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Adaptive synchronization of uncertain Liu system via nonlinear input*

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This paper addresses the adaptive synchronization for uncertain Liu system via a nonlinear input. By using a single nonlinear controller, the approach is utilized to implement the synchronization of Liu system with total parameters unknown. This method is simple and can be easily designed. What is more, it improves the existing conclusions in Ref [12]. Simulation results prove that the controller is effective and feasible in the end.

Keywords: adaptive synchronization, Liu system, uncertain parameters, nonlinear controller
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1. Introduction

The topic of the chaos synchronization has received increasing attention since it was studied by Pecora and Carroll[1] in 1990. Chaos synchronization can be applied to many fields such as secure communication, chemical reactions, biological systems and information processing.

Nowadays, a wide variety of approaches have been proposed for the synchronization of chaotic systems, which include linear and nonlinear feedback control, time-delay feedback control, adaptive control, observer design, among many others.[2−8] In general, in the above publications regarding chaotic systems control, most of them are under the assumption of linear input. However, in many practical physical systems, due to physical limitations, there exist nonlinearities in the control input. Their presence may lead to serious degradation of system performance and obvious decrease in speed of response and possibly cause chaotic perturbation to original regular behaviour if the controller is not well designed. Their effect cannot be ignored in analysis or control design and realization for chaotic systems. Therefore, the control design for uncertain systems subject to sector nonlinear inputs is considered in this paper.

This paper presents a nonlinear feedback adaptive synchronization for the Liu system. In the existing literatures, many methods about adaptive synchronization that based on the drive system and response system are all definite. Adaptive synchronization of an uncertain chaotic system is also studied mostly through a linear feedback controller.[9−13] In this paper, adaptive nonlinear feedback synchronization with only one controller is designed for the completely uncertain Liu system. This improves the work of Tu *et al*,[14] in which three controllers are needed for the synchronization of the completely uncertain Liu system. In the end, numerical simulations show the effectiveness and feasibility of the developed method.

2. Problem formulation and preliminaries

In 2004, Liu *et al* found a new chaotic dynamical system.[15] It is a three-dimensional autonomous system which relies on one multiplier and one quadratic term to introduce the non-linearity necessary for folding trajectories. The Liu system is described by the following differential equations:

$$\begin{aligned} \dot{\tilde{x}} &= a(\tilde{y} - \tilde{x}), \\ \dot{\tilde{y}} &= b\tilde{x} - k\tilde{x}\tilde{z}, \\ \dot{\tilde{z}} &= -c\tilde{z} + h\tilde{x}\tilde{z}, \end{aligned} \tag{1}$$

(a, b, c, k, h are nonzero positive constants),

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for any $a, b, c \in R$ and $k, h \in R\backslash\{0\}$. When $a = 10$, $b = 40$, $k = 1$, $c = 2.5$, $h = 4$, it has a chaotic attractor. System (1) has three unstable equilibria $O = (0, 0, 0)$, $O_+ = (5, 5, 40)$ and $O_- = (-5, -5, 40)$. Worth mentioning, it has been shown that the new system (1) is state-equivalent to the generalized Lorenz canonical form with $\tau = -1$.

System (1) is equivalent to

$$\begin{aligned} \dot{x} &= a(y - x), \\ \dot{y} &= bx - xz, \\ \dot{z} &= -cz + x^2, \end{aligned} \tag{2}$$

under the following linear transform of coordinates:

$$\begin{aligned} x &= \tilde{x}\sqrt{hk}, \\ y &= \tilde{y}\sqrt{hk}, \\ z &= k\tilde{z}, \end{aligned} \tag{3}$$

as $a = 10$, $b = 40$, $c = 2.5$, a chaotic attractor can be generated.

Let Eq.(2) be the drive system, the response system is as follows:

$$\begin{aligned} \dot{\hat{x}} &= \hat{a}(\hat{y} - \hat{x}), \\ \dot{\hat{y}} &= \hat{b}\hat{x} - \hat{x}\hat{z} - k\phi(u(t)) + \hat{x}(\hat{z} - z), \\ \dot{\hat{z}} &= -\hat{c}\hat{z} + \hat{x}^2, \end{aligned} \tag{4}$$

where $\hat{a}, \hat{b}, \hat{c}$ are estimated parameters to a, b, c and k is a constant to be designed later. $\phi(u(t))$ is a continuous nonlinear function, $\phi{:}R \to R$ with the law $u(t) \to \phi(u(t))$ and inside sector $[k_1, k_2]$, i.e.

$$k_1 u^2(t) \le \phi(u(t))u(t) \le k_2 u^2(t), \tag{5}$$

where k_1 and k_2 are nonzero positive constants.

To fulfil the synchronization of systems (1) and (3), we define the state error dynamic equation

$$\begin{aligned} e_1(t) &= \hat{x}(t) - x(t), \\ e_2(t) &= \hat{y}(t) - y(t), \\ e_3(t) &= \hat{z}(t) - z(t). \end{aligned} \tag{6}$$

From Eqs.(2) and (4), we have

$$\begin{aligned} \dot{e}_1(t) &= \hat{a}(\hat{y} - \hat{x}) - a(y - x), \\ \dot{e}_2(t) &= \hat{b}\hat{x} - \hat{x}\hat{z} - bx + xz - k\phi(u(t)) + \hat{x}(\hat{z} - z), \\ \dot{e}_3(t) &= -\hat{c}\hat{z} + cz + \hat{x}^2 - x^2. \end{aligned} \tag{7}$$

The goal of this article is to determine $u(t)$ such that the error dynamic system (7) is asymptotically stable at origin, and hence the drive system (2) and response system (4) are globally asymptotically synchronized.

3. Synchronization via a single nonlinear feedback input

First, we let $u(t) = e_2(t)$, and then we can rewrite Eq.(7) in the following form:

$$\begin{aligned} \dot{e}_1(t) &= \hat{a}(\hat{y} - \hat{x}) - a(y - x), \\ \dot{e}_2(t) &= \hat{b}\hat{x} - \hat{x}\hat{z} - bx + xz - k\phi(e_2) + \hat{x}(\hat{z} - z), \\ \dot{e}_3(t) &= -\hat{c}\hat{z} + cz + \hat{x}^2 - x^2, \end{aligned} \tag{8}$$

where k is the constant to be determined, and $\hat{a}$, $\hat{b}$, $\hat{c}$ are also needed to be determined. Let

$$\begin{aligned} e_a(t) &= \hat{a} - a, \\ e_b(t) &= \hat{b} - b, \\ e_c(t) &= \hat{c} - c. \end{aligned} \tag{9}$$

It is assumed that

$$\begin{aligned} \dot{e}_a(t) &= -(\hat{y} - \hat{x})\lambda e_1, \\ \dot{e}_b(t) &= -\hat{x}e_2, \\ \dot{e}_c(t) &= \hat{z}e_3, \end{aligned} \tag{10}$$

where λ is a proper positive constant. As the state trajectories of chaotic system are all bounded, in the following text, we let $M = \max\{|x|, |y|, |z|\}$. For the sake of convenience, we also suppose $|x| + |\hat{x}| < M_x$.

Theorem 1 For systems (2) and (4), we choose λ, θ as proper positive numbers. If $k > \dfrac{\xi}{k_1}$ under the conditions that

$$4\lambda ac - \theta M_x^2 > 0, \tag{11}$$

$$\xi = \max\left\{\frac{[\lambda a + \theta(b + M)]^2}{4\lambda a\theta}, \frac{c[\lambda a + \theta(b + M)]^2}{4\lambda\theta ac - \theta^2 M_x^2}\right\}, \tag{12}$$

then the two systems (2) and (4) are globally asymptotically synchronized.

Proof Choose the Lyapunov function as

$$V(e) = \frac{1}{2}(\lambda e_1^2 + \theta e_2^2 + \beta e_3^2 + \rho e_a^2 + \gamma e_b^2 + \mu e_c^2), \tag{13}$$

where $\lambda, \theta, \beta, \rho, \gamma, \mu$ are positive numbers, then the differentiation of V along trajectories (8) and (10) is

$$\begin{aligned}\dot{V}(e) =&\lambda e_1\dot{e}_1 + \theta e_2\dot{e}_2 + \beta e_3\dot{e}_3 + \rho\dot{e}_a e_a + \gamma\dot{e}_b e_b + \mu\dot{e}_c e_c\\ =&\lambda e_1\hat{a}(\hat{y}-\hat{x}) - \lambda e_1 a(y-x) + \theta e_2(\hat{b}\hat{x} - bx) + (xz - \hat{x}\hat{z})\theta e_2 - k\phi(e_2)\theta e_2 + \hat{x}(\hat{z}-z)\theta e_2\\ &+ \beta e_3(cz - \hat{c}\hat{z}) + (\hat{x}^2 - x^2)\beta e_3 + \rho\dot{e}_a e_a + \gamma\dot{e}_b e_b + \mu\dot{e}_c e_c\\ =&e_a(\hat{y}-\hat{x})\lambda e_1 + a(e_2-e_1)\lambda e_1 + e_b\hat{x}\theta e_2 + b\theta e_1 e_2 + (-ze_1 - \hat{x}e_3)\theta e_2 - k\phi(e_2)\theta e_2\\ &+ \hat{x}(\hat{z}-z)\theta e_2 - e_c\hat{z}\beta e_3 - c\beta e_3^2 + (\hat{x}+x)\beta e_1 e_3 + \rho\dot{e}_a e_a + \gamma\dot{e}_b e_b + \mu\dot{e}_c e_c. \end{aligned} \tag{14}$$

Obviously, if we choose $\theta = \beta = \gamma = \mu, \rho = 1$, from system (10), it can be found that

$$\begin{aligned}\dot{V}(e) \le& -\lambda a e_1^2 - c\beta e_3^2 + (\lambda a + \theta(b-z))e_1 e_2 + (\hat{x}+x)\beta e_1 e_3 - k\theta\phi(e_2)e_2\\ \le& -\lambda a e_1^2 - c\beta e_3^2 + (\lambda a + \theta(b+M))\left|e_1 e_2\right| + M_x\beta\left|e_1 e_3\right| - kk_1\theta e_2^2,\end{aligned}$$

where $e = [|e_1|, |e_2|, |e_3|]^{\mathrm{T}}$. Let

$$A = \begin{pmatrix} \lambda a & -\frac{1}{2}[\lambda a + \beta(b+M)] & -\frac{1}{2}\beta M_x \\ -\frac{1}{2}[\lambda a + \beta(b+M)] & kk_1\beta & 0 \\ -\frac{1}{2}\beta M_x & 0 & \beta c \end{pmatrix}, \tag{15}$$

then

$$\dot{V}(e) \le -e^{\mathrm{T}}Ae. \tag{16}$$

Obviously, to ensure that the origin of error system (8) is asymptotically stable, the matrix A should be positive definite, this is the case if and only if the following three inequalities hold:

$$\begin{aligned}&a)\lambda a > 0,\\ &b)\psi = \lambda a k k_1\beta - \frac{1}{4}(\lambda a + \theta(b+M))^2 > 0,\\ &c)c\psi - \frac{1}{4}\beta^2 M_x^2 k k_1 > 0.\end{aligned} \tag{17}$$

From the above inequalities, we get the formula $k > \dfrac{\xi}{k_1}$ which holds under the conditions (11) and (12). Since $\dot{V}$ is negative semi-definite, we cannot immediately obtain that the origin of error system (8) is asymptotically stable. In fact, as $\dot{V} \le 0$, then $e_1, e_2, e_3 \in L_\infty$ and $e_a, e_b, e_c \in L_\infty$. From the error system (8), we have $\dot{e}_1, \dot{e}_2, \dot{e}_3 \in L_\infty$. Since $\dot{V}(e) \le -e^{\mathrm{T}}Ae$, then we have

$$\begin{aligned}&\int_0^t \lambda_{\min}(A)\left\|e\right\|^2 \mathrm{d}t \le \int_0^t e^{\mathrm{T}}Ae\mathrm{d}t \le \int_0^t -\dot{V}\mathrm{d}t\\ =&V(0) - V(t) \le V(0).\end{aligned}$$

Where $\lambda_{\min}(A)$ is the minimum eigenvalue of positive-definite matrix A. Thus $e_1, e_2, e_3 \in L_2$. According to Barbalat's lemma, we have $e_1, e_2, e_3 \to 0$ as $t \to \infty$, i.e., $\lim_{t\to\infty}\left\|e(t)\right\| = 0$. Therefore, systems (2) and (4) are globally asymptotically synchronized if $k > \dfrac{\xi}{k_1}$ under the conditions (11) and (12). This completes the proof.

In fact, the linear feedback is a special case of our problem. Obviously, if $\phi(u(t)) = u(t)$, we have $k_1 = k = 1$, so this theorem can also be applied to the linear feedback.

4. Simulation results

For error dynamic system (7), the nonlinear input is defined as

$$\phi(u(t)) = (0.8 + 0.2\sin(u(t)))u(t).$$

For theorem 1, in Eqs.(11) and (12) , we let $\lambda = 2.2$, $\theta = 1$, $k = 16$ and choose the initial states as $x(0) = 3$, $y(0) = 5$, $z(0) = 7$, $\hat{x}(0) = 4$, $\hat{y}(0) = 6$, $\hat{z}(0) = 8$. The synchronization errors varying with time are shown in Fig.1 from which we find that the synchronization occurs. Fig.2 shows trajectories of parameters for the response system (4).

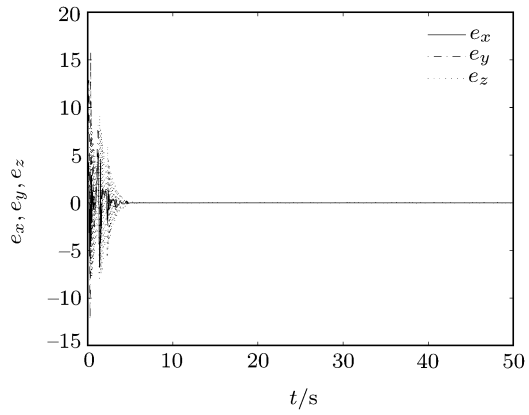


Fig.1. The synchronous errors versus time t.

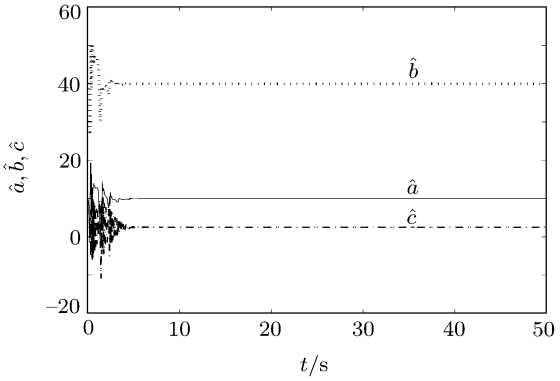


Fig.2. The identification results of parameters versus time t.

5. Conclusion

This paper investigates adaptive synchronization of the Liu chaotic system with all uncertain parameters via a single nonlinear input. The method is simple and can be easily realized, which improves some existing conclusions in Ref.[14]. Finally, simulation results verified the effectiveness of the controller and the feasibility of our method.

References

[1] Pecora L M and Carroll T L 1990 *Phys. Rev. Lett.* **64** 821
[2] Jiang G P and Tang K S 2002 *Int. J. Bifur. Chaos* **12** 2239
[3] Jiang G and Chen G R 2003 *Int. J. Bifur. Chaos* **13** 2243
[4] Zhang X Y, Guan X P and Li H G 2005 *Chin. Phys.* **14** 0279
[5] Liao T L 1998 *Chaos, Solitons and Fractals* **9** 1555
[6] Nijmeijer H and Mareels I M Y 1997 *IEEE.Trans. Circuits. Sys.* **14** 882
[7] Tao C H and Lu J A 2005 *Acta Phys. Sin.* **54** 5058 (in Chinese)
[8] Yue L J and Shen K 2005 *Chin. Phys.* **14** 1760
[9] Chen S H and Lü J H 2002 *Chaos, Solitons and Fractals* **14** 643
[10] Lü L, Guo Z A, L Y and Xia X L 2007 *Acta Phys. Sin.* **56** 0095 (in Chinese)
[11] Yassen M T 2005 *Chaos, Solitons and Fractals* **23** 1527
[12] Chen S H, Zhao L M and Liu J 2002 *Chin. Phys.* **11** 0543
[13] Yu Y G and Zhang S C 2004 *Chaos, Solitons and Fractals* **21** 0643
[14] Tu L L and Lu J A 2005 *Chin. Phys.* **14** 1755
[15] Liu C, Liu T, Liu L and Liu k 2004 *Chaos, Solitons and Fractals* **22** 1031