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Controlling chaos based on an adaptive nonlinear compensator mechanism*

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The control problems of chaotic systems are investigated in the presence of parametric uncertainty and persistent external disturbances based on nonlinear control theory. By using a designed nonlinear compensator mechanism, the system deterministic nonlinearity, parametric uncertainty and disturbance effect can be compensated effectively. The renowned chaotic Lorenz system subjected to parametric variations and external disturbances is studied as an illustrative example. From the Lyapunov stability theory, sufficient conditions for choosing control parameters to guarantee chaos control are derived. Several experiments are carried out, including parameter change experiments, set-point change experiments and disturbance experiments. Simulation results indicate that the chaotic motion can be regulated not only to steady states but also to any desired periodic orbits with great immunity to parametric variations and external disturbances.

Keywords: chaotic system, nonlinear compensator mechanism, Lorenz chaotic system
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1. Introduction

Chaotic motion, as a peculiar behaviour in deterministic systems, may be undesirable in many cases, so suppressing such a phenomenon has been intensively studied in recent years. Generally speaking, chaos suppression and chaos synchronization[1−4] are two active research subjects in the field of chaos control and are both crucial in application of chaos. In the following we only deal with the problem of chaos suppression and will not discuss the chaos synchronization problem.

Since the early 1990s, the small time-dependent parameter perturbation has been introduced by Ott *et al*[5] to eliminate chaos. Varieties of effective control methods have been reported in the literature.[1−4,6−45] There are two lines in these methods. One is to introduce parameter perturbations into an accessible system parameter,[5,6,8−13] and the other is to introduce an additive external force into an original uncontrolled chaotic system.[14−36,38−42,45] Along the first line, when system parameters are not accessible or cannot be changed easily, or the environmental perturbations are not avoided, these methods are inapplicable. Recently, using an additive external force to suppress chaos is in the ascendant. As to the second line of the approaches, it places emphasis on chaos elimination as illustrated in a few examples as follows:

1.1. Linear state feedback control

In Ref.[14] a conventional feedback controller was designed to drive the chaotic Duffing equation to one of its inherent multiperiodic orbits. Recently a linear feedback control law based on the Lyapunov–Krasovskii (LK) method was developed for the suppression of chaotic oscillations.[15] A linear state feedback controller was designed to solve the chaos control problem of a class of new chaotic system in Ref.[16].

1.2. Structure variation control [12−16]

Since Yu[17] proposed the structure variation method to control chaos of the Lorenz system, some improved sliding-mode control strategies have been adopted in chaos control. In Ref.[18] the author used a newly developed sliding mode controller with a time-varying manifold dynamic to compensate the external

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excitation in chaotic systems. In Ref.[19] the design schemes of integration fuzzy sliding-mode control were addressed, in which the reaching law was proposed by a set of linguistic rules. A radial basis function sliding mode controller was employed for chaos control in Ref.[20].

1.3. Nonlinear geometric control

Nonlinear geometric control theory was applied to chaos control in Ref.[22], in which a Lorenz system model slightly different from the original Lorenz system model was studied by considering only the Prandtl number variation and process noise. In Ref.[23] the state space exact linearization method was used to stabilize the equilibrium of the Lorenz system with a controllable Rayleigh number.

1.4. Intelligence control [24−27]

An intelligent control method based on radical basis function (RBF) neural network was proposed for chaos control in Ref.[24]. Liu *et al* in Ref.[25] proposed to use least square support vector machines to drive the chaotic system to desirable points. A switching static output-feedback fuzzy-model-based controller was studied in Ref.[27], which was capable of handling chaos.

Other methods are also extensively studied, such as the entrainment and migration control, impulsive control method, optimal control method, stochastic control method, robust control method, adaptive control method, backstepping design method and so on. A detailed survey of recent publications on the control of chaos can be found in Refs.[28-34] and the references therein.

Among most of the existing control strategies, it is considered essential to know the model parameters for the derivation of a controller, and the control aims often to stabilize the embedded unstable period orbits of chaotic systems or to control the system to its equilibrium point. In the case of controlling the system to its equilibrium point, a general approach is to linearize the system at a given equilibrium point, then design a controller with local stability, which restricts the usage of the control scheme. According to the machine learning methods, such as neural network method[24] or support vector machine method,[25] the control performance often depends largely on the training samples, and sometimes better generalization capability cannot be guaranteed.

For chaos, as a particular phenomenon of deterministic nonlinear system, nonlinearity is essential. So if a nonlinear real-time compensator can eliminate the effect of the system nonlinearities, the chaotic motion is expected to be suppressed. Consequently the chaotic system can be controlled to a desired state. Under the guidance of nonlinear control theory, the objective of this paper is to design a control system to drive the chaotic systems not only to steady states but also to periodic trajectories. In the next section the controller architecture is introduced. In Section 3, a Lorenz system with considering parametric uncertainties and external disturbances is studied as an illustrative example. Two control schemes are designed for the chaotic system under study. By constructing appropriate Lyapunov functions, after rigorous analysis from Lyapunov stability theory sufficient conditions for choosing control parameters are deduced for each scheme. Then in Section 4 we present the numerical simulation results to illustrate the effectiveness of the design techniques. Finally some conclusions are provided in Section 5.

2. Controller architecture

The differential equation of the system under study is only an approximate description of the actual plant due to various uncertainties and disturbances. Without loss of generality we consider a nonlinear continuous dynamical system, in which there appear strange attractors under certain parameter conditions. With the relative degree $r \leq n$ (n is the dimension of the system), it can be directly described by being transformed into the following normal form:

$$\begin{aligned}
&\dot{z}_1 = z_2,\\
&\qquad \vdots\\
&\dot{z}_{r-1} = z_r,\\
&\dot{z}_r = a(z, v, \theta + \Delta\theta) + w\\
&\qquad\quad +(b(z, v, \theta + \Delta\theta) - 1)u + u\\
&\quad = d(z, v, \theta + \Delta\theta, u) + u,\\
&\dot{v} = c(z, v, \theta + \Delta\theta),\\
&y = z_1,
\end{aligned} \tag{1}$$

where θ is the parameter vector, $\Delta\theta$ denotes the parameter uncertainty, and w represents the external

disturbance, such that $|w| \leq M$ with M being positive.

In Eq.(1) $z = (z_1, \cdots, z_r)^{\mathrm{T}}$ can be called external state variable vector, and $v = (v_{r+1}, \cdots, v_n)^{\mathrm{T}}$ is called internal state variable vector. As we can see from Eq.(1),

$$\begin{aligned} &d(z, v, \theta + \Delta\theta, w, u) \\ &= a(z, v, \theta + \Delta\theta) + w + (b(z, v, \theta + \Delta\theta) - 1)u \end{aligned} \quad (2)$$

includes system nonlinearities, uncertainties, external disturbances and so on.

According to chaotic system (1), the following assumptions are made in order to obtain the results concerning the controller design (for more details see Ref.[37]):

Assumption 1 Relative degree r of the chaotic system is finite and known.

Assumption 2 Output variable y and its time derivatives y^i up to order $r-1$ are measurable.

Assumption 3 The zero dynamics of the system is asymptotically stable, i.e. $\dot{v} = c(0, v, \theta + \Delta\theta)$ is asymptotically stable.

Assumption 4 The sign of function $b(z, v, \theta + \Delta\theta)$is known, and it is always positive or negative.

Since maybe not all the state vectors are measurable, $a(z, v, \theta + \Delta\theta)$ and $b(z, v, \theta + \Delta\theta)$ are not known. Thus a controller with integral action is introduced to compensate the influence of $d(z, v, \theta + \Delta\theta, w, u)$, namely

$$u \triangleq -h_0 z_1 - h_1 z_2 - \cdots - h_{r-1} z_r - \hat{d}, \quad (3)$$

where

$$\begin{aligned} \hat{d} &= \xi + \sum_{i=0}^{r-1} k_i z_{i+1}, \\ \dot{\xi} &= -k_{r-1}\xi - k_{r-1} \sum_{i=0}^{r-1} k_i z_{i+1} \\ &\quad - \sum_{i=0}^{r-2} k_i z_{i+2} - k_{r-1} u, \end{aligned} \quad (4)$$

with $\hat{d}$ being the estimation of $d(z, v, \theta + \Delta\theta, w, u)$.

The controller parameters include h_i and k_i $(i = 0, \cdots, r-1)$. Here $H = [h_0, h_1, \cdots, h_{r-1}]^{\mathrm{T}}$ is the Hurwitz vector, such that all eigenvalues of the polynomial

$$P(s) = s^r + h_{r-1} s^{r-1} + \cdots + h_2 s^2 + h_1 s + h_0 \quad (5)$$

have negative real parts. The suitable positive constants h_i $(i = 0, \cdots, r-1)$ can be chosen according to the expected dynamic characteristic. In most cases they are determined according to different design requirements.

Define $k_{r-1} = \mathrm{sign}(b(z, v, \theta))\mu$, where μ denotes the suitable positive constant, and the other parameters k_i $(i = 0, \cdots, r-2)$ can be chosen arbitrarily. After $H = [h_0, h_1, \cdots, h_{r-1}]^{\mathrm{T}}$ has been determined, we can adjust k_i $(i = 0, \cdots, r-1)$ to achieve satisfactory static performances.

Remark In this section, we consider an n-dimensional nonlinear continuous dynamical system with strange attractors. By using an appropriate coordinate transformation, it can be represented in a normal form. Then a control system with a nonlinear compensator can be designed easily. In particular, the control parameters can be divided into two parts, which correspond to the dynamic characteristic and the static performance respectively (The theoretic analysis and more details about the controller can be found in Ref.[37]).

3. Illustrative example—the Lorenz system

The Lorenz system captures many of the features of chaotic dynamics, and many control methods have been tested on it.[17,20,22,23,27,30,32–35,41] However most of the existing methods are model-based and do not take the influence of persistent external disturbances into consideration.

The uncontrolled original Lorenz system can be described by

$$\begin{aligned} \dot{x}_1 &= -(P + \Delta P)x_1 + (P + \Delta P)x_2 + w_1, \\ \dot{x}_2 &= (R + \Delta R)x_1 - x_1 x_3 - x_2 + w_2, \\ \dot{x}_3 &= x_1 x_2 - (b + \Delta b)x_3 + w_3, \end{aligned} \quad (6)$$

where P and R are related to the Prandtl number and Rayleigh number respectively; b is the geometric factor; ΔP, ΔR and Δb are the variations of parameters P, R and b respectively; state variables, x_1 represents the convective fluid motion, x_2 denotes the horizontal temperature variation and x_3 refers to the vertical temperature variation; w_i $(i = 1, 2, 3)$ represent external disturbances. In the Lorenz system the desired variable of time response of state is x_1. It is desired that x_1 is regulated to x_{1r}, where x_{1r} is a given constant. In this section we consider two control schemes for system (6).

3.1. Control schemes for Lorenz chaotic system

3.1.1. Control scheme 1

Let the control act on the right-side of the first equation ($\dot{x}_1$), thus the controlled Lorenz system without any disturbance can be depicted as

$$\begin{aligned} \dot{x}_1 &= -Px_1 + Px_2 + u, \\ \dot{x}_2 &= Rx_1 - x_1x_3 - x_2, \\ \dot{x}_3 &= x_1x_2 - bx_3, \\ y &= x_1. \end{aligned} \tag{7}$$

By simple computation we know that system (7) has relative degree 1 (i.e. the lowest order of time-derivative of output y, which is directly related to the control u, is 1), and it can be rewritten as

$$\begin{aligned} \dot{z}_1 &= -Pz_1 + Pv_2 + u, \\ \dot{v}_2 &= Rz_1 - z_1v_3 - v_2, \\ \dot{v}_3 &= z_1v_2 - bv_3, \\ y &= z_1. \end{aligned} \tag{8}$$

According to Section 2, the following control strategy is introduced:

$$u = -h_0z_1 - \hat{d}, \tag{9}$$

$$\begin{aligned} \hat{d} &= \xi + k_0z_1, \\ \dot{\xi} &= -k_0\xi - k_0^2z_1 \text{ - } k_0u. \end{aligned} \tag{10}$$

Theorem 1 Under Assumptions 1–4 there exists a constant value $\mu^* > 0$, such that if $\mu > \mu^*$, then closed-loop system (7), (9) and (10) is asymptotically stable.

Proof Define $d = -Pz_1 + Pv_2$, then Eq.(8) can be easily rewritten as

$$\begin{aligned} \dot{z}_1 &= d + u, \\ \dot{v}_2 &= Rz_1 - z_1v_3 - v_2, \\ \dot{v}_3 &= z_1v_2 - bv_3. \end{aligned} \tag{11}$$

Substituting Eq.(9) into Eq.(11) yields

$$\begin{aligned} \dot{z}_1 &= -h_0z_1 + d - \hat{d}, \\ \dot{v}_2 &= Rz_1 - z_1v_3 - v_2, \\ \dot{v}_3 &= z_1v_2 - bv_3. \end{aligned} \tag{12}$$

Computing the time derivatives of d and $\hat{d}$ and considering Eq.(12) yield

$$\begin{aligned} \dot{d} &= -P\dot{z}_1 + P\dot{v}_2 \\ &= -P(-h_0z_1 + d - \hat{d}) \\ &\quad +P(Rz_1 - z_1v_3 - v_2), \end{aligned} \tag{13}$$

$$\begin{aligned} \dot{\hat{d}} &= \dot{\xi} + k_0\dot{z}_1 \\ &= -k_0\xi - k_0^2z_1 \text{ - } k_0u + k_0(d + u) \\ &= k_0(d - \xi - k_0z_1) \\ &= k_0(d - \hat{d}) \\ &= k_0\tilde{d}. \end{aligned} \tag{14}$$

Defining $\tilde{d} = d - \hat{d}$, we have

$$\begin{aligned} \dot{\tilde{d}} &= \dot{d} - \dot{\hat{d}} \\ &= (Ph_0 + PR)z_1 - Pz_1v_3 \\ &\quad -Pv_2 - (P + k_0)\tilde{d}. \end{aligned} \tag{15}$$

Then, we can obtain the following closed-loop system:

$$\begin{aligned} \dot{z}_1 &= -h_0z_1 + \tilde{d}, \\ \dot{v}_2 &= Rz_1 - z_1v_3 - v_2, \\ \dot{v}_3 &= z_1v_2 - bv_3, \\ \dot{\tilde{d}} &= (Ph_0 + PR)z_1 - Pz_1v_3 - Pv_2 \\ &\quad -(P + k_0)\tilde{d}. \end{aligned} \tag{16}$$

To stabilize closed-loop system (16), a Lyapunov function is defined as

$$V(\varsigma) = \frac{1}{2}\left\|\varsigma\right\|^2, \tag{17}$$

where ς denotes state vector $\left(z_1, v_2, v_3, \tilde{d}\right)^{\mathrm{T}}$, and $\|\cdot\|$ is the Euclidean norm, i.e.

$$V(\varsigma) = \frac{1}{2}(z_1^2 + v_2^2 + v_3^2 + \tilde{d}^2). \tag{18}$$

We define the following compact domain, which is composed of all the points internal to the super ball with radius $\sqrt{2M}$.

$$U \triangleq \left\{ \left(z_1, v_2, v_3, \tilde{d}\right) \middle| z_1^2 + v_2^2 + v_3^2 + \tilde{d}^2 \leq 2M \right\}. \tag{19}$$

By taking the time derivative of $V(\varsigma)$ and replacing the system expressions, we have

$$\begin{aligned} \dot{V}(\varsigma) &= z_1\dot{z}_1 + v_2\dot{v}_2 + v_3\dot{v}_3 + \tilde{d}\dot{\tilde{d}} \\ &= -h_0z_1^2 - v_2^2 - bv_3^2 - (k_0 + P)\tilde{d}^2 + Rz_1v_2 \\ &\quad +(1 + PR + Ph_0)z_1\tilde{d} - Pv_2\tilde{d} \\ &\quad -Pz_1v_3\tilde{d}. \end{aligned} \tag{20}$$

For any $\left(z_1, v_2, v_3, \tilde{d}\right) \in U$, we have

$$\begin{aligned} \dot{V}(\varsigma) \leq\ & -h_0\left|z_1\right|^2 - \left|v_2\right|^2 - b\left|v_3\right|^2 - (k_0 + P)\left|\tilde{d}\right|^2 \\ & +R\left|z_1\right|\left|v_2\right| + (1 + PR + Ph_0)\left|z_1\right|\left|\tilde{d}\right| \\ & +P\left|v_2\right|\left|\tilde{d}\right| + P\sqrt{2M}\left|v_3\right|\left|\tilde{d}\right|, \end{aligned} \tag{21}$$

namely

$$\dot{V}(\varsigma) \le -\left[|z_1|\,|v_2|\,|v_3|\,\left|\tilde{d}\right|\right] \times \begin{bmatrix} h_0 & -\frac{R}{2} & 0 & -\frac{(1+PR+Ph_0)}{2} \\ -\frac{R}{2} & 1 & 0 & -\frac{P}{2} \\ 0 & 0 & b & -\frac{P\sqrt{2M}}{2} \\ -\frac{(1+PR+Ph_0)}{2} & -\frac{P}{2} & -\frac{P\sqrt{2M}}{2} & (k_0+P) \end{bmatrix} \times \left[|z_1|\,|v_2|\,|v_3|\,\left|\tilde{d}\right|\right]^{\mathrm{T}}. \tag{22}$$

So if the above symmetrical parameter matrix of Eq.(22) is positive definite, then $\dot{V}$ is negative and definite, which implies that system (16) is asymptotically stable based on the Lyapunov stability theory.

By defining the principal minor determinant of symmetrical matrix in Eq.(22) as D_i $(i = 1, 2, 3, 4)$, from the well-known Sylvester theorem it is straightforward to obtain the following inequations:

$$D_1 = h_0 > 0, \tag{23}$$

$$D_2 = h_0 - \frac{R^2}{4} > 0, \tag{24}$$

$$D_3 = bh_0 - \frac{R^2 b}{4} > 0, \tag{25}$$

$$D_4 = (k_0+P)D_3 - \frac{1}{8}b(1+PR+Ph_0)(PR+2) - \frac{P^2M}{2}D_2 - \frac{1}{8}Pb[2Ph_0 + R(1+PR+Ph_0)] > 0. \tag{26}$$

After h_0 has been determined by solving inequalities (23)–(25), undoubtedly, inequality (26) can serve effectively as the constraints for choosing k_0, i.e.

$$k_0 > \frac{\frac{1}{8}b(1+PR+Ph_0)(PR+2) + \frac{P^2M}{2}D_2 + \frac{1}{8}Pb[2Ph_0 + R(1+PR+Ph_0)]}{D_3} - P. \tag{27}$$

Here

$$\mu^* = \frac{\frac{1}{8}b(1+PR+Ph_0)(PR+2) + \frac{P^2M}{2}D_2 + \frac{1}{8}Pb[2Ph_0 + R(1+PR+Ph_0)]}{D_3} - P.$$

Thus the proof of theorem 1 is completed.

3.1.2. Control scheme 2

Adding the control signal to the second equation $(\dot{x}_2)$, the system under control can be derived as

$$\begin{aligned} \dot{x}_1 &= -Px_1 + Px_2, \\ \dot{x}_2 &= Rx_1 - x_1x_3 - x_2 + u, \\ \dot{x}_3 &= x_1x_2 - bx_3. \end{aligned} \tag{28}$$

From Eq.(28), for a target constant $x_1(t) = x_{1r}$, $\dot{x}_1(t) = 0$. By solving the above differential equation, we obtain $x_{2r} = x_{1r}$. Moreover, when $t \to \infty$, x_{3r} converges to x_{1r}^2/b. Since x_1 and x_2 have the same equilibrium points, then the measured state can also be chosen as x_2.

To determine u, using the following coordinate transform:

$$\begin{aligned} z_1 &= x_2, \\ v_2 &= x_1, \\ v_3 &= x_3, \end{aligned}$$

to convert Eq.(28) into the following normal

$$\begin{aligned} \dot{z}_1 &= Rv_2 - v_2v_3 - z_1 + u, \\ \dot{v}_2 &= Pz_1 - Pv_2, \\ \dot{v}_3 &= z_1v_2 - bv_3, \\ y &= z_1, \end{aligned} \tag{29}$$

thus the controller can be derived, which has the same

expression as that in scheme 1.

Theorem 2 Under Assumptions 1, 2, 3 and 4, there exists a constant value $\mu^* > 0$, such that if $\mu > \mu^*$, then closed-loop system (9), (10) and (28) is asymptotically stable.

Proof In order to acquire compact analysis, Eq.(29) can be rewritten as

$$\begin{aligned} \dot{z}_1 &= d + u, \\ \dot{v}_2 &= Pz_1 - Pv_2, \\ \dot{v}_3 &= z_1 v_2 - bv_3, \end{aligned} \tag{30}$$

where $d = Rv_2 - v_2 v_3 - z_1$.

Substituting Eq.(9) into Eq.(30),we obtain

$$\begin{aligned} \dot{z}_1 &= -h_0 z_1 + d - \hat{d}, \\ \dot{v}_2 &= Pz_1 - Pv_2, \\ \dot{v}_3 &= z_1 v_2 - bv_3. \end{aligned} \tag{31}$$

Giving the following definition:

$$\tilde{d} = d - \hat{d} \tag{32}$$

then we can have

$$\begin{aligned} \dot{d} &= R\dot{v}_2 - \dot{v}_2 v_3 - v_2 \dot{v}_3 - \dot{z}_1 \\ &= R(Pz_1 - Pv_2) - (Pz_1 - Pv_2)v_3 \\ &\quad -v_2(z_1 v_2 - bv_3) - (-h_0 z_1 + \tilde{d}), \end{aligned} \tag{33}$$

$$\begin{aligned} \dot{\hat{d}} &= \dot{\xi} + k_0 \dot{z}_1 \\ &= -k_0 \xi - k_0^2 z_1 \text{ - } k_0 u + k_0(d + u) \\ &= k_0 \tilde{d}, \end{aligned} \tag{34}$$

$$\begin{aligned} \dot{\tilde{d}} &= \dot{d} - \dot{\hat{d}} \\ &= R(Pz_1 - Pv_2) - (Pz_1 - Pv_2)v_3 \\ &\quad -v_2(z_1 v_2 - bv_3) + h_0 z_1 - (k_0 + 1)\tilde{d}. \end{aligned} \tag{35}$$

Thus the closed-loop system can be represented as the following compact form:

$$\begin{aligned} \dot{z}_1 &= -h_0 z_1 + \tilde{d}, \\ \dot{v}_2 &= Pz_1 - Pv_2, \\ \dot{v}_3 &= z_1 v_2 - bv_3, \\ \dot{\tilde{d}} &= R(Pz_1 - Pv_2) - (Pz_1 - Pv_2)v_3 \\ &\quad -v_2(z_1 v_2 - bv_3) + h_0 z_1 - (k_0 + 1)\tilde{d}. \end{aligned} \tag{36}$$

The quadratic Lyapunov function is chosen as follows:

$$V(\varsigma) = \frac{1}{2} \|\varsigma\|^2, \tag{37}$$

where ς denotes state vector $\left(z_1, v_2, v_3, \tilde{d}\right)^{\mathrm{T}}$, and $\|\cdot\|$ is the Euclidean norm, i.e.

$$V(\varsigma) = \frac{1}{2}(z_1^2 + v_2^2 + v_3^2 + \tilde{d}^2). \tag{38}$$

We can also define the following compact domain, which is composed of all the points internal to the super ball with radius $\sqrt{2M}$.

$$U = \left\{ \left(z_1, v_2, v_3, \tilde{d}\right) | z_1^2 + v_2^2 + v_3^2 + \tilde{d}^2 \le 2M \right\}. \tag{39}$$

Differentiating V with respect to t and using Eq.(36) yield

$$\begin{aligned} \dot{V}(\varsigma) = z_1 \dot{z}_1 + v_2 \dot{v}_2 + v_3 \dot{v}_3 + \tilde{d}\dot{\tilde{d}} &= -h_0 z_1^2 - Pv_2^2 - bv_3^2 - (k_0 + 1)\tilde{d}^2 + (1 + PR + h_0) z_1 \tilde{d} + Pz_1 v_2 \\ &\quad + z_1 v_2 v_3 + (P + b) v_2 v_3 \tilde{d} - Pv_2 \tilde{d} - Pz_1 v_3 \tilde{d} - z_1 v_2^2 \tilde{d}. \end{aligned} \tag{40}$$

Similarly, for any $\left(z_1, v_2, v_3, \tilde{d}\right) \in U$, we have

$$\begin{aligned} \dot{V}(\varsigma) \le &-h_0 |z_1|^2 - P |v_2|^2 - b |v_3|^2 - (k_0 + 1) \left|\tilde{d}\right|^2 + (1 + PR + h_0) |z_1| \left|\tilde{d}\right| + P |z_1| |v_2| \\ &+ \sqrt{2M} |z_1| |v_3| + (P + b)\sqrt{2M} |v_3| \left|\tilde{d}\right| + P |v_2| \left|\tilde{d}\right| + P\sqrt{2M} |v_3| \left|\tilde{d}\right| + 2M |z_1| \left|\tilde{d}\right|, \end{aligned} \tag{41}$$

i.e.

$$\dot{V}(\varsigma) \le -\left[|z_1| |v_2| |v_3| \left|\tilde{d}\right|\right] \times \begin{bmatrix} h_0 & -\frac{P}{2} & -\frac{\sqrt{2M}}{2} & -\frac{(1 + PR + 2M + h_0)}{2} \\ -\frac{P}{2} & P & 0 & -\frac{P}{2} \\ -\frac{\sqrt{2M}}{2} & 0 & b & -\frac{\sqrt{2M}(2P + b)}{2} \\ -\frac{(1 + PR + 2M + h_0)}{2} & -\frac{P}{2} & -\frac{\sqrt{2M}(2P + b)}{2} & (k_0 + 1) \end{bmatrix}$$

$$\times \left[|z_1|\,|v_2|\,|v_3|\,\left|\tilde{d}\right|\right]^{\mathrm{T}}. \tag{42}$$

For brevity, let

$$\alpha_1 = \frac{1}{8}P(1+PR+2M+h_0)[b(2PR+P+2+2h_0) +2M(2P+3b)], \tag{43}$$

$$\alpha_2 = \frac{1}{8}P^2[M(2P+3b-1)+b(1+PR+3h_0)], \tag{44}$$

$$\alpha_3 = \frac{1}{8}PM(2P+b)[P^2+2P(1+PR+2M+h_0) +(2P+b)(4Ph_0-P^2)]. \tag{45}$$

According to the Sylvester theorem the following inequations are obtained:

$$D_1 = h_0 > 0, \tag{46}$$

$$D_2 = h_0P - \frac{P^2}{4} > 0, \tag{47}$$

$$D_3 = bD_2 - \frac{PM}{2} > 0, \tag{48}$$

$$D_4 = (k_0+1)D_3 - \alpha_1 - \alpha_2 - \alpha_3 > 0, \tag{49}$$

where $D_i \quad (i = 1, 2, 3, 4)$ are the principal minor determinants of the symmetrical matrix of Eq.(42).

$$k_0 > \mu^*,$$
$$\mu^* = \frac{\alpha_1+\alpha_2+\alpha_3}{D_3} - 1. \tag{50}$$

Thus theorem 2 is proved.

Remark In this section we give two control schemes for controlling chaos in a Lorenz system. For each scheme the control depends on the observed variable only, and two control parameters need adjusting viz h_0 and k_0. According to Lyapunov stability theory, after h_0 has been fixed, the sufficient condition for choosing parameter k_0 is also obtained.

4. Simulation results

Choosing $P = 10$, $R = 28$, and $b = 8/3$, the uncontrolled Lorenz system exhibits chaotic behaviours, as plotted in Fig.1. In simulation the initial values of the state of the system have been chosen to be $x_1(0) = 10$, $x_2(0) = 10$, and $x_3(0) = 10$.

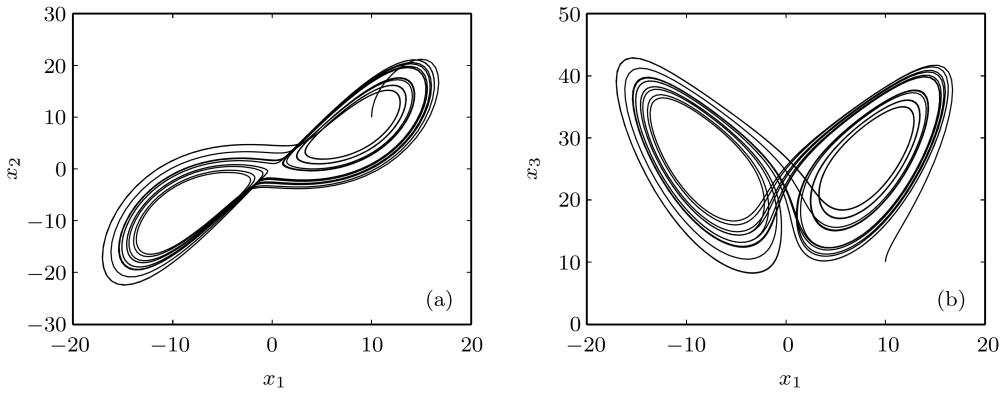


Fig.1. Chaotic trajectories of Lorenz system, projected on $x_1 - x_2$ plane (a) and on $x_1 - x_3$ plane (b).

4.1. Simulation results of controlling the trajectory to a steady state

In this section only the simulation results of control scheme 2 are depicted. The simulation results of control scheme 1 will be given in Appendix. In the first five seconds the control input is not active; at $t = 5\,\mathrm{s}$, a control signal is input and the system trajectory turns into set point $(x_{1r}, x_{2r}, x_{1r}^2/b)^{\mathrm{T}} = (8.5, 8.5, 27.1)^{\mathrm{T}}$, as can be seen in Fig.2(a). The time history of the Lyapunov function is illustrated in Fig.2(b).

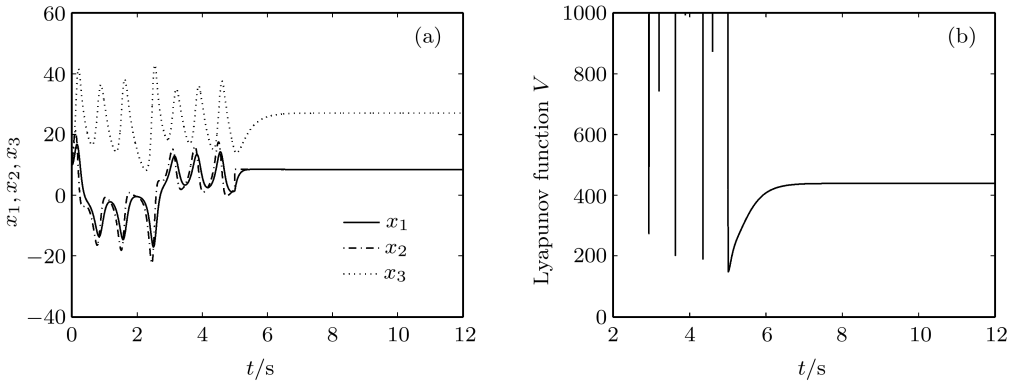


Fig.2. Time responses of state under control (a) and time history of the Lyapunov function (b).

4.1.1. Simulation results with parameters changing

At $t = 9\,\mathrm{s}$, system parameters are abruptly changed to $P = 15$, $R = 35$, and $b = 12/3$. Accordingly the new equilibrium is established at $(x_{1r}, x_{2r}, x_{1r}^2/b)^{\mathrm{T}} = (8.5, 8.5, 18.1)^{\mathrm{T}}$. Obviously, after very short transient duration, system state converges to a new point as shown in Fig.3(a). Figure 4(a) represents the evolution of the Lyapunov function with time.

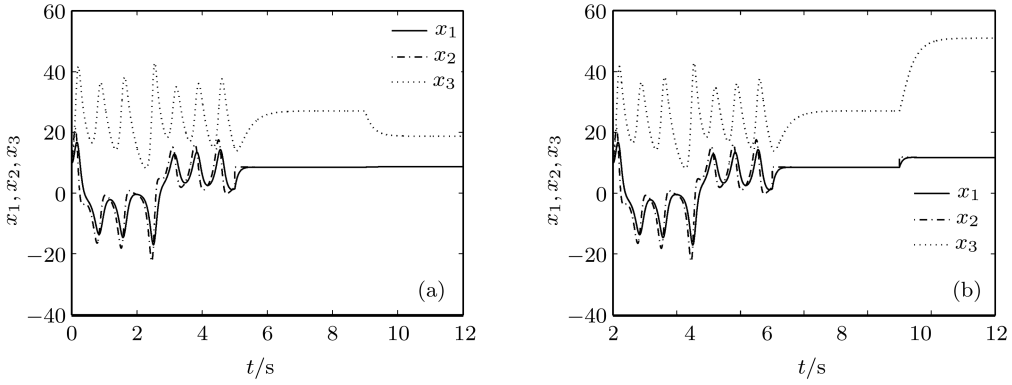


Fig.3. Time responses of state with parameters varying (a) and set points changing (b).

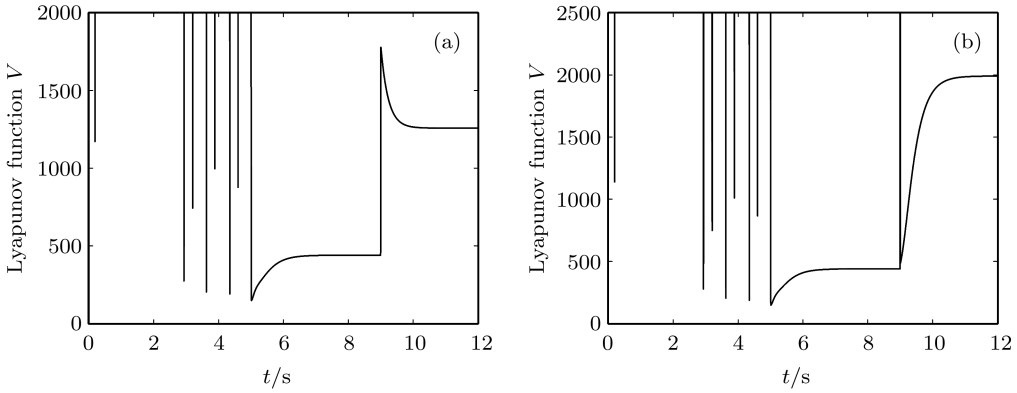


Fig.4. Time history of the Lyapunov function with parameters varying (a) and set points changing (b).

4.1.2. Simulation results with set point changing

At $t = 9\,\mathrm{s}$, the target is abruptly changed to $(x_{1r}, x_{2r}, x_{1r}^2/b)^{\mathrm{T}} = (12, 12, 54)^{\mathrm{T}}$. The time responses of the system state are shown in Fig.3(b). Indicated in Fig.4(b) is the time history of the Lyapunov function.

4.1.3. Simulation results with disturbance present

In Eq.(5) external periodic disturbances $w_i = 3\cos(5\pi t)$ $(i = 1, 2, 3)$ are considered. The time responses of the system states are given in Fig.5. After being controlled, the steady-state phase plane trajectory describes a limit cycle as shown in Fig.6.

4.1.4. Simulation results with random noise present

Under the influence of random noise,

$$\begin{aligned} \dot{x}_1 &= -Px_1 + Px_2 + \varepsilon\delta_1, \\ \dot{x}_2 &= Rx_1 - x_1x_3 - x_2 + \varepsilon\delta_2 + u, \\ \dot{x}_3 &= x_1x_2 - bx_3 + \varepsilon\delta_3, \end{aligned} \tag{51}$$

where δ_i $(i = 1, 2, 3)$ are normally distributed with mean value 0 and variance 0.5, and $\varepsilon = 5$. The results of the numerical simulation are depicted in Fig.7, showing that the steady responses are hardly affected by the perturbations.

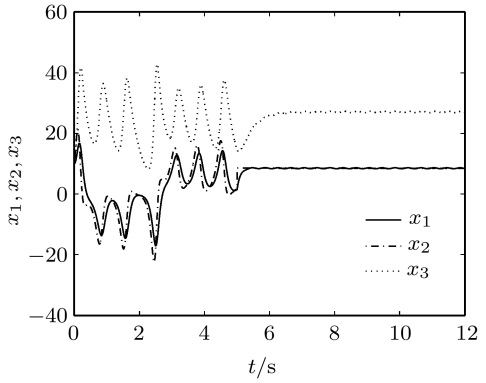


Fig.5. Time responses of state in the presence of periodic disturbance.

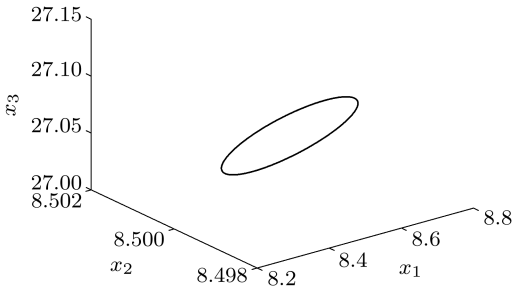


Fig.6. The state space trajectory at $t \in [10, 12]$ in the presence of periodic disturbance.

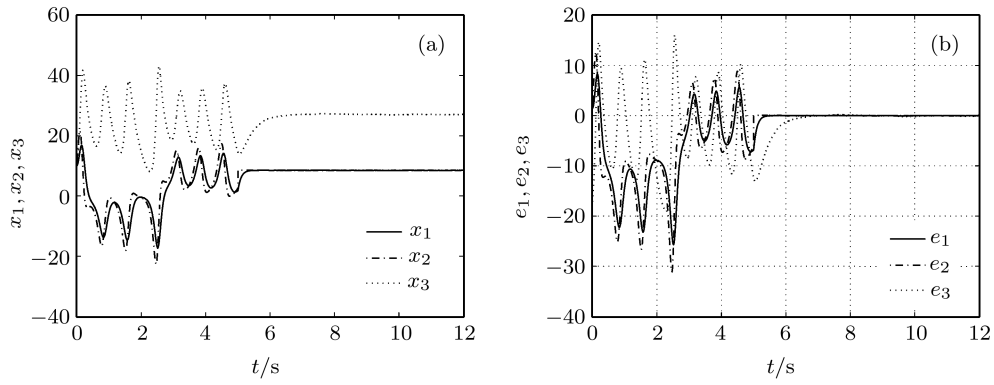


Fig.7. Time responses in the presence of random noise for state responses (a) and state tracking error responses (b).

4.2. Simulation results of controlling the trajectory to periodic orbit

If the reference signal is periodic, then the system output will also be periodic due to tracking this signal. Figures 8(a) to 8(d) show time responses of $x_1(t)$ and the 3-period and 4-period tracking trajectories respectively.

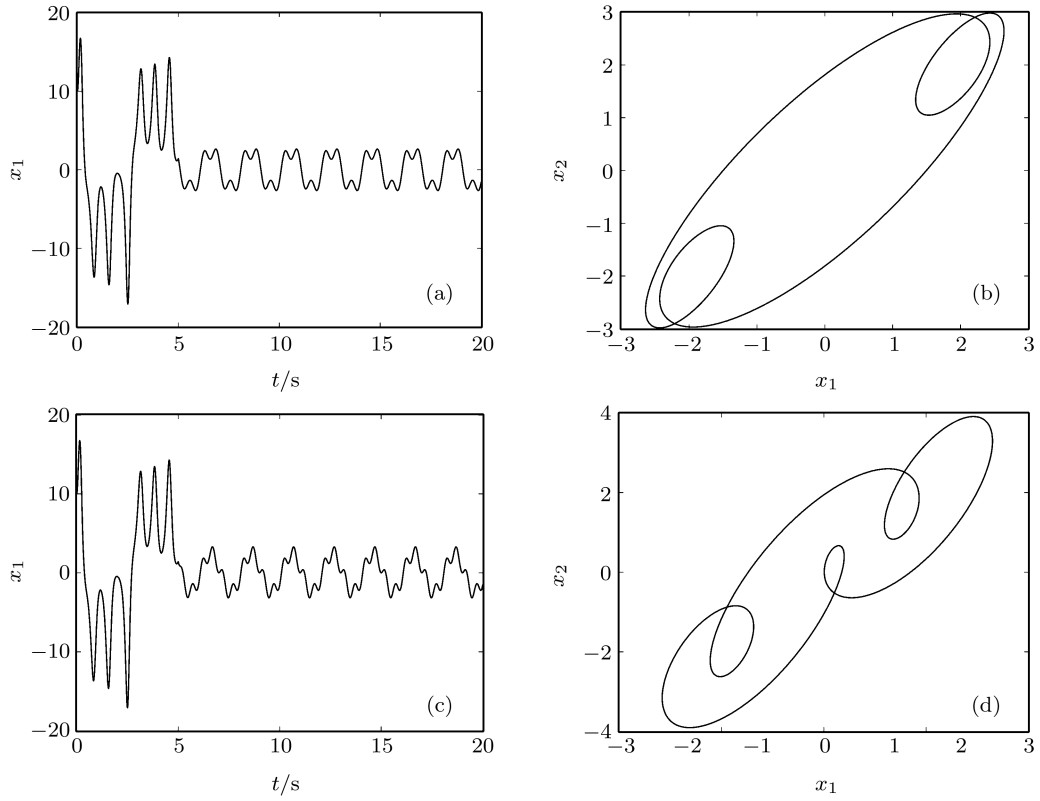


Fig.8. Time responses of state and 3-period ((a) and (b)) and 4-period ((c) and (d)) tracking periodic orbits.

Remark The two controllers designed above can solve the chaos control problems of Lorenz chaotic system, and the controller design method can also be extended to solving the chaos suppression problems of the whole Lorenz system family, namely the unified chaotic system.[43,44,46] The detailed design process and close-loop system analysis can be found in Ref.[47]. In Ref.[47] according to different positions that the scalar control input was added to, three controllers were designed to reject the chaotic behaviours from the unified chaotic system. Taking the first state x_1 as the system output, by transforming the system equation into the normal form, the relative degree $r(r \leq 3)$ of the controlled systems is obtained. Then we can design the controller with the expressions of Eqs.(3) and (4). Three effective adaptive nonlinear compensator mechanisms are derived to compensate the nonlinearities and external disturbances of the chaotic system. According to the Lyapunov stability theory sufficient conditions for choosing control parameters are deduced so that designers can adjust the design parameters in an explicit way to obtain the required closed loop behaviour. Numerical simulation results show that the designed three controllers can successfully regulate the chaotic motion of the whole family of the system to a given point or make the output state track a given bounded signal with great robustness.

5. Conclusions

In this paper we introduce a promising tool to design a control system for a chaotic system subjected to persistent disturbances, of which the entire dynamics is assumed to be unknown and the state variables are not completely measurable. By integral action the nonlinearities, including system structure nonlinearity and the various disturbances are compensated successfully. This control system can handle, therefore, a large class of chaotic systems, which satisfy the four assumptions made above. Taking a chaotic Lorenz system for example, it has been shown that the designed control scheme is robust in the sense that the unmeasured states, parameter uncertainties and external disturbance effects are all compensated and chaos suppression is achieved. Some advantages of this control strategy can be summarized as follows:

1. It is not restricted to stabilizing the embedded periodic orbits, but it can stabilize any desired set points and multiperiodic orbits even when the desired trajectories are not located on the embedded orbits of the chaotic system.

2. The parameter uncertainty and the external disturbance are allowed to exist. The controller can be designed according to the nominal system.

3. The dynamic characteristics of the controlled systems are approximately linear and the transient responses can be regulated by the designer through controller parameters h_i $(i = 0, \cdots, r-1)$.

4. From the Lyapunov stability theory sufficient conditions for choosing control parameters can be derived easily.

5. The error-converging speed is very fast even when the initial state is far from the target one without waiting for the actual state to reach the neighbourhood of the target state.

Appendix

Simulation results of control scheme 1.

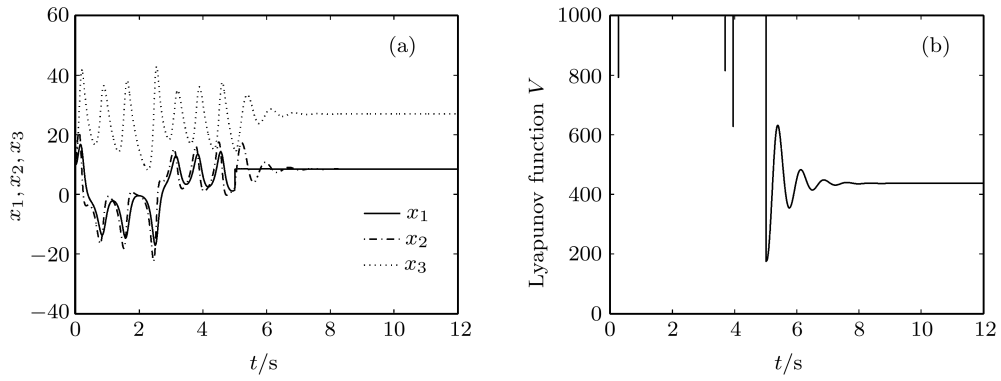


Fig.A1. Time responses of state under control (a) and time history of the Lyapunov function (b).

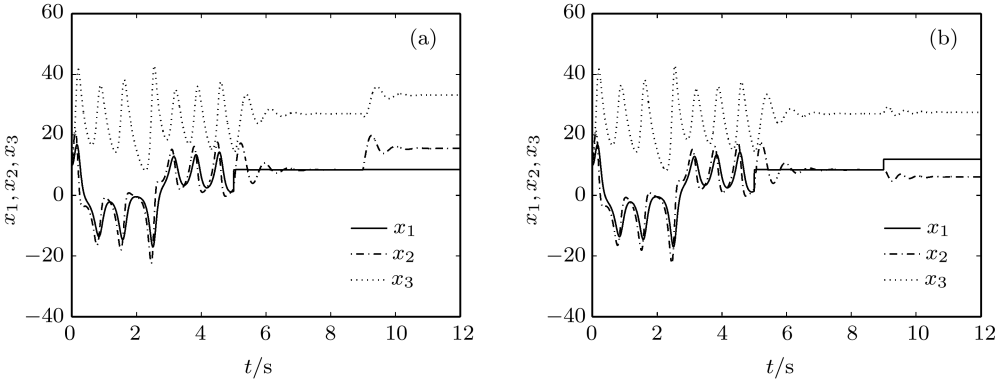


Fig.A2. Time responses of state with the parameters varying (a) and set points changing (b).

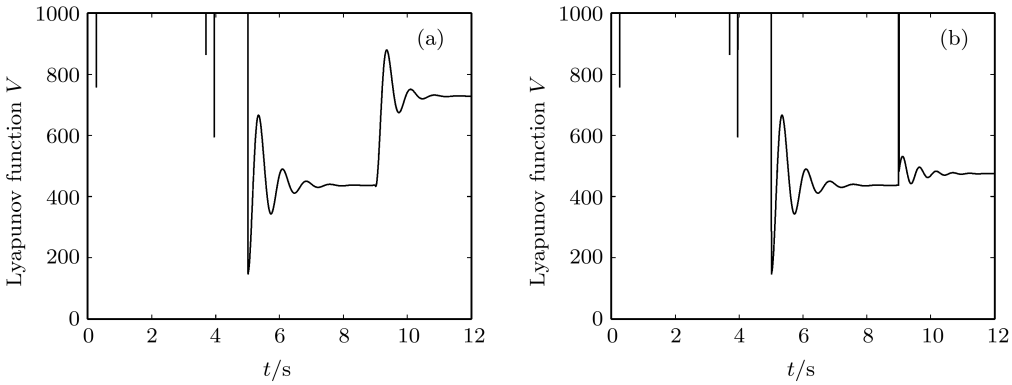


Fig.A3. Time history of the Lyapunov function with parameters varying (a) and set points changing (b).

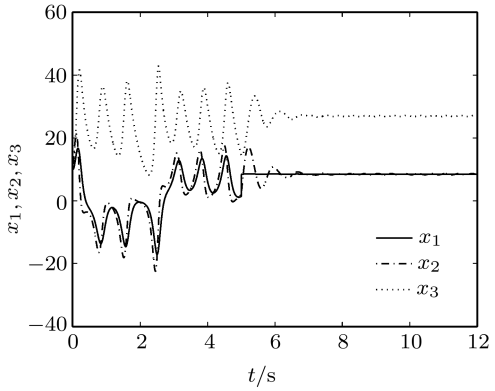


Fig.A4. Time responses of state in the presence of periodic disturbance.

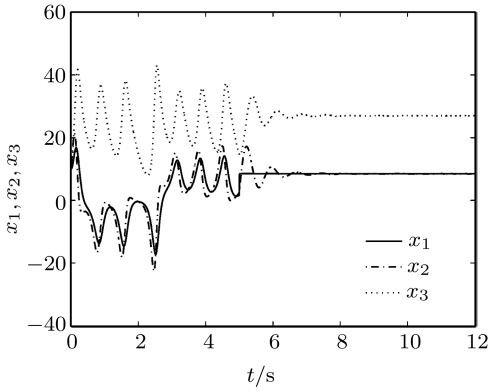


Fig.A5. Time responses of state in the presence of random noise.

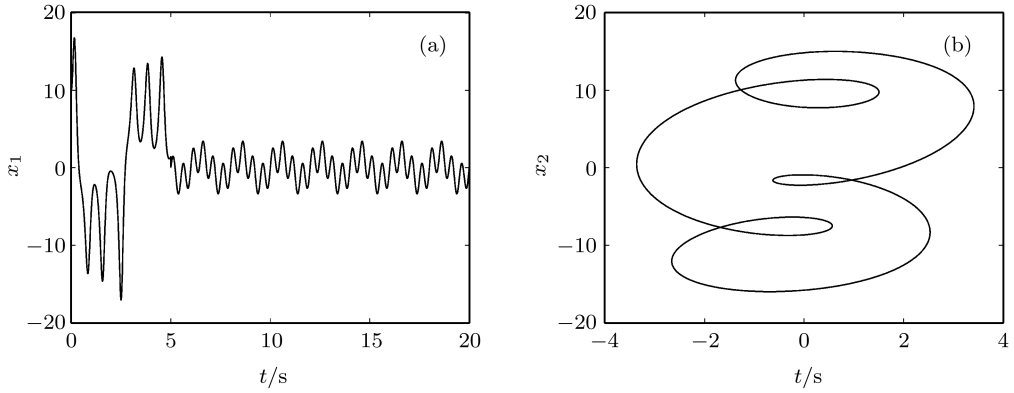


Fig.A6. Time response of state and the 4-period tracking periodic orbit.

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