Vol 17 No 2, February 2008
1674-1056/2008/17(02)/0498-05

Chinese Physics B

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Backstepping synchronization of uncertain chaotic systems by a single driving variable*

Lü Ling(吕 翎)[a)b)†], Zhang Qing-Ling(张庆灵)[a)], and Guo Zhi-An(郭治安)[c)]

[a)] *Institute of System Science, Northeastern University, Shenyang* 110004, *China*
[b)] *College of Physics and Electronic Technology, Liaoning Normal University, Dalian* 116029, *China*
[c)] *Department of Mathematics and Physics, Dalian Jiaotong University, Dalian* 116028, *China*

(Received 16 January 2007; revised manuscript received 21 May 2007)

In this paper a parameter observer and a synchronization controller are designed to synchronize unknown chaotic systems with diverse structures. Based on stability theory the structures of the observer and the controller are presented. The unknown Coullet system and Rossler system are taken for examples to demonstrate that the method is effective and feasible. The artificial simulation results show that global synchronization between the unknown Coullet system and the Rossler system can be achieved by a single driving variable with co-operation of the observer and the controller, and all parameters of the Coullet system can be identified at the same time.

Keywords: backstepping synchronization, parameter identification, uncertain Coullet system, Rossler system

PACC: 0545

1. Introduction

Chaos synchronization has attracted much attention for its great potential applications in many fields, such as in security communications, auto control, etc., and it has become an important subject of research in modern science. So far, many methods and techniques for synchronization have been developed, such as the Pecora Carroll (PC) method, variable coupling method, adaptive control method, variable feedback method and so on.[1−16] But most methods are used to synchronize two identical chaotic systems with certain parameters, and they are not effective to synchronize systems with uncertain parameters. However, chaotic systems are so complex that it is almost impossible practically to find two identical systems. Furthermore, system parameters may be unstable or cannot be well known in advance due to complication of the system or limitation of technology. Therefore, an adaptive method is proposed by Elabbasy for synchronization between two Liu systems with unknown system parameters;[17] and a variable coupling method is proposed by Marino for synchronization between two Lorenz systems with unknown system parameters.[18] These are certainly great developments in the research of synchronization of chaotic systems.

Though these methods mentioned above can synchronize chaotic systems with unknown parameters, they fail when the systems are of different structures. We know that the systems have the same nonlinear functions when they have the same structures. Then their difference is only due to the initial condition. Therefore it is obviously difficult to synchronize chaotic systems with diverse structures. However, synchronization between systems with diverse structures and unknown parameters is of great value in practice. In this paper a parameter observer and a synchronization controller are designed to synchronize unknown chaotic systems with diverse structures. Based on stability theory, the structures of the observer and the controller are presented. The unknown Coullet system and Rossler system are taken for examples to demonstrate that the method is effective and feasible. The artificial simulation results show that global synchronization between the unknown Coullet system and the Rossler system can be achieved by a single driving variable with co-operation of the observer and the controller, and all parameters of the Coullet system can be identified at the same time.

*Project supported by the National Natural Science Foundation of China (Grant No 60574011).
†Corresponding author. E-mail: luling1960@yahoo.com.cn

2. Design of the parameter observer and synchronization controller

Though the dynamic equations of different chaotic systems are not the same, some typical systems, such as Coullet system, Rossler system, Chua's circuit, Van der Pol system, Genesio system, and Duffing's system can be written in the following form:

$$\begin{aligned}
&\dot{x}_1 = m_1 x_2 + G_1(x_1, t),\\
&\dot{x}_2 = m_2 x_3 + G_2(x_1, x_2, t),\\
&\qquad\vdots\\
&\dot{x}_{n-1} = m_{n-1} x_n + G_{n-1}(x_1, x_2, \cdots, x_{n-1}, t),\\
&\dot{x}_n = G_n(x_1, x_2, \cdots, x_n, t),
\end{aligned} \tag{1}$$

where $x_i (i = 1, 2, \cdots, n)$ are system state variables, $m_i (i = 1, 2, \cdots, n-1)$ are system parameters, and $G_i (i = 1, 2, \cdots, n)$ are smooth functions.

With Coullet system and Rossler system as prototypes, a parameter observer and a synchronization controller are proposed. The Coullet system is taken as the drive system, which can be described as follows[19]

$$\begin{aligned}
&\dot{x}_1 = x_2,\\
&\dot{x}_2 = x_3,\\
&\dot{x}_3 = \alpha x_3 + \beta x_2 + \gamma x_1 - x_1^3,
\end{aligned} \tag{2}$$

and the Rossler system as the response system in the form[20]

$$\begin{aligned}
&\dot{y}_1 = y_2 + a y_1 + u_1(t),\\
&\dot{y}_2 = -y_3 - y_1 + u_2(t),\\
&\dot{y}_3 = y_3(y_2 - c) + b + u_3(t),
\end{aligned} \tag{3}$$

where $u_1(t)$, $u_2(t)$, $u_3(t)$ as the controllers are designed to fulfill the synchronization between the two systems. When $\alpha = -0.45$, $\beta = -1.1$, $\gamma = 0.8$, $a = 0.2$, $b = 0.2$, $c = 5.7$, the two systems are in chaos. There are two nonzero balance points in the Coullet system, and the trace surrounding them is a chaotic attractor of double helixes, while there is one nonzero balance point in the Rossler system, around which is an attractor of a single helix. They are different in shape and type, as shown in Fig.1. Furthermore, the nonlinear functions of the two systems are different, and the traces of the two systems are quite different, as is shown in Fig.2. Therefore they are two different chaotic systems.

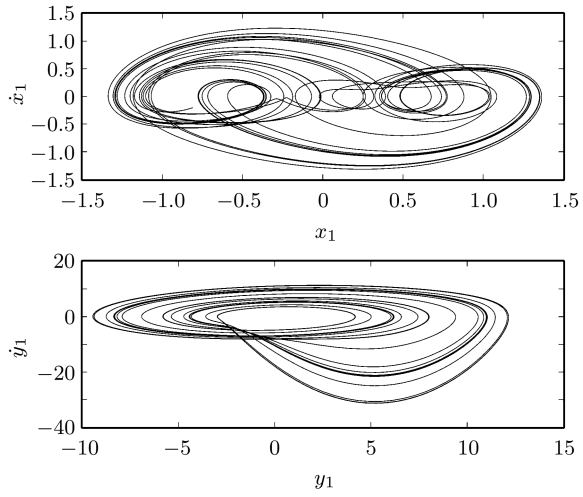


Fig.1. The phase maps of system (1) and (2).

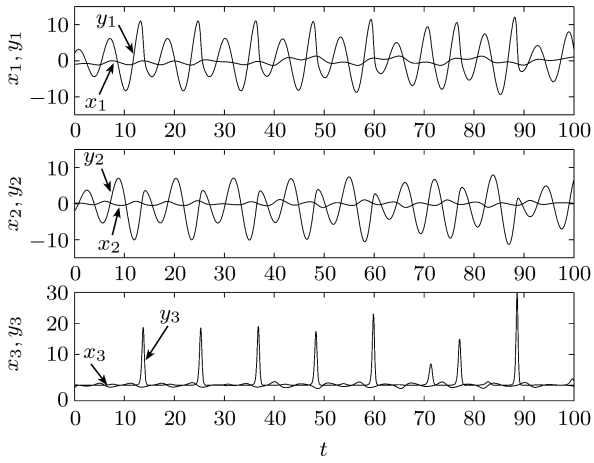


Fig.2. State variables of system (1) and (2) vs time t.

Suppose that the drive system (2) is a chaotic system with uncertain parameters, α, β, γ. x_1 is the only driving variable that is easy to separate. The error variable is $e_1 = x_1 - y_1$, where y_1 is a variable in the response system (3). The purpose of the article is to propose a controller which can synchronize systems (2) and (3) by the variable x_1 in the condition that the unknown parameters of the system (2), α, β, γ can be identified. The structure of the controller is designed in the form

$$\begin{aligned}
&u_1(t) = f_1(t) e_1,\\
&u_2(t) = f_2(t) e_1,\\
&u_3(t) = f_3(t) e_1 + W(t),
\end{aligned} \tag{4}$$

where $f_1(t)$, $f_2(t)$, $f_3(t)$ are error adjusting functions to be determined, and $W(t)$ is an assistant quantity. They are taken as

$$f_1(t) = \bar{\alpha} + 3,$$

$$
\begin{aligned}
f_2(t) &= (f_1 - a - 3)f_1 - 2\dot{f}_1 + \bar{\beta} + 5,\\
f_3(t) &= (4 - 3f_1 + f_1^2 - f_2 - 2af_1 + 3a + a^2)f_1\\
&\quad +(3 + a - f_1)f_2 + (3 - 3f_1 + a)\dot{f}_1\\
&\quad +\ddot{f}_1 + \dot{f}_2 + 3y_1^2 - \bar{\gamma} - 3, \qquad (5)
\end{aligned}
$$

$$
\begin{aligned}
W(t) = &-[(2 - f_1) + \bar{\gamma} + a\dot{f}_1\\
&-a(2 - 2f_1 + f_1^2 - \dot{f}_1 - f_2 - af_1)\\
&+(1 + a)(1 - 2a - a^2 + af_1)]y_1\\
&-(\bar{\beta} + 1 - 3a - a^2 + af_1)y_2\\
&-(6 - 2f_1 + \bar{\alpha} + a + y_2 - c)y_3\\
&+3e_1^2y_1 + e_1^3 + y_1^3 - b. \qquad (6)
\end{aligned}
$$

The structure of a parameter observer is designed as

$$
\begin{aligned}
\dot{\bar{\alpha}} &= x_3e_z,\\
\dot{\bar{\beta}} &= x_2e_z,\\
\dot{\bar{\gamma}} &= (e_1 + y_1)e_z, \qquad (7)
\end{aligned}
$$

where $e_z = (2 - 2f_1 + f_1^2 - \dot{f}_1 - f_2 - af_1)e_1 + (2 - f_1)(x_2 - y_2) + x_3 + (1 - 2a - a^2 + af_1)y_1 - ay_2 + y_3$ and $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$ are quantities to be identified for parameters α, β, γ, respectively.

Theorem When the controller is designed as in Eq.(4), and the structure of the parameter observer as in (7), the systems (2) and (3) can be synchronized for any initial condition, and all the parameters of the uncertain Coullet system can be identified.

Proof Introduce $e_x = e_1$, the first partial Lyapunov function is constructed as

$$
V_1 = \frac{1}{2}e_x^2. \qquad (8)
$$

The derivative of V_1 is

$$
\begin{aligned}
\dot{V}_1 &= e_x\dot{e}_x\\
&= -e_x^2 + e_x[(1 - f_1)e_1 + x_2 - y_2 - ay_1]. \qquad (9)
\end{aligned}
$$

Taking $e_y = (1 - f_1)e_1 + x_2 - y_2 - ay_1$ and introducing function k_2 as

$$
e_y = k_2 - y_2, \qquad (10)
$$

we have

$$
k_2 = (1 - f_1)e_1 + x_2 - ay_1. \qquad (11)
$$

The second partial Lyapunov function is constructed as

$$
V_2 = V_1 + \frac{1}{2}e_y^2. \qquad (12)
$$

The derivative of V_2 is

$$
\begin{aligned}
\dot{V}_2 &= \dot{V}_1 + e_y\dot{e}_y\\
&= -e_x^2 - e_y^2 + e_y[(2 - 2f_1 + f_1^2 - \dot{f}_1 - f_2 - af_1)e_1 + (2 - f_1)(x_2 - y_2)\\
&\quad +x_3 + (1 - 2a - a^2 + af_1)y_1 - ay_2 + y_3]. \qquad (13)
\end{aligned}
$$

Now letting

$$
e_z = (2 - 2f_1 + f_1^2 - \dot{f}_1 - f_2 - af_1)e_1 + (2 - f_1)(x_2 - y_2) + x_3 + (1 - 2a - a^2 + af_1)y_1 - ay_2 + y_3 \qquad (14)
$$

and introducing another function k_3 as

$$
e_z = k_3 - y_3, \qquad (15)
$$

we have

$$
k_3 = (2 - 2f_1 + f_1^2 - \dot{f}_1 - f_2 - af_1)e_1 + (2 - f_1)(x_2 - y_2) + x_3 + (1 - 2a - a^2 + af_1)y_1 - ay_2 + 2y_3. \qquad (16)
$$

The Lyapunov function is constructed as

$$
V_3 = V_2 + \frac{1}{2}e_z^2 + \frac{1}{2}(\bar{\alpha} - \alpha)^2 + \frac{1}{2}(\bar{\beta} - \beta)^2 + \frac{1}{2}(\bar{\gamma} - \gamma)^2. \qquad (17)
$$

The derivative of V_3 is

$$
\begin{aligned}
\dot{V}_3 &= \dot{V}_2 + e_z\dot{e}_z + (\bar{\alpha} - \alpha)\dot{\bar{\alpha}} + (\bar{\beta} - \beta)\dot{\bar{\beta}} + (\bar{\gamma} - \gamma)\dot{\bar{\gamma}}\\
&= -e_x^2 - e_y^2 - e_z^2 + e_z\eta(t) + (\bar{\alpha} - \alpha)\dot{\bar{\alpha}} + (\bar{\beta} - \beta)\dot{\bar{\beta}} + (\bar{\gamma} - \gamma)\dot{\bar{\gamma}}, \qquad (18)
\end{aligned}
$$

where

$$\begin{aligned}\eta(t) = {} & [(1-f_1)+(2-2f_1+f_1^2-\dot{f}_1-f_2-af_1)+(-2\dot{f}_1+2f_1\dot{f}_1-\ddot{f}_1-\dot{f}_2-a\dot{f}_1)-f_1(2-2f_1\\ & +f_1^2-\dot{f}_1-f_2-af_1)-(2-f_1)f_2+\gamma-3y_1^2-f_3+(1-2a-a^2+af_1)f_1-af_2+2f_3]e_1\\ & +[1+(2-f_1)+(2-2f_1+f_1^2-\dot{f}_1-f_2-af_1)-\dot{f}_1+\beta](x_2-y_2)+(3-f_1+\alpha)(x_3-y_3)\\ & +[(2-f_1)+\gamma+a\dot{f}_1-a(2-2f_1+f_1^2-\dot{f}_1-f_2-af_1)+(1+a)(1-2a-a^2+af_1)]y_1\\ & +(\beta+1-3a-a^2+af_1)y_2+(6-2f_1+\alpha+a+y_2-c)y_3-3e_1^2y_1-e_1^3-y_1^3+b+W(t). \end{aligned} \tag{19}$$

For certain $f_1(t)$, $f_2(t)$, $f_3(t)$, $W(t)$ and the form of the parameter observer, let

$$e_z\eta(t)+(\bar{\alpha}-\alpha)\dot{\bar{\alpha}}+(\bar{\beta}-\beta)\dot{\bar{\beta}}+(\bar{\gamma}-\gamma)\dot{\bar{\gamma}}=0. \tag{20}$$

It's easy to obtain

$$\begin{aligned} f_1(t) &= \bar{\alpha}+3,\\ f_2(t) &= (f_1-a-3)f_1-2\dot{f}_1+\bar{\beta}+5,\\ f_3(t) &= (4-3f_1+f_1^2-f_2-2af_1+3a+a^2)f_1+(3+a-f_1)f_2+(3-3f_1+a)\dot{f}_1+\ddot{f}_1\\ &\quad +\dot{f}_2+3y_1^2-\bar{\gamma}-3, \end{aligned} \tag{21}$$

$$\begin{aligned} W(t) = {} & -[(2-f_1)+\bar{\gamma}+a\dot{f}_1-a(2-2f_1+f_1^2-\dot{f}_1-f_2-af_1)+(1+a)(1-2a-a^2+af_1)]y_1\\ & -(\bar{\beta}+1-3a-a^2+af_1)y_2-(6-2f_1+\bar{\alpha}+a+y_2-c)y_3+3e_1^2y_1+e_1^3+y_1^3-b, \end{aligned} \tag{22}$$

$$\begin{aligned} \dot{\bar{\alpha}} &= x_3e_z,\\ \dot{\bar{\beta}} &= x_2e_z,\\ \dot{\bar{\gamma}} &= (e_1+y_1)e_z. \end{aligned} \tag{23}$$

Then, according to response system (2), the controllers are in the following form:

$$\begin{aligned} u_1(t) &= f_1(t)e_1,\\ u_2(t) &= f_2(t)e_1,\\ u_3(t) &= f_3(t)e_1+W(t), \end{aligned} \tag{24}$$

and the structure of the parameter observer is as follows:

$$\begin{aligned} \dot{\bar{\alpha}} &= x_3e_z,\\ \dot{\bar{\beta}} &= x_2e_z,\\ \dot{\bar{\gamma}} &= (e_1+y_1)e_z, \end{aligned} \tag{25}$$

we obtain

$$\dot{V}_3=-e_x^2-e_y^2-e_z^2\le 0 \tag{26}$$

According to Lyapunov stability theory,[21] the synchronization of the two systems is then achieved when Eq.(26) is fulfilled.

3. Simulation

In the simulation, the initial values of the two systems are taken as $x_1(0)=-1$, $x_2(0)=0.1$, $x_3(0)=0.1$, $y_1(0)=2$, $y_2(0)=-2$, $y_3(0)=-1$ respectively, the systems are synchronized as shown in Fig.3 and Fig.4 when the parameter observer and the controller are taken as those in Eq.(4) and Eq.(7). We see that all the three variables of Coullet system are synchronized with those of Rossler system, and the

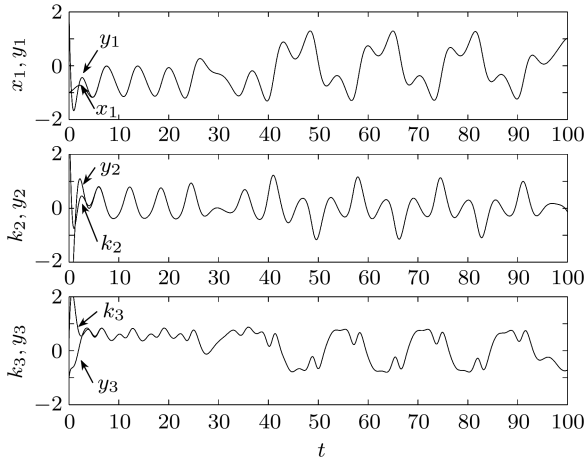


Fig.3. Synchronization state variables vs time t.

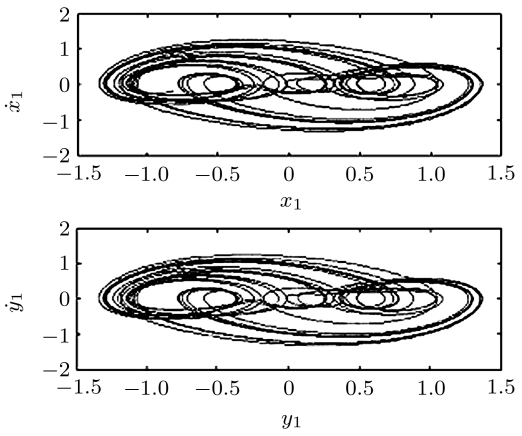


Fig.4. Synchronization phase map of system (2) and (3).

phase maps change into exactly the same in shape and type. When control is added, the error signals approach zero smoothly and rapidly as shown in Fig.5.

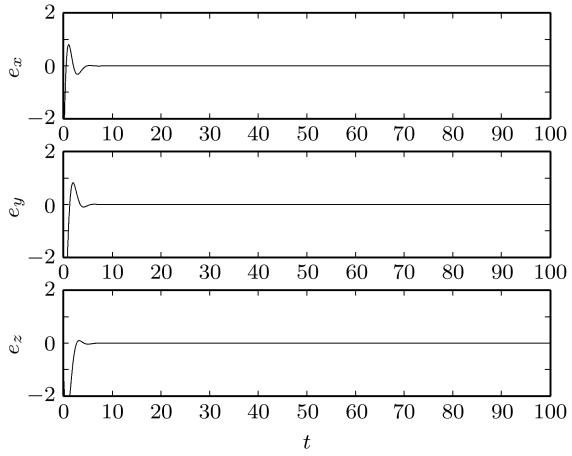


Fig.5. Error variable vs time t.

From Fig.6, we see that when time reaches to 5s, the parameters α, β, γ approach to values -0.45, -1.1 and 0.8. This shows that the observer proposed in the article can identify the unknown parameters.

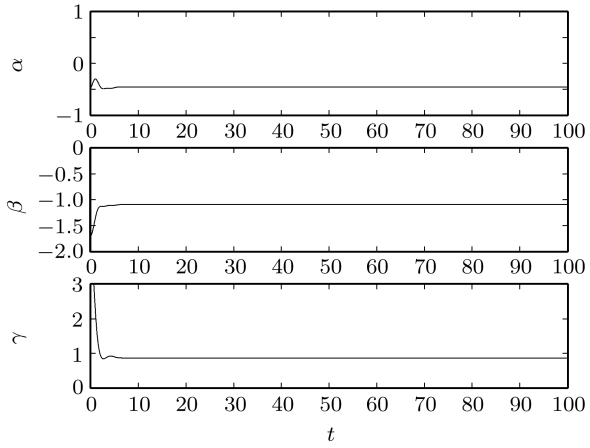


Fig.6. Organization of parameters α, β, γ.

4. Conclusion

A parameter observer and a synchronization controller are proposed in the article. For Coullet system and Rossler system to be synchronized, the unknown parameters in Coullet system are identified by the observer. The parameters α, β, γ approach to the values -0.45, -1.1 and 0.8, respectively.Backstepping method is used to fulfill the synchronization between uncertain Coullet system and Rossler system. The single driving variable x_1 in Rossler is used to synchronize the two systems. Simulation results show that the method is effective and feasible.

References

[1] Pecora L M and Carroll T L 1990 *Phys. Rev. Lett.* **64** 821
[2] Lü L, Luan L and Guo Z A 2007 *Chin.Phys.* **16** 346
[3] Lü L, Guo Z A, Li Y and Xia X L 2007 *Acta Phys. Sin.* **56** 95 (in Chinese)
[4] Awad E G 2006 *Chaos, Solitons and Fractals* **27** 345
[5] Lu J G 2006 *Chin. Phys.* **15** 83
[6] Wang Y W, GuanZ H and Wang H O 2005 *Phys. Lett.* A **339** 325
[7] Tsimring L S, Rulkov N F, Larsen M L and Gabbay M 2005 *Phys. Rev. Lett.* **95** 14101
[8] Yan W W, Zhi H G and Hua O W 2005 *Phys. Lett.* A **339** 325
[9] Yue L J and Shen K 2005 *Chin. Phys.* **14** 1760
[10] Wang X Y and Shi Q J 2005 *Acta Phys. Sin.* **54** 5591 (in Chinese)
[11] Tao C H and Lu J A 2005 *Acta Phys. Sin.* **54** 5058 (in Chinese)
[12] Yu H J and Liu Y Z 2005 *Acta Phys. Sin.* **54** 3029 (in Chinese)
[13] Ma J, Liao G H, Mo X H, Li W X and Zhang P W 2005 *Acta Phys. Sin.* **54** 5585 (in Chinese)
[14] Park J H 2005 *Chaos, Solitons and Fractals* **26** 959
[15] Li S H and Cai H X 2004 *Acta Phys. Sin.* **53** 1687 (in Chinese)
[16] Cheng L, Zhang R Y and Peng J H 2003 *Acta Phys. Sin.* **52** 536 (in Chinese)
[17] Elabbasy E M, Agiza H N and EI-Dessoky M M 2004 *Chaos, Solitons and Fractals* **21** 657
[18] Marino I P and Miguez J 2006 *Phys. Lett.* A **351** 262
[19] Wu C W and Chua L O 1996 *Int. J. Bif. Chaos* **6** 801
[20] Lü L, Luan L, Du Z, Qiu D C, Liu Y and Li Y 2005 *Int. J. Infor. & Sys. Sci.* **1** 237
[21] Lü L 2000 *Nonlinear dynamics and chaos* (Dalian: Dalian Publishing House) (in Chinese)