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Robust adaptive synchronization of chaotic neural networks by slide technique*

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In this paper, we focus on the robust adaptive synchronization between two coupled chaotic neural networks with all the parameters unknown and time-varying delay. In order to increase the robustness of the two coupled neural networks, the key idea is that a sliding-mode-type controller is employed. Moreover, without the estimate values of the network unknown parameters taken as an updating object, a new updating object is introduced in the constructing of controller. Using the proposed controller, without any requirements for the boundedness, monotonicity and differentiability of activation functions, and symmetry of connections, the two coupled chaotic neural networks can achieve global robust synchronization no matter what their initial states are. Finally, the numerical simulation validates the effectiveness and feasibility of the proposed technique.

Keywords: robust adaptive synchronization, slide technique, chaotic neural networks, time-varying delay

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1. Introduction

In the past few years, there has been increasing the interest in potential applications of the dynamics of artificial neural networks in many areas.[1−17] Most of the publications are devoted mainly to the analyses of stability and periodic oscillations of neural networks of different kinds. It is well known that delays often occur in the interaction between the neurons, and the introduction of delays into neural networks may make their dynamical behaviours much more complicated even in the presence of only one strange attractor if the network parameters and time delays are appropriately chosen.[18,19]

A chaotic system has complex dynamical behaviours, i.e. extremely sensitive to any tiny variation of initial conditions, trajectories bounded in phase space, and so on.[20] Since a drive-response conception was used in the pioneering work by Pecora and Carrol,[21] the increasing interest has been devoted to the study of chaos synchronization. Research on the synchronization of chaotic activity has increased considerably in the last few years. Besides the original master-slave mechanism for chaos synchronization, a wide variety of approaches have been proposed to the synchronization of chaotic systems, which include the PC method,[18] OGY (Ott, Grebogi, and Yorke) method,[22] scalar driving method, coupling control,[23] manifold-based method,[24] fuzzy control,[25] impulsive control method,[26] active control, adaptive control,[27−30] time-delay feedback approach,[31] etc. In most of the previous studies the proposed schemes are often only applicable to very specific scenarios; in addition, the added controller may sometimes be too big to be implemented physically. One practical method is linear feedback, but it is difficult to find the suitable feedback constant, and thus numerical calculation has to be used to calculate, for example, the conditional Lyapunov exponents.

Artificial neural network models can exhibit chaotic behaviours,[18,27,32−37] so the synchronization of chaotic neural networks has also become an important subject of study. Very recently, some investigators have paid attention to the synchronization of neural networks.[18,19,38−44] Synchronization of coupled delayed neural networks and applications to chaotic

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cellular neural networks in Ref. [38] have led to a theoretical condition for synchronization under the assumption that the coupling matrix is irreducible. In Ref. [41], the problem of global exponential synchronization between two identical chaotic neural networks was considered via output or state coupling. Cao and Lu [19] proposed a simple adaptive feedback scheme for the synchronization of coupled uncertain neural networks with or without time-varying delay based on the invariant principles of functional differential equations. However, there exists a restriction on network unknown parameters, i.e. they must be constant in most of the above studies. When the network unknown parameters are time-varying, how do we deal with the problem about the synchronization of nonlinear chaotic systems? Solving this problem is more meaningful, for in practical circumstances, the parameters of some systems cannot be exactly known in advance, while these uncertainties can destroy, or even break, the synchronization.[20,27,45]

In this paper, we deal with the synchronization in a similar broad class of chaotic neural networks with time-varying delay and unknown parameters. As we can see, the synchronization dynamics of recurrently delayed neural networks with only the slave system parameters unknown has been studied in Refs.[40,43]. However, some novel approaches to robust adaptive synchronization and parameters identification for chaotic neural networks are proposed in this paper, when both the master system parameters and the slave system parameters are unknown. In order to increase the robustness of the two coupled neural networks, an adaptive sliding-mode-type controller and an updating law are adopted. Moreover, without the estimate values of the network unknown parameters taken as an updating object, a new updating object is introduced in the constructing of controllers. Some numerical simulations are also given to validate the effectiveness and feasibility of the proposed technique.

The rest of this paper is organized as follows. In Section 2, the preparatory knowledge and model description are given. In Section 3, some criteria for the robust adaptive synchronization of two chaotic neural networks are presented based on the Lyapunov stability theory and the slide technique. In Section 4, a numerical example is given to validate the effectiveness of our results. Finally, conclusions are drawn from the present study in Section 5.

2. Preparatory knowledge and problem formulation

Let $\boldsymbol{R}$ denote the set of real numbers. Consider two chaotic neural networks with different time-varying unknown parameters and initial conditions, in which the master (drive) system (1) and the slave (response) system (3) are described respectively as follows:

$$\dot{x}_i(t) = -c_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t)f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(t)f_j(x_j(t-\tau(t))) + J_i \quad (1)$$

$(i = 1, 2, \cdots, n)$ with initial conditions

$$x_i(t) = \phi_i(t) \in \mathcal{C}([-\tau^*, 0], \boldsymbol{R}), i = 1, 2, \cdots, n, \quad (2)$$

and

$$\dot{\widetilde{x}}_i(t) = -c_i^m(t)\widetilde{x}_i(t) + \sum_{j=1}^{n} a_{ij}^m(t)f_j(\widetilde{x}_j(t)) + \sum_{j=1}^{n} b_{ij}^m(t)f_j(\widetilde{x}_j(t-\tau(t))) + J_i + u_i \quad (3)$$

$(i = 1, 2, \cdots, n)$ with initial conditions

$$\widetilde{x}_i(t) = \psi_i(t) \in \mathcal{C}([-\tau^*, 0], \boldsymbol{R}), i = 1, 2, \cdots, n, \quad (4)$$

where $\mathcal{C}([-\tau^*, 0], \boldsymbol{R})$ denotes the set of all continuous functions from $[-\tau^*, 0]$ to $\boldsymbol{R}$ with $\tau^* = \max_t\{\tau(t)\}$; $x_i(t)$ and $\widetilde{x}_i(t)$ $(i = 1, 2, \cdots, n)$ represent the state variables; decay functions $c_i(t) > 0$ and $c_i^m(t) > 0$; $a_{ij}(t)$ and $a_{ij}^m(t)$ are the feedback weight functions; $b_{ij}(t)$ and $b_{ij}^m(t)$ are the delayed feedback weight functions; J_i $(i = 1, 2, \cdots, n)$ are external inputs; $f_j(\cdot)$ $(i = 1, 2, \cdots, n)$ are activation functions. The systems reduce to the neural networks without delay if $\tau(t) = 0$ or all the $b_{ij}(t) = 0$ and $b_{ij}^m(t) = 0$. Obviously, the well-known Hopfield neural network and cellular neural network belong to this category.

Here, it should be noted that $c_i(t), a_{ij}(t), b_{ij}(t)$ and $c_i^m(t), a_{ij}^m(t), b_{ij}^m(t)$ are time-varying unknown parameters which vary in bound intervals and their bounds of the intervals are unknown. $u(t) = [u_1, \cdots, u_n]^{\mathrm{T}}$ is the driving signal which can be in different forms. Therefore, the goal of control is to design and implement an appropriate controller $u(t)$ for the slave system such that controlled slave system (3) could be synchronous with master system (1).

Let

$$\begin{aligned} c_i^m(t) &= c_i(t) + \delta c_i(t), \\ a_{ij}^m(t) &= a_{ij}(t) + \delta a_{ij}(t), \\ b_{ij}^m(t) &= b_{ij}(t) + \delta b_{ij}(t). \end{aligned} \tag{5}$$

It is obvious that $\delta c_i(t), \delta a_{ij}(t)$ and $\delta b_{ij}(t)$ are also time-varying unknown parameters.

Then, error dynamical system between Eqs.(1) and (3) can be expressed as

$$\begin{aligned} \dot{e}_i(t) =& - c_i(t)e_i(t) - \delta c_i(t)\widetilde{x}_i(t) \\ &+ \sum_{j=1}^{n} a_{ij}(t)g_j(e_j(t)) \\ &+ \sum_{j=1}^{n} \delta a_{ij}(t)f_j(\widetilde{x}_j(t)) \\ &+ \sum_{j=1}^{n} b_{ij}(t)g_j(e_j(t-\tau(t))) \\ &+ \sum_{j=1}^{n} \delta b_{ij}(t)f_j(\widetilde{x}_j(t-\tau(t))) + u_i(t), \end{aligned} \tag{6}$$

$(i = 1, 2, \cdots, n)$ where

$$\begin{aligned} &e_i(t) = \widetilde{x}_i(t) - x_i(t), \\ &g_j(e_j(t)) = f_j(\widetilde{x}_j(t)) - f_j(x_j(t)), \\ &g_j(e_j(t-\tau(t))) = f_j(\widetilde{x}_j(t-\tau(t))) - f_j(x_j(t-\tau(t))). \end{aligned}$$

Throughout the paper, we have made the following assumptions:

Assumption 1

(A1) $0 \le \tau(t) \le \tau^*$ and $\dot{\tau} \le \sigma < 1$.

(A2) There exist constants $k_j > 0$ $(j = 1, 2, \cdots, n)$ for any two different variables $x, y \in \boldsymbol{R}$ such that

$$0 \le \frac{f_j(x) - f_j(y)}{x - y} \le k_j,\ j = 1, 2, \cdots, n. \tag{7}$$

$(A2^*)$ There exist constants $k_j > 0$ $(j = 1, 2, \cdots, n)$ for any two different variables $x, y \in \boldsymbol{R}$ such that

$$|f_j(x) - f_j(y)| \le k_j|x - y|\ (j = 1, 2, \cdots, n). \tag{8}$$

Obviously, condition $(A2^*)$ is less conservative than condition (A2), and some general activation functions in conventional neural networks, such as the standard sigmoidal functions and piecewise-linear function, satisfy condition (A2) or $(A2^*)$.

According to the properties of condition (A2), $g_i(\cdot)$ possess the following properties:

$$|g_i(e_i(t))| \le k_i|e_i(t)|,\ 0 \le e_i(t)g_i(e_i(t)) \le k_i e_i^2(t), \tag{9}$$

$(i = 1, 2, \cdots, n)$.

Now, our goal of control is to find a controller $u(t)$ such that the states of response system (3) and the states of drive system (1) are globally robustly asymptotically synchronized, that is,

$$\lim_{t\to\infty} \|e(t)\| = 0 \tag{10}$$

holds, where $e(t) = [e_1(t), e_2(t), \cdots, e_n(t)]^{\mathrm{T}}$.

Since the time-derivatives of time-varying unknown parameters in Eqs.(1) and (3) are not always equal to zero, their estimate values cannot directly be taken as an updating object when constructing a controller for the goal as described by expression (10). In order to achieve this goal, we must find a new updating object. Therefore, we make the following assumption about Eqs.(1) and (3):

Assumption 2 Let

$$\begin{aligned} &c_i(t), c_i^m(t) \in [\underline{c}_i, \overline{c}_i], \\ &a_{ij}(t), a_{ij}^m(t) \in [\underline{a}_{ij}, \overline{a}_{ij}], \\ &b_{ij}(t), b_{ij}^m(t) \in [\underline{b}_{ij}, \overline{b}_{ij}], \end{aligned} \tag{11}$$

where $\underline{c}_i$, $\overline{c}_i$, $\underline{a}_{ij}$, $\overline{a}_{ij}$, $\underline{b}_{ij}$ and $\overline{b}_{ij}$ are unknown constants.

From Assumption 2, we have

$$\begin{aligned} &|c_i(t)|, |c_i^m(t)| \le \alpha_i, \\ &|a_{ij}(t)|, |a_{ij}^m(t)| \le \beta_{ij}, \\ &|b_{ij}(t)|, |b_{ij}^m(t)| \le \gamma_{ij}, \end{aligned} \tag{12}$$

and

$$\begin{aligned} &|\delta c_i(t)| \le \alpha_i - \underline{c}_i, \\ &|\delta a_{ij}(t)| \le \beta_{ij} - \underline{a}_{ij}, \\ &|\delta b_{ij}(t)| \le \gamma_{ij} - \underline{b}_{ij}, \end{aligned} \tag{13}$$

where

$$\begin{aligned} \alpha_i &= \max\{|\underline{c}_i|, |\overline{c}_i|\}, \\ \beta_{ij} &= \max\{|\underline{a}_{ij}|, |\overline{a}_{ij}|\}, \\ \gamma_{ij} &= \max\{|\underline{b}_{ij}|, |\overline{b}_{ij}|\}. \end{aligned} \tag{14}$$

Notice that $\alpha_i, \beta_{ij}, \gamma_{ij}, \underline{c}_i, \underline{a}_{ij}$ and $\underline{b}_{ij}$ are unknown constants due to Assumption 2. So their estimate values can be chosen as a new updating object when constructing the controller for the goal described by expression (10).

3. Synchronization between chaotic neural networks

Based on a Lyapunov functional and slide technique, we can provide design rules for controller $u(t)$, with which global robust synchronization is ensured.

Theorem 1 Under assumption 2, drive system (1) will synchronize with response system (3) if we choose the following adaptive sliding-mode-type controller and updating law:

$$\begin{aligned}
u_i(t) =& -\operatorname{sign}(e_i(t))\Big[-\hat{\alpha}_i|\widetilde{x}_i(t)| + \sum_{j=1}^{n}\hat{\beta}_{ij}|f_j(\widetilde{x}_j(t))| \\
&+ \sum_{j=1}^{n}\hat{\gamma}_{ij}|f_j(\widetilde{x}_j(t-\tau(t)))| \\
&+ \hat{\underline{c}}_i|\widetilde{x}_i(t)| - \sum_{j=1}^{n}\hat{\underline{a}}_{ij}|f_j(\widetilde{x}_j(t))| \\
&- \sum_{j=1}^{n}\hat{\underline{a}}_{ij}|f_j(\widetilde{x}_j(t-\tau(t)))|\Big], \\
\dot{\hat{\alpha}}_i =& -\frac{p_i}{q_i}|e_i(t)||\widetilde{x}_i(t)|, \\
\dot{\hat{\beta}}_{ij} =& \frac{p_i}{r_{ij}}|e_i(t)||f_j(\widetilde{x}_j(t))|, \\
\dot{\hat{\gamma}}_{ij} =& \frac{p_i}{s_{ij}}|e_i(t)||f_j(\widetilde{x}_j(t-\tau(t)))|, \\
\dot{\hat{\underline{c}}}_i =& \frac{p_i}{q_i}|e_i(t)||\widetilde{x}_i(t)|, \\
\dot{\hat{\underline{a}}}_{ij} =& -\frac{p_i}{r_{ij}}|e_i(t)||f_j(\widetilde{x}_j(t))|, \\
\dot{\hat{\underline{b}}}_{ij} =& -\frac{p_i}{s_{ij}}|e_i(t)||f_j(\widetilde{x}_j(t-\tau(t)))|,
\end{aligned} \tag{15}$$

where $\hat{\alpha}_i, \hat{\beta}_{ij}, \hat{\gamma}_{ij}$ $\hat{\underline{c}}_i, \hat{\underline{a}}_{ij}$ and $\hat{\underline{b}}_{ij}$ are, respectively, the estimate values of unknown constants $\alpha_i, \beta_{ij}, \gamma_{ij}$ $\underline{c}_i, \underline{a}_{ij}$ and $\underline{b}_{ij}$ which appear in expressions (14) and (11); p_i, q_i, r_{ij}, s_{ij} $(i, j = 1, 2, \cdots, n)$ are positive real values and one of the following conditions is satisfied:

(B1) Assume that conditions (A1) and (A2) hold, and that there exist two positive numbers $r_1 \in [0, 1]$ and $r_2 \in [0, 1]$. Let

$$\begin{aligned}
\Phi_i \triangleq& -p_i c_i(t) + \frac{1}{2}\sum_{j=1, j\neq i}^{n} p_i\big(|a_{ij}(t)|^{2r_1} \\
&+ k_i^2 p_j |a_{ji}(t)|^{2(1-r_1)}\big) \\
&+ \frac{1}{2}\sum_{j=1}^{n} p_i\big(\hat{\gamma}_{ij}^{2r_2} + \frac{1}{1-\sigma}k_i^2 p_j \hat{\gamma}_{ji}^{2(1-r_2)}\big) \\
&(i = 1, 2, \cdots, n)
\end{aligned}$$

such that

$$p_i k_i \big(a_{ii}(t)\big)^+ + \Phi_i < 0 \quad (i = 1, 2, \cdots, n),$$

where $\big(a_{ii}(t)\big)^+ = \max\big\{a_{ii}(t), 0\big\}$.

(B2) Assume that condition $(A1)$ holds and

$$p_i k_i |a_{ii}(t)| + \Phi_i < 0 \quad (i = 1, 2, \cdots, n).$$

Proof. (B1) Choose the Lyapunov functional

$$\begin{aligned}
V(t) =& \frac{1}{2}\sum_{i=1}^{n}\Big\{p_i e_i^2(t) \\
&+ \frac{p_i}{1-\sigma}\sum_{j=1}^{n}\hat{\gamma}_{ij}^{2(1-r_2)}\int_{t-\tau(t)}^{t} g_j^2(e_j(s))\mathrm{d}s \\
&+ \Big[q_i(\alpha_i - \hat{\alpha}_i)^2 + \sum_{j=1}^{n} r_{ij}(\beta_{ij} - \hat{\beta}_{ij})^2 \\
&+ \sum_{j=1}^{n} s_{ij}(\gamma_{ij} - \hat{\gamma}_{ij})^2\Big] \\
&+ \Big[q_i(\underline{c}_i - \hat{\underline{c}}_i)^2 + \sum_{j=1}^{n} r_{ij}(\underline{a}_{ij} - \hat{\underline{a}}_{ij})^2 \\
&+ \sum_{j=1}^{n} s_{ij}(\underline{b}_{ij} - \hat{\underline{b}}_{ij})^2\Big]\Big\},
\end{aligned} \tag{16}$$

where $\hat{\alpha}_i, \hat{\beta}_{ij}, \hat{\gamma}_{ij}$ $\hat{\underline{c}}_i, \hat{\underline{a}}_{ij}$ and $\hat{\underline{b}}_{ij}$ are estimate values of unknown constants $\alpha_i, \beta_{ij}, \gamma_{ij}$ $\underline{c}_i, \underline{a}_{ij}$ and $\underline{b}_{ij}$ respectively.

Differentiating V with respect to time and using the solution of expression (6), we obtain

$$\begin{aligned}
\dot{V}(t) \leq& -\sum_{i=1}^{n} p_i c_i(t) e_i^2(t) - \sum_{i=1}^{n} p_i|\delta c_i(t)||e_i(t)||\widetilde{x}_i(t)| \\
&+ \sum_{i=1}^{n} p_i a_{ii}(t) e_i(t) g_i(e_i(t)) \\
&+ \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} p_i|a_{ij}(t)||e_i(t)||g_j(e_j(t))| \\
&+ \sum_{i=1}^{n}\sum_{j=1}^{n} p_i|\delta a_{ij}(t)||e_i(t)||f_j(\widetilde{x}_j(t))| \\
&+ \sum_{i=1}^{n}\sum_{j=1}^{n} p_i\Big(|b_{ij}(t)||e_i(t)|\Big)|g_j(e_j(t-\tau(t)))| \\
&+ \sum_{i=1}^{n}\sum_{j=1}^{n} p_i|\delta b_{ij}(t)||e_i(t)||f_j(\widetilde{x}_j(t-\tau(t)))| \\
&+ \sum_{i=1}^{n} p_i e_i(t) u_i(t) + \sum_{i=1}^{n} q_i(\alpha_i - \hat{\alpha}_i)(-\dot{\hat{\alpha}}_i) \\
&+ \sum_{i=1}^{n}\sum_{j=1}^{n} r_{ij}(\beta_{ij} - \hat{\beta}_{ij})(-\dot{\hat{\beta}}_{ij})
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{i=1}^{n}\sum_{j=1}^{n}s_{ij}(\gamma_{ij}-\hat{\gamma}_{ij})(-\dot{\hat{\gamma}}_{ij})\\
&+\sum_{i=1}^{n}q_i(\underline{c}_i-\hat{\underline{c}}_i)(-\dot{\hat{\underline{c}}}_i)\\
&+\sum_{i=1}^{n}\sum_{j=1}^{n}r_{ij}(\underline{a}_{ij}-\hat{\underline{a}}_{ij})(-\dot{\hat{\underline{a}}}_{ij})\\
&+\sum_{i=1}^{n}\sum_{j=1}^{n}s_{ij}(\underline{b}_{ij}-\hat{\underline{b}}_{ij})(-\dot{\hat{\underline{b}}}_{ij})\\
&+\frac{1}{2(1-\sigma)}\sum_{i=1}^{n}\sum_{j=1}^{n}p_i\hat{\gamma}_{ij}^{2(1-r_2)}g_j^2(e_j(t))\\
&-\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}p_i\hat{\gamma}_{ij}^{2(1-r_2)}g_j^2(e_j(t-\tau(t))). \qquad (17)
\end{aligned}
$$

In the above estimations, using the inequality $2ab\le a^2+b^2$ and inequalities of expression (12), one can have

$$
\begin{aligned}
&\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}p_i|a_{ij}(t)||e_i(t)||g_j(e_j(t))|\\
=&\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}p_i\Big(|a_{ij}(t)|^{r_1}|e_i(t)|\Big)\\
&\times\Big(|a_{ij}(t)|^{1-r_1}|g_j(e_j(t))|\Big)\\
\le&\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}p_i|a_{ij}(t)|^{2r_1}e_i^2(t)\\
&+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}p_i|a_{ij}(t)|^{2(1-r_1)}g_j^2(e_j(t)) \qquad (18)
\end{aligned}
$$

and

$$
\begin{aligned}
&\sum_{i=1}^{n}\sum_{j=1}^{n}p_i\Big(|b_{ij}(t)||e_i(t)|\Big)|g_j(e_j(t-\tau(t)))|\\
\le&\sum_{i=1}^{n}\sum_{j=1}^{n}p_i\Big(\hat{\gamma}_{ij}^{r_2}|e_i(t)|\Big)\Big(\hat{\gamma}_{ij}^{1-r_2}|g_j(e_j(t-\tau(t)))|\Big)\\
\le&\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}p_i\hat{\gamma}_{ij}^{2r_2}e_i^2(t)\\
&+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}p_i\hat{\gamma}_{ij}^{2(1-r_2)}g_j^2(e_j(t-\tau(t))). \qquad (19)
\end{aligned}
$$

Combining inequalities (17)–(19), we obtain

$$
\begin{aligned}
\dot{V}(t)\le&-\sum_{i=1}^{n}p_ic_i(t)e_i^2(t)\\
&-\sum_{i=1}^{n}p_i(\alpha_i-\underline{c}_i)|e_i(t)||\widetilde{x}_i(t)|\\
&+\sum_{i=1}^{n}p_ik_i\Big(a_{ii}(t)\Big)^{+}e_i^2(t)\\
&+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}p_i|a_{ij}(t)|^{2r_1}e_i^2(t)\\
&+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}p_i|a_{ij}(t)|^{2(1-r_1)}g_j^2(e_j(t))\\
&+\sum_{i=1}^{n}\sum_{j=1}^{n}p_i(\beta_{ij}-\underline{a}_{ij})|e_i(t)||f_j(\widetilde{x}_j(t))|\\
&+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}p_i\hat{\gamma}_{ij}^{2r_2}e_i^2(t)\\
&+\sum_{i=1}^{n}\sum_{j=1}^{n}p_i(\gamma_{ij}-\underline{b}_{ij})|e_i(t)||f_j(\widetilde{x}_j(t-\tau(t)))|\\
&-\sum_{i=1}^{n}p_i|e_i(t)|\Big[-\hat{\alpha}_i|\widetilde{x}_i(t)|+\sum_{j=1}^{n}\hat{\beta}_{ij}|f_j(\widetilde{x}_j(t))|\\
&+\sum_{j=1}^{n}\hat{\gamma}_{ij}|f_j(\widetilde{x}_j(t-\tau(t)))|+\hat{\underline{c}}_i|\widetilde{x}_i(t)|\\
&-\sum_{j=1}^{n}\hat{\underline{a}}_{ij}|f_j(\widetilde{x}_j(t))|-\sum_{j=1}^{n}\hat{\underline{a}}_{ij}|f_j(\widetilde{x}_j(t-\tau(t)))|\Big]\\
&+\sum_{i=1}^{n}p_i(\alpha_i-\hat{\alpha}_i)\Big(|e_i(t)||\widetilde{x}_i(t)|\Big)\\
&-\sum_{i=1}^{n}\sum_{j=1}^{n}p_i(\beta_{ij}-\hat{\beta}_{ij})\Big(|e_i(t)||f_j(\widetilde{x}_j(t))|\Big)\\
&-\sum_{i=1}^{n}\sum_{j=1}^{n}p_i(\gamma_{ij}-\hat{\gamma}_{ij})\Big(|e_i(t)||f_j(\widetilde{x}_j(t-\tau(t)))|\Big)\\
&-\sum_{i=1}^{n}p_i(\underline{c}_i-\hat{\underline{c}}_i)\Big(|e_i(t)||\widetilde{x}_i(t)|\Big)\\
&+\sum_{i=1}^{n}\sum_{j=1}^{n}p_i(\underline{a}_{ij}-\hat{\underline{a}}_{ij})\Big(|e_i(t)||f_j(\widetilde{x}_j(t))|\Big)\\
&+\sum_{i=1}^{n}\sum_{j=1}^{n}p_i(\underline{b}_{ij}-\hat{\underline{b}}_{ij})\Big(|e_i(t)||f_j(\widetilde{x}_j(t-\tau(t)))|\Big)\\
&+\frac{1}{2(1-\sigma)}\sum_{i=1}^{n}\sum_{j=1}^{n}p_i\hat{\gamma}_{ij}^{2(1-r_2)}g_j^2(e_j(t)). \qquad (20)
\end{aligned}
$$

Arranging the above inequalities, it follows that

$$
\begin{aligned}
\dot{V}(t)\le&-\sum_{i=1}^{n}p_ic_i(t)e_i^2(t)+\sum_{i=1}^{n}p_ik_i\Big(a_{ii}(t)\Big)^{+}e_i^2(t)\\
&+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}p_i|a_{ij}(t)|^{2r_1}e_i^2(t)\\
&+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}p_i|a_{ij}(t)|^{2(1-r_1)}g_j^2(e_j(t))\\
&+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}p_i\hat{\gamma}_{ij}^{2r_2}e_i^2(t)
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{2(1-\sigma)}\sum_{i=1}^{n}\sum_{j=1}^{n}p_i\hat{\gamma}_{ij}^{2(1-r_2)}g_j^2(e_j(t))\\
&\le\sum_{i=1}^{n}\Big[-p_ic_i(t)+p_ik_i\Big(a_{ii}(t)\Big)^{+}\\
&\quad+\frac{1}{2}\sum_{j=1,j\neq i}^{n}p_i\Big(|a_{ij}(t)|^{2r_1}+k_i^2p_ja_{ji}(t)|^{2(1-r_1)}\Big)\\
&\quad+\frac{1}{2}\sum_{j=1}^{n}p_i\Big(\hat{\gamma}_{ij}^{2r_2}\quad+\frac{1}{1-\sigma}k_i^2p_j\hat{\gamma}_{ji}^{2(1-r_2)}\Big)\Big]e_i^2(t)\\
&<0. \qquad (21)
\end{aligned}
$$

It is obvious that $\dot{V}=0$ if and only if $e_i=0$ $(i=1,2,\cdots,n)$. Note that the construction of Lyapunov functional (16) and inequality (21) implies the boundedness of all the $e_i(t)$. According to the well-known Lyapunov–LaSalle-type theorem for functional differential equations,[46] the trajectories of the error dynamical system (6), starting with an arbitrary initial value, converges asymptotically to the largest invariant set contained in $\dot{V}=0$ as $t\to\infty$. So, if the condition (B1) of theorem 1 is satisfied, by using the adaptive sliding-mode-type controller and parameter estimation updating law (15), we conclude that $e_i(t)\to 0$ when $t\to\infty$, i.e. $\widetilde{x}_i(t)\to x_i(t)$.

(B2) The proof of condition (B2) is exactly the same as that for (B1) by employing the same Lyapunov functional as that in condition (A2)*.

In summary, this completes the proof of Theorem 1.

Corollary 1 Under assumption 2, drive system (1) will synchronize with response system (3) if we choose the following adaptive sliding-mode-type controller and updating law:

$$
\begin{aligned}
u_i(t)=&-\operatorname{sign}(e_i(t))\Big[-\hat{\alpha}_i|\widetilde{x}_i(t)|+\sum_{j=1}^{n}\hat{\beta}_{ij}|f_j(\widetilde{x}_j(t))|\\
&+\sum_{j=1}^{n}\hat{\gamma}_{ij}|f_j(\widetilde{x}_j(t-\tau(t)))|\\
&+\hat{\underline{c}}_i|\widetilde{x}_i(t)|-\sum_{j=1}^{n}\hat{\underline{a}}_{ij}|f_j(\widetilde{x}_j(t))|\\
&-\sum_{j=1}^{n}\hat{\underline{a}}_{ij}|f_j(\widetilde{x}_j(t-\tau(t)))|\Big],
\end{aligned}
$$

$$
\begin{aligned}
&\dot{\hat{\alpha}}_i=-p_i|e_i(t)||\widetilde{x}_i(t)|,\\
&\dot{\hat{\beta}}_{ij}=p_i|e_i(t)||f_j(\widetilde{x}_j(t))|,\\
&\dot{\hat{\gamma}}_{ij}=p_i|e_i(t)||f_j(\widetilde{x}_j(t-\tau(t)))|,\\
&\dot{\hat{\underline{c}}}_i=p_i|e_i(t)||\widetilde{x}_i(t)|,\\
&\dot{\hat{\underline{a}}}_{ij}=-p_i|e_i(t)||f_j(\widetilde{x}_j(t))|,\\
&\dot{\hat{\underline{b}}}_{ij}=-p_i|e_i(t)||f_j(\widetilde{x}_j(t-\tau(t)))|,
\end{aligned}
$$

where p_i $(i=1,2,\cdots,n)$ are positive real values, and one of the following conditions is satisfied:

(B1) Assume that conditions (A1) and (A2) hold, and that there exist two positive numbers $r_1\in[0,1]$ and $r_2\in[0,1]$. Let

$$
\begin{aligned}
\Phi_i\triangleq&-p_ic_i(t)+\frac{1}{2}\sum_{j=1,j\neq i}^{n}p_i\Big(|a_{ij}(t)|^{2r_1}\\
&+k_i^2p_j|a_{ji}(t)|^{2(1-r_1)}\Big)\\
&+\frac{1}{2}\sum_{j=1}^{n}p_i\Big(\hat{\gamma}_{ij}^{2r_2}+\frac{1}{1-\sigma}k_i^2p_j\hat{\gamma}_{ji}^{2(1-r_2)}\Big)\\
&(i=1,2,\cdots,n)
\end{aligned}
$$

such that

$$p_ik_i\Big(a_{ii}(t)\Big)^{+}+\Phi_i<0\quad(i=1,2,\cdots,n)$$

where $\Big(a_{ii}(t)\Big)^{+}=\max\Big\{a_{ii}(t),0\Big\}$.

(B2) Assume that condition (A1) holds and

$$p_ik_i|a_{ii}(t)|+\Phi_i<0\quad(i=1,2,\cdots,n).$$

Proof Let $q_i=r_{ij}=s_{ij}=1$ $(i,j=1,2,\cdots,n)$, we can obtain corollary 1 directly from theorem 1. Here we omit it.

Corollary 2 Under assumption 2, drive system (1) will synchronize with response system (3) if we choose the following adaptive sliding-mode-type controller and updating law:

$$
\begin{aligned}
u_i(t)=&-\operatorname{sign}(e_i(t))\Big[-\hat{\alpha}_i|\widetilde{x}_i(t)|\\
&+\sum_{j=1}^{n}\hat{\beta}_{ij}|f_j(\widetilde{x}_j(t))|\\
&+\sum_{j=1}^{n}\hat{\gamma}_{ij}|f_j(\widetilde{x}_j(t-\tau(t)))|\\
&+\hat{\underline{c}}_i|\widetilde{x}_i(t)|-\sum_{j=1}^{n}\hat{\underline{a}}_{ij}|f_j(\widetilde{x}_j(t))|\\
&-\sum_{j=1}^{n}\hat{\underline{a}}_{ij}|f_j(\widetilde{x}_j(t-\tau(t)))|Big],
\end{aligned}
\qquad (22)
$$

$$
\begin{aligned}
&\dot{\hat{\alpha}}_i=-|e_i(t)||\widetilde{x}_i(t)|,\\
&\dot{\hat{\beta}}_{ij}=|e_i(t)||f_j(\widetilde{x}_j(t))|,\\
&\dot{\hat{\gamma}}_{ij}=|e_i(t)||f_j(\widetilde{x}_j(t-\tau(t)))|,\\
&\dot{\hat{\underline{c}}}_i=|e_i(t)||\widetilde{x}_i(t)|,\\
&\dot{\hat{\underline{a}}}_{ij}=-|e_i(t)||f_j(\widetilde{x}_j(t))|,\\
&\dot{\hat{\underline{b}}}_{ij}=-|e_i(t)||f_j(\widetilde{x}_j(t-\tau(t)))|,
\end{aligned}
\qquad (23)
$$

and one of the following conditions is satisfied:

(B1) Assume that conditions (A1) and (A2) hold, and that there exist two positive numbers $r_1 \in [0,1]$ and $r_2 \in [0,1]$. Let

$$\begin{aligned}\Phi_i \triangleq & -c_i(t) + \frac{1}{2}\sum_{j=1,j\neq i}^{n}\Big(|a_{ij}(t)|^{2r_1} \\ & + k_i^2|a_{ji}(t)|^{2(1-r_1)}\Big) \\ & + \frac{1}{2}\sum_{j=1}^{n}\Big(\hat{\gamma}_{ij}^{2r_2} + \frac{1}{1-\sigma}k_i^2\hat{\gamma}_{ji}^{2(1-r_2)}\Big) \\ & (i=1,2,\cdots,n)\end{aligned}$$

such that

$$k_i\Big(a_{ii}(t)\Big)^{+} + \Phi_i < 0 \quad (i=1,2,\cdots,n),$$

where $\Big(a_{ii}(t)\Big)^{+} = \max\Big\{a_{ii}(t), 0\Big\}$.

(B2) Assume that condition (A1) holds and

$$k_i|a_{ii}(t)| + \Phi_i < 0 \quad (i=1,2,\cdots,n).$$

Proof Let $p_i = q_i = r_{ij} = s_{ij} = 1$ $(i,j = 1,2,\cdots,n)$, we can obtain corollary 2 directly from theorem 1. Here we omit it.

Remark 1 If we choose variable parameters $r_1 \in [0,1]$ and $r_2 \in [0,1]$ appropriately, we can obtain a series of corollaries.

Remark 2 In Ref.[40], the authors have studied a particular type of chaotic system with constant delays. However, in this paper, we have considered a general model with time-varying delays. Moreover, this paper studies the robust adaptive synchronization for chaotic neural networks, when both the master system parameters and the slave system parameters are unknown.

Remark 3 Many approaches have been proposed to the synchronization of chaotic neural networks, such as the coupling matrix,[39,41,42] adaptive control scheme,[19,40,43] impulsive control method.[44] In order to increase the robustness of the two chaotic neural networks, we introduce the slide technique and adopt an adaptive sliding-mode-type controller and updating law. Moreover, without the estimate values of the network unknown parameters taken as an updating object, a new updating object is introduced in the constructing of controller.

Remark 4 For the robust synchronization of uncertain delayed neural networks, there are only a few results available and it is still a challenging problem (see, Refs.[40,43]). Zhou *et al*[40] obtained some simple yet generic criteria for determining the robust synchronization of uncertain chaotic delayed neural networks by combining the adaptive control and linear feedback with the updated law. The presented controller and updated law are based on the unknown parameters. While the derived controller and updated law here possess vigorous robustness, and they are based on the bounds of intervals of the unknown parameters and allow robust synchronization to extend to all the variables of a coupled neural network.

4. Simulation

In this section, we show the effectiveness of the robust adaptive synchronization method proposed in Section 3.

Example Consider the following two-order Hopfield neural network with time-varying delay:

$$\dot{x}(t) = -Cx(t) + Af(x(t)) + Bf(x(t-\tau(t))) + J, \quad (24)$$

with

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ A = \begin{bmatrix} 2.1 & -0.12 \\ -5.1 & 3.2 \end{bmatrix},$$

$$B = \begin{bmatrix} -1.6 & -0.1 \\ -0.2 & -2.4 \end{bmatrix},\ J = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

and

$\tau(t) = e^t/(1+e^t)$, where $x(t) = [x_1(t), x_2(t)]^{\mathrm{T}}$ and $f(x(t)) = [\tanh(x_1(t)), \tanh(x_2(t))]^{\mathrm{T}}$.

We can see $k_1 = k_2 = 1$. Moreover,

$$\tau^* = 1, \dot{\tau}(t) = \frac{e^t}{(1+e^t)^2} \in [0, 0.5],$$

i.e. $\sigma = 0.5$. Obviously, assumption 1 holds.

It should be noted that this neural network is actually a chaotic delayed Hopfield neural network. When the initial values are $\phi_1(s) = -1$, $\phi_2(s) = 1.5$, and $\forall s \in [-1, 0]$, Fig.1 shows the attractor of neural network model of system (24). By using the Jacobi method to calculate Lyapunov exponents of system (24), we have

$$\lambda_1 = 0.3037, \text{ and } \lambda_2 = -0.3608.$$

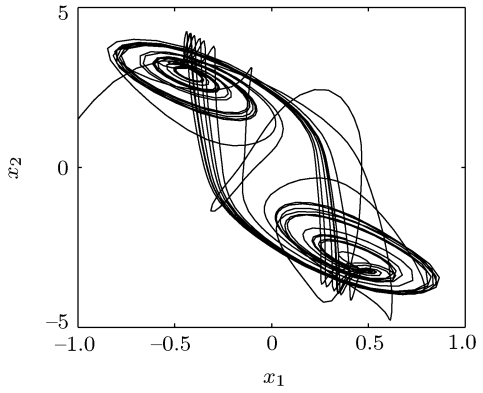


Fig.1. Attractor of neural network model of system (24).

In order to verify the effectiveness of the proposed method, we consider the following two coupled neural networks with different time-varying unknown parameters:

$$\begin{aligned}\dot{x}_i(t) =& - c_i(t)x_i(t) + \sum_{j=1}^{2} a_{ij}(t)f_j(x_j(t)) \\ &+ \sum_{j=1}^{2} b_{ij}(t)f_j(x_j(t-\tau(t))), \\ \dot{y}_i(t) =& - c_i^m(t)y_i(t) + \sum_{j=1}^{2} a_{ij}^m(t)f_j(y_j(t)) \\ &+ \sum_{j=1}^{2} b_{ij}^m(t)f_j(y_j(t-\tau(t))) + u_i, \end{aligned} \quad (25)$$

$(i = 1, 2)$ with time-varying unknown parameters taken as follows:

$$c_1(t) = c_2(t) = c_1^m(t) = c_2^m(t) = 1,$$

$$a_{11}(t) = 2.1(1+\cos t), a_{12}(t) = -0.12(1+\cos t),$$

$$a_{21}(t) = -5.1(1+\cos t), a_{22}(t) = 3.2(1+\cos t),$$

$$b_{11}(t) = -1.6(1+\cos t), b_{12}(t) = -0.1(1+\cos t),$$

$$b_{21}(t) = -0.2(1+\cos t), b_{22}(t) = -2.4(1+\cos t),$$

$$a_{11}^m(t) = 2.1(1+\sin t), a_{12}^m(t) = -0.12(1+\sin t),$$

$$a_{21}^m(t) = -5.1(1+\sin t), a_{22}^m(t) = 3.2(1+\sin t),$$

$$b_{11}^m(t) = -1.6(1+\sin t), b_{12}^m(t) = -0.1(1+\sin t),$$

$$b_{21}^m(t) = -0.2(1+\sin t), b_{22}^m(t) = -2.4(1+\sin t).$$

So from expression (5), we have
$\delta c_1(t) = \delta c_2(t) = 0,$

$$\delta a_{11}(t) = 2.1(\sin t - \cos t), \delta a_{12}(t) = -0.12(\sin t - \cos t),$$

$$\delta a_{21}(t) = -5.1(\sin t - \cos t), \delta a_{22}(t) = 3.2(\sin t - \cos t),$$

$$\delta b_{11}(t) = -1.6(\sin t - \cos t), \delta b_{12}(t) = -0.1(\sin t - \cos t),$$

$$\delta b_{21}(t) = -0.2(\sin t - \cos t), \delta b_{22}(t) = -2.4(\sin t - \cos t).$$

In order to achieve the goal of expression (10), let $p_i = q_i = r_{ij} = s_{ij} = 1$ $(i, j = 1, 2, \cdots, n)$, and we can construct a robust adaptive controller with a sliding mode according to Eqs. (22) and (23). In our simulation, the initial states of two neural networks and the initial values of unknown parameters are taken as

$$\phi_1(s) = 0.2,\ \phi_2(s) = 0.5,$$

$$\psi_1(s) = -0.3,\ \phi_2(s) = 2.0,\ s \in [-1, 0],$$

$$\hat{\alpha}_i(0) = \hat{\beta}_{ij}(0) = \hat{\gamma}_{ij}(0) = 0.8,$$

$$\hat{\underline{c}}_i(0) = \hat{\underline{a}}_{ij}(0) = \hat{\underline{b}}_{ij}(0) = 0.8.$$

Figures 2(a) and 2(b) show the synchronization errors of the two coupled neural networks, which imply that the synchronization is achieved successfully, where

$$\begin{aligned} x_i(t) &= \phi_i(t) \in \mathcal{C}([-\tau^*, 0], \boldsymbol{R}),\ (i = 1, 2), \\ y_i(t) &= \psi_i(t) \in \mathcal{C}([-\tau^*, 0], \boldsymbol{R}),\ (i = 1, 2). \end{aligned} \quad (26)$$

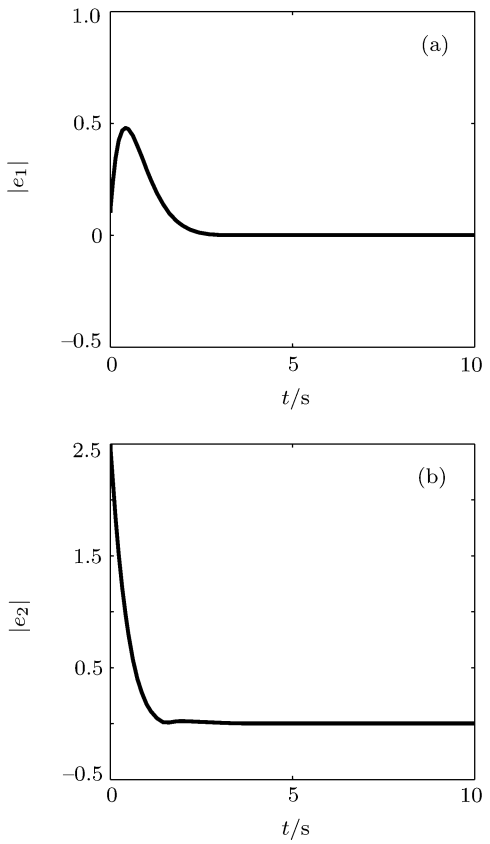


Fig.2. The synchronized errors (a) between $x_1(t)$ and $y_1(t)$ and (b) between $x_2(t)$ and $y_2(t)$ for the illustrative example.

5. Conclusion

In this paper, we obtain some criteria for the robust adaptive synchronization between two coupled chaotic neural networks with all the parameters unknown and time-varying delay based on the Lyapunov stability theory and slide technique. A sliding-mode-type controller is employed to increase the robustness of the two coupled neural networks. Moreover, without the estimate values of the network unknown parameters taken as an updating object, a new updating object is introduced in the constructing of controllers. It has been demonstrated that our criteria extend and improve those results reported in the literature. We also give a numerical example and computational simulations to validate our results.

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