_i+\pi\beta$. Therefore, as long as one choose an appropriate β, such that all the following inequalities are met,

$$\begin{aligned}(a_{ii}+\Delta a_{ii})s_i\ &+\pi\beta-\sum_{j=1,j\neq i}^{n}|(a_{ij}+\Delta a_{ij})s_j|>1\\&\text{for}\ \ i=1,2,\dots,n,\end{aligned}\tag{9}$$

it is guaranteed that all $\lambda_i(\boldsymbol{g}'(\boldsymbol{x}^*))$ exceed 1 in magnitude for $i=1,2,\dots,n$.

Indeed, according to the definition of infinite norm, and noting that $\boldsymbol{S}$ is a diagonal matrix, one has

$$\begin{aligned}&\|(\boldsymbol{A}+\Delta\boldsymbol{A})\cdot\boldsymbol{S}\|_\infty\\&=\max_i\left\{\sum_{j=1}^{n}|(a_{ij}+\Delta a_{ij})s_j|\right\}.\end{aligned}\tag{10}$$

Therefore, the following inequalities are satisfied

$$\begin{aligned}&\sum_{j=1,j\neq i}^{n}|(a_{ij}+\Delta a_{ij})s_j|\\&\le\|(\boldsymbol{A}+\Delta\boldsymbol{A})\cdot\boldsymbol{S}\|_\infty-|(a_{ii}+\Delta a_{ii})s_i|\\&\qquad\text{for}\ \ i=1,2,\dots,n.\end{aligned}\tag{11}$$

If $(a_{ii}+\Delta a_{ii})s_i\ge0$, based on the inequalities (11), when $\beta>\frac{1}{\pi}(1+\|(\boldsymbol{A}+\Delta\boldsymbol{A})\cdot\boldsymbol{S}\|_\infty)$, one can get

$$\begin{aligned}&(a_{ii}+\Delta a_{ii})s_i+\pi\beta-\sum_{j=1,j\neq i}^{n}|(a_{ij}+\Delta a_{ij})s_j|\\&>1+2(a_{ii}+\Delta a_{ii})s_i\ \ \text{for}\ \ i=1,2,\dots,n.\end{aligned}\tag{12}$$

Consequently, the inequalities (9) are satisfied.

Otherwise, for $(a_{ii}+\Delta a_{ii})s_i<0$, when $\beta>\frac{1}{\pi}(1+\|(\boldsymbol{A}+\Delta\boldsymbol{A})\cdot\boldsymbol{S}\|_\infty)$, the inequalities (9) are satisfied directly. Then, According to Assumption 1 and the property of infinite norm, one has

$$\|(\boldsymbol{A}+\Delta\boldsymbol{A})\cdot\boldsymbol{S}\|_\infty\le\|\boldsymbol{A}+\Delta\boldsymbol{A}\|_\infty\cdot\|\boldsymbol{S}\|_\infty=\rho\eta.\tag{13}$$

Therefore, if $\beta>\dfrac{1+\rho\eta}{\pi}$, one can guarantee that all the eigenvalues of $\boldsymbol{g}'(\boldsymbol{x}^*)$ exceed 1 in magnitude.

Next, it is shown that there exists a point $\boldsymbol{x}^0\in B(\boldsymbol{x}^*,r)$ with $\boldsymbol{x}^0\neq\boldsymbol{x}^*$, where r is a sufficiently small positive constant (that is, this r is small enough to guarantee that the closed ball $B(\boldsymbol{x}^*,r)$ defined by Euclidean norm is a repelling neighbourhood of $\boldsymbol{x}^*$), such that $\boldsymbol{g}(\boldsymbol{x}^0)=\boldsymbol{0}=\boldsymbol{x}^*$ and $\det(\boldsymbol{g}'(\boldsymbol{x}^0))\neq0$. In the sequel, for simplicity, $\boldsymbol{x}<\boldsymbol{y}$ denotes that every component of $\boldsymbol{x}-\boldsymbol{y}$ is negative; $\boldsymbol{x}>\boldsymbol{y}$ denotes that every component of $\boldsymbol{x}-\boldsymbol{y}$ is positive. First, we choose two points $\boldsymbol{x}_1=((\delta/2\beta),\dots,(\delta/2\beta))^{\mathrm{T}}$ and $\boldsymbol{x}_2=((3\delta/2\beta),\dots,(3\delta/2\beta))^{\mathrm{T}}$. Obviously, if $\beta>\dfrac{3\sqrt{n}\delta}{2r}$, then it can be guaranteed that $\boldsymbol{x}_1\in B(\boldsymbol{x}^*,r)$ and $\boldsymbol{x}_2\in B(\boldsymbol{x}^*,r)$ simultaneously.

Note that $s_j>0$ (for $j=1,\dots,n$), $\boldsymbol{x}_1>\boldsymbol{0}$, $\boldsymbol{x}_2>\boldsymbol{0}$, $\tanh(x)<x$ for $x>0$, then we can derive the following two inequalities.

$$\boldsymbol{g}_i(\boldsymbol{x}_1)\ge-\sum_j^n|a_{ij}+\Delta a_{ij}|\tanh(s_jx_{1j})+\delta\sin\left(\frac{\beta\pi}{\delta}x_{1i}\right)$$

$$\geq -\|\boldsymbol{A}+\Delta\boldsymbol{A}\|_\infty \cdot \|S\|_\infty \cdot \frac{\delta}{2\beta} + \delta \geq -\frac{\rho\eta\delta}{2\beta} + \delta, \quad \text{for} \quad i = 1, 2, \ldots, n, \tag{14}$$

$$\begin{aligned} \boldsymbol{g}_i(\boldsymbol{x}_2) &\leq \sum_j^n |a_{ij} + \Delta a_{ij}| \tanh(s_j x_{2j}) + \delta \sin\left(\frac{\beta\pi}{\delta} x_{2i}\right) \\ &\leq \|\boldsymbol{A}+\Delta\boldsymbol{A}\|_\infty \cdot \|S\|_\infty \cdot \frac{3\delta}{2\beta} - \delta \leq \frac{3\rho\eta\delta}{2\beta} - \delta, \quad \text{for} \quad i = 1, 2, \ldots, n. \end{aligned} \tag{15}$$

From Eq.(14) we can infer that if $\beta > \dfrac{\rho\eta}{2}$ then $\boldsymbol{g}(\boldsymbol{x}_1) > \boldsymbol{0}$ can be guaranteed. From Eq.(15) we can infer that if $\beta > \dfrac{3\rho\eta}{2}$ then $\boldsymbol{g}(\boldsymbol{x}_2) < \boldsymbol{0}$ can be guaranteed. Combining the above two results, we can conclude that if $\beta > \dfrac{3\rho\eta}{2}$, then there exist two points $\boldsymbol{x}_1 = ((\delta/2\beta), \ldots, (\delta/2\beta))^{\mathrm{T}}$ and $\boldsymbol{x}_2 = ((3\delta/2\beta), \ldots, (3\delta/2\beta))^{\mathrm{T}}$, such that $\boldsymbol{g}(\boldsymbol{x}_1) > \boldsymbol{0}$, $\boldsymbol{g}(\boldsymbol{x}_2) < \boldsymbol{0}$.

Therefore, by the Mean Value Theorem in calculus, there exists a point $\boldsymbol{x}^0 \in B(\boldsymbol{x}^*, r)$, $\boldsymbol{x}_1 < \boldsymbol{x}^0 < \boldsymbol{x}_2$, such that $\boldsymbol{g}(\boldsymbol{x}^0) = \boldsymbol{0} = \boldsymbol{x}^*$. Then, as long as an appropriate β satisfying $\det(\boldsymbol{g}'(\boldsymbol{x}^0)) \neq 0$ is found out, we can conclude that $\boldsymbol{x}^*$ is a snap-back repeller of the map $\boldsymbol{g}$ defined in Eq.(7). In fact, considering that β can be chosen sufficiently large, we can always find out an appropriate constant β, such that $\det(\boldsymbol{g}'(\boldsymbol{x}^0)) = \det\left((\boldsymbol{A}+\Delta\boldsymbol{A}) \cdot \boldsymbol{S} \cdot \boldsymbol{\Phi}(\boldsymbol{S}\boldsymbol{x}^0) + \pi\beta \cos\left(\dfrac{\beta\pi}{\delta}\boldsymbol{x}^0\right)\right) \neq 0$. Then it can be inferred that $\boldsymbol{x}^*$ is a snap-back repeller of the map $\boldsymbol{g}$ defined in Eq.(7).

To conclude, if an appropriate β is selected, such that $\beta > \bar{\beta} \equiv \max\left\{\dfrac{1+\rho\eta}{\pi}, \dfrac{3\rho\eta}{2}, \dfrac{3\sqrt{n}\delta}{2r}\right\}$, furthermore, $\det(\boldsymbol{g}'(\boldsymbol{x}^0)) \neq 0$ is satisfied, then $\boldsymbol{x}^* = \boldsymbol{0}$ is a snap-back repeller of the map $\boldsymbol{g}$ defined in Eq.(7). Then, according to Lemma 1, it can be concluded that the controlled system is chaotic in the sense of Li–Yorke.

Remark 1 On the one hand, there exists the possibility that the selected β is not very appropriate, which makes the equality $\det(\boldsymbol{g}'(\boldsymbol{x}^0)) = 0$ met, though the probability of the event $\det(\boldsymbol{g}'(\boldsymbol{x}^0)) = 0$ is very small for a certain β. Then, the controlled system maybe unable to generate chaos in this condition. On the other hand, the exact value of r in $B(\boldsymbol{x}^*, r)$ is not easy to determine. Based on the two facts above, a feasible method is presented here. Actually we can regard β as an adjustable parameter, i.e., the value of β is increased gradually from the initial value 0. In this process, we can see that a series of period-doubling bifurcation behaviours occur continually. In the end, chaos will be generated in the controlled system.

Remark 2 Assume that the original system to be chaotified is fully unknown. However, if it can be approximated by DFHM with adequate accuracy, then we can believe that the controller for chaotification of DFHM can chaotify the original system at the same time.

Remark 3 In Ref.[30], the authors claim that clearly there exists a constant $\beta_1 > 0$, such that if $\beta > \beta_1$, then $\boldsymbol{g}(\boldsymbol{0}) = \boldsymbol{0} < \boldsymbol{x}_1^*$ and $\boldsymbol{g}(\tilde{\boldsymbol{x}}) > \boldsymbol{x}_1^*$, where $\boldsymbol{x}_1 < \boldsymbol{x}_1^* < \boldsymbol{x}_2$, $\boldsymbol{x}_1 = [(\sigma/2\beta), \ldots, (\sigma/2\beta)]^{\mathrm{T}}$, $\boldsymbol{x}_2 = [(3\sigma/2\beta), \ldots, (3\sigma/2\beta)]^{\mathrm{T}}$. Without doubt, the inequality $\boldsymbol{g}(\boldsymbol{0}) = \boldsymbol{0} < \boldsymbol{x}_1^*$ is met at any time. But the inequality $\boldsymbol{g}(\tilde{\boldsymbol{x}}) > \boldsymbol{x}_1^*$ is not always right. Obviously, we can get $\boldsymbol{g}(\tilde{\boldsymbol{x}}) = \left[\sum\limits_{i=1}^{r} \mu_i(kT_s)\boldsymbol{G}_i\right] \tilde{\boldsymbol{x}} + \boldsymbol{\Phi}(\beta\tilde{\boldsymbol{x}})$. Evidently, the value of the first item of the former equality is fixed. However, the last item is a periodical function with respect to β. So it is impossible that there exists a constant $\beta_1 > 0$, such that $\boldsymbol{g}(\tilde{\boldsymbol{x}}) > \boldsymbol{x}_1^*$ for all $\beta > \beta_1$. Next, we point out another mistake in Ref.[30]. To prove that $\boldsymbol{x}_0$ is a snap-back repeller, the inequality $\det((\boldsymbol{g}^2)'(\boldsymbol{x}_0)) \neq 0$ should be met. However, the authors in Ref.[30] give the following process of verification:

There exists a constant $\beta_2 > \boldsymbol{0}$ such that $\boldsymbol{g}'(\boldsymbol{x}_1^*) < \boldsymbol{0}$ for $\cos\left(\dfrac{\beta\pi}{\sigma}x_1^*\right) < \boldsymbol{0}$. Therefore

$$(\boldsymbol{g}^2)'(\boldsymbol{x}_0) = \boldsymbol{g}'(\boldsymbol{x}_1^*)\boldsymbol{g}'(\boldsymbol{x}_0) \neq \boldsymbol{0}. \tag{16}$$

Indeed, we can get

$$\begin{aligned} \boldsymbol{g}'(\boldsymbol{x}_1^*) = \sum_{i=1}^{r} \Bigg(& \boldsymbol{G}_i \boldsymbol{x}_1^* \frac{\partial \mu_i(\boldsymbol{x})}{\partial \boldsymbol{x}}\bigg|_{\boldsymbol{x}_1^*} \\ & + \mu_i(\boldsymbol{x}_1^*)\boldsymbol{G}_i \Bigg) + \beta\pi \cos\left(\frac{\beta\pi}{\sigma}\boldsymbol{x}_1^*\right), \end{aligned} \tag{17}$$

where

$$\begin{aligned} & \cos\left(\frac{\beta\pi}{\sigma}\boldsymbol{x}_1^*\right) \\ & \equiv \operatorname{diag}\left(\cos\left(\frac{\beta\pi}{\sigma}\boldsymbol{x}_{11}^*\right), \ldots, \cos\left(\frac{\beta\pi}{\sigma}\boldsymbol{x}_{1n}^*\right)\right). \end{aligned}$$

For $\boldsymbol{x}_1 < \boldsymbol{x}_1^* < \boldsymbol{x}_2$, the fact that $\cos\left(\frac{\beta\pi}{\sigma}\boldsymbol{x}_1^*\right)$ is negative definite is true. But we cannot conclude that for sufficiently large β, $\boldsymbol{g}'(\boldsymbol{x}_1^*) < \boldsymbol{0}$, i.e. $\boldsymbol{g}'(\boldsymbol{x}_1^*)$ is also negative definite.

Furthermore, we can derive

$$\boldsymbol{g}'(\boldsymbol{x}_0) = \sum_{i=1}^{r}\left(\boldsymbol{G}_i\boldsymbol{x}_0\frac{\partial\mu_i(\boldsymbol{x})}{\partial\boldsymbol{x}}\bigg|_{x_0} + \mu_i(\boldsymbol{x}_0)\boldsymbol{G}_i\right) + \beta\pi\cos\left(\frac{\beta\pi}{\sigma}\boldsymbol{x}_0\right). \tag{18}$$

Since the last item of Eq.(18) is a periodical function with respect to β and its magnitude increases with the increase of β, it is not possible that there exists a constant $\beta_2 > 0$, such that if $\beta > \beta_2$, then $\det(\boldsymbol{g}'(\boldsymbol{x}_0)) \neq 0$ is met all the time. So, we can not find such a constant $\beta_2 > 0$, for all $\beta > \beta_2$, $\det((\boldsymbol{g}^2)'(\boldsymbol{x}_0)) = \det(\boldsymbol{g}'(\boldsymbol{x}_1^*)) \cdot \det(\boldsymbol{g}'(\boldsymbol{x}_0)) \neq 0$ is always met at any time. Based on the above major mistakes, it can be concluded that the theorem given by authors in Refs.[30, 31] is not accurate.

4. Simulation study

To visualize the effectiveness of the theoretical analysis and controller design, an example is included for illustration. Consider an uncertain DFHM as follows:

$$\boldsymbol{x}(k+1) = (\boldsymbol{A} + \Delta\boldsymbol{A})\tanh(\boldsymbol{S}\boldsymbol{x}(k)) + \boldsymbol{u}(k), \tag{19}$$

where $\boldsymbol{x}(k) = (x_1, x_2, x_3)^{\mathrm{T}}$,

$$\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 1.5 \\ 0 & 2 & 1 \\ 1.5 & 1 & 3 \end{pmatrix}, \quad \Delta\boldsymbol{A} = \begin{pmatrix} 0.07\sin\left(\frac{k\pi}{2}\right) & 0.1\cos\left(\frac{k\pi}{2}\right) & 0 \\ 0.01\cos\left(\frac{k\pi}{2}\right) & 0 & 0.05\cos\left(\frac{k\pi}{2}\right) \\ 0.09\cos\left(\frac{k\pi}{2}\right) & 0.08\sin\left(\frac{k\pi}{2}\right) & 0 \end{pmatrix},$$

$$\boldsymbol{S} = \mathrm{diag}(s_1, s_2, s_3) = \mathrm{diag}(1, 1, 1), \quad \boldsymbol{u}(k) = \delta\sin\left(\frac{\beta\pi}{\delta}\boldsymbol{x}(k)\right).$$

In the simulation, the magnitude of the control input is arbitrarily chosen to be $\delta = 0.1$. The initial value of the states of the controlled uncertain DFHM is chosen as $\boldsymbol{x}(0) = (0, 1, 2)^{\mathrm{T}}$. β can be regarded as adjustable control parameter. Figure 1 presents the periodical orbits of the uncertain DFHM without control. When β takes values from 0 to 1 gradually, the period-doubling bifurcation diagram of x_1 is shown in Fig.2. For clarity, Fig.3 describes the first time bifurcation process of x_1. The bifurcation processes of x_2 and x_3 are similar to that of x_1, therefore are omitted. The chaotic attractor of the controlled system is presented in Fig.4. From these numerical results, it can be verified that the designed controller realizes the chaotification of the above uncertain DFHM.

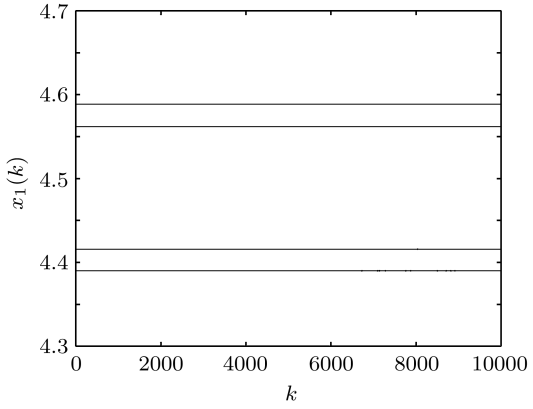


Fig.1. The periodic orbits of x_1 at $\beta = 0$.

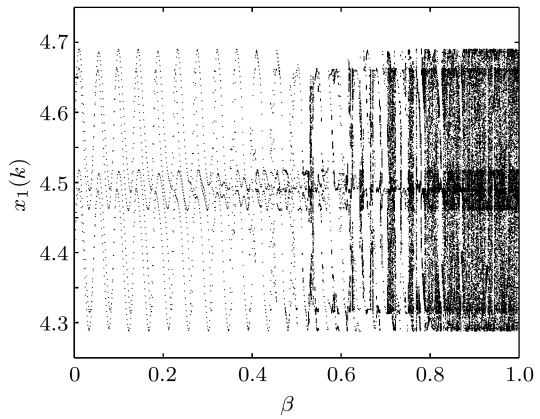


Fig.2. The period-doubling bifurcation diagram of x_1-β.

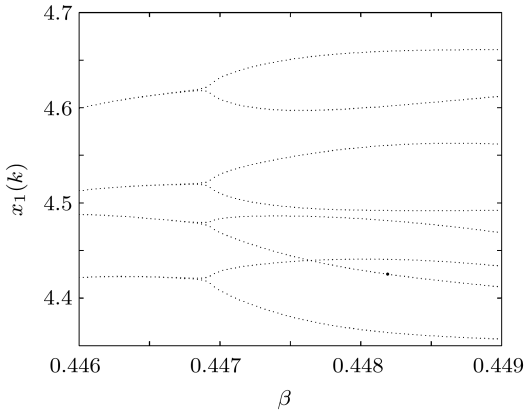


Fig.3. The first time period-doubling bifurcation diagram of x_1-β.

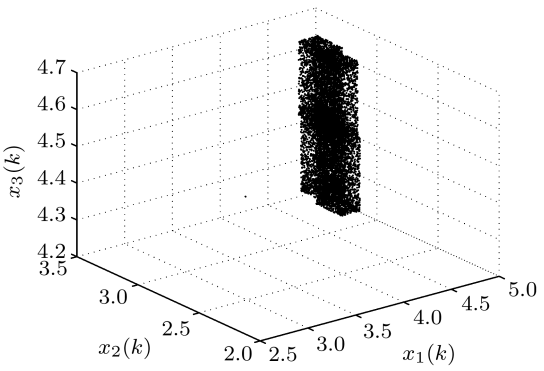


Fig.4. The chaotic attractor of the controlled system at $\beta = 200$.

5. Conclusions

A new approach is developed in this paper for chaotification of a DFHM with uncertain parameters. A simple state feedback controller with arbitrarily small magnitude is designed for this purpose. It is proven that the controlled system indeed becomes chaotic in the sense of Li–Yorke by rigorous mathematical analysis. Since applications of chaos have enormous potential in many fields, such as life systems, biological systems, medical systems, etc., the result of this paper is valuable in future theoretical researches and engineering applications.

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