Vol 17 No 2, February 2008
1674-1056/2008/17(02)/0492-06
Chinese Physics B
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Passive control and synchronization of hyperchaotic Chen system*

Zhang Qun-Jiao(张群娇) and Lu Jun-An(陆君安)†

School of Mathematics and Statistics, Wuhan University, Wuhan 430072, *China*

(Received 29 December 2006; revised manuscript received 23 June 2007)

This paper investigates the control and synchronization of hyperchaotic Chen system based on the passive theory. By using two outputs, novel passive controllers are respectively designed to realize the globally asymptotical stability of the hyperchaotic Chen system and the error dynamical system, which avoids mistakes in Ref.[11], where function $W(z)$ cannot guarantee that $f_0(z)$ is globally asymptotically stable via only one output and $W(z)$ is the Lyapunov function of $f_0(z)$. Furthermore, numerical simulations are given to show the effectiveness of our method.

Keywords: hyperchaotic Chen system, passive control, stability, synchronization
PACC: 0545

1. Introduction

Since the chaos control and synchronization was first introduced by Pecora and Carroll,[1,2] it has been an active research topic in nonlinear science, and has been extensively studied. In the past two decades, a variety of approaches have been proposed for the chaos synchronization, including manifold-based method,[3] adaptive method,[4] time delay feedback approach,[5] backstepping method,[6] nonlinear control scheme[7,8] and many others. Recently, the concept of passivity[9] of nonlinear systems has aroused the new interest in nonlinear control field. Qi,[10] Wang and Liu[11] and Sun *et al*[12] have applied it to studying the control and synchronization of some chaotic or hyperchaotic systems. Generally, a hyperchaotic system is defined as a chaotic system with more than one positive Lyapunov exponent, which is more complex and plays a more important role in many practical applications. In Ref.[11], the authors studied the synchronization of hyperchaotic Lorenz system by only one output. But there exists a main drawback that the function $W(z)$ in their work cannot guarantee that the Lyapunov function $f_0(z)$ is globally asymptotically stable, because the derivative of $W(z)$ is not strictly negative definite but semi-negative definite.

In this letter, based on the properties of a passive system, we investigate the control and synchronization of hyperchaotic Chen system via two outputs. Some novel passive controllers are respectively designed to realize the globally asymptotical stability and synchronization of the hyperchaotic Chen system.

The rest of this paper is organized as follows. In Section 2, we briefly introduce the passive theory and the hyperchaotic Chen system. Some controllers for the passive control and synchronization of the hyperchaotic Chen system are obtained in Section 3. Section 4 is devoted to some numerical simulations for the illustration and verification of our results. Finally, some conclusions are given in Section 5.

2. Passive theory and the hyperchaotic Chen system

Consider the following continuous differential equation:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u, \\ y &= h(x), \end{aligned} \tag{1}$$

where $x \in R^n$ is the state variable, $u \in R^m$ is the input, $y \in R^m$ is the output. $f(x)$ and $g(x)$ are the smooth vector fields, $h(x)$ is a smooth mapping. We suppose that the vector field f has at least one equilibrium point. Without loss of generality, we can assume that the equilibrium point is $x = 0$. If the equilibrium point is not at $x = 0$, we can shift the equilibrium point to $x = 0$ by coordinate transform.

*Project supported by the National Natural Science Foundation of China (Grant Nos 60574045 and 70771084).
†E-mail addresses: qunjiao99@163.com, jalu@whu.edu.cn

Definition 1 System (1) is a minimum phase system if $L_g h(0)$ is nonsingular and $x = 0$ is one of the asymptotically stabilized equilibrium points of $f(x)$.

Definition 2 System (1) is said to be passive if there exists a real-valued constant β such that for any $t \geq 0$,

$$\int_0^t u^{\mathrm{T}}(\tau) y(\tau) \mathrm{d}\tau \geq \beta, \tag{2}$$

or there exist a constant $\rho > 0$ and a real-valued constant β such that for any $t \geq 0$,

$$\int_0^t u^{\mathrm{T}}(\tau) y(\tau) \mathrm{d}\tau + \beta \geq \rho \int_0^t y^{\mathrm{T}}(\tau) y(\tau) \mathrm{d}\tau. \tag{3}$$

Let $z = \varphi(x)$, then system (1) will be changed into the following normal form:

$$\begin{aligned} \dot{z} &= f_0(z) + p(z, y) y, \\ \dot{y} &= b(z, y) + a(z, y) u, \end{aligned} \tag{4}$$

where $a(z, y)$ is nonsingular for any (z, y).

Nonlinear system (4) may be rendered passive by using a state feedback in the form[13]

$$u = \phi(x) + \psi(x) v. \tag{5}$$

Next we will use this idea to control the new hyperchaotic system,[14] which is created from Chen system[15,16] by adding a state feedback controller. The controlled hyperchaotic Chen system can be described by

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_4, \\ \dot{x}_2 &= d x_1 - x_1 x_3 + c x_2 + u_1, \\ \dot{x}_3 &= x_1 x_2 - b x_3, \\ \dot{x}_4 &= x_2 x_3 + r x_4 + u_2, \end{aligned} \tag{6}$$

where x_1, x_2, x_3, and x_4 are state variables and a, b, c, d, and r are real constants, u_1 and u_2 are controllers to be designed. The system ($u_1 = u_2 = 0$) is found to be hyperchaotic for parameters $a = 36, b = 3, c = 20, d = 7$ and $r \in (0.085, 0.798]$, because it has two positive Lyapunov exponents and the sum of all the Lyapunov exponents is negative. For instance, when the parameters are $a = 36$, $b = 3$, $c = 20$, $d = 1.3$, and $r = 0.58$, the Lyapunov exponents are $\lambda_1 = 0.5011$, $\lambda_2 = 0.1858$, $\lambda_3 = 0$ and $\lambda_4 = -26.1010$ in this case. The simulated results with initial conditions $(x_1(0), x_2(0), x_3(0), x_4(0)) = (1.0, -2.0, 3.0, -4.0)$ are depicted in Fig.1. It displays the projections of the hyperchaotic attractor of system (6) in the $x_1 - x_2 - x_3$ space, $x_1 - x_2 - x_4$ space, $x_1 - x_3 - x_4$ space, and $x_2 - x_3 - x_4$ space respectively.

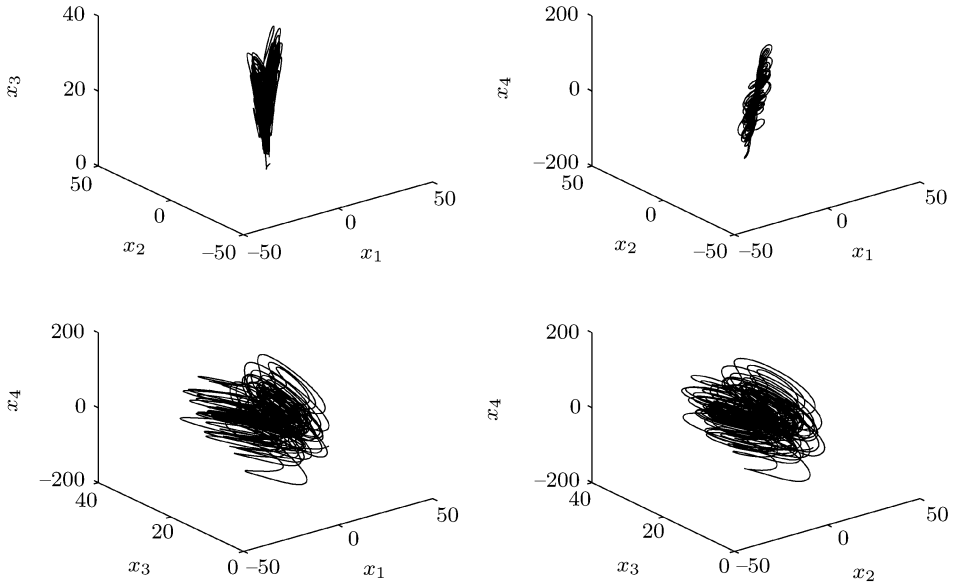


Fig.1. Hyperchaotic Chen attractor.

3. Passive control and synchronization of hyperchaotic Chen system

For controlled hyperchaotic system (6), introduce

$$z \triangleq \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_3 \end{pmatrix},$$

$$y \triangleq \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ x_4 \end{pmatrix},$$

then system (6) can be rewritten as the normal form

$$\begin{aligned} \dot{z}_1 &= -az_1 + ay_1 + y_2, \\ \dot{z}_2 &= -bz_2 + z_1 y_1, \\ \dot{y}_1 &= dz_1 - z_1 z_2 + cy_1 + u_1, \\ \dot{y}_2 &= z_2 y_1 + ry_2 + u_2. \end{aligned} \tag{7}$$

We have

$$\begin{aligned} \dot{z} &= f_0(z) + p(z,y)y, \\ \dot{y} &= b(z,y) + a(z,y)u, \end{aligned} \tag{8}$$

where

$$\begin{aligned} f_0(z) &= [-az_1, -bz_2]^{\mathrm{T}}, \\ p(z,y) &= \begin{pmatrix} a & 1 \\ z_1 & 0 \end{pmatrix}, \\ b(z,y) &= [dz_1 - z_1 z_2 + cy_1, z_2 y_1 + ry_2]^{\mathrm{T}}, \\ a(z,y) &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \end{aligned} \tag{9}$$

Theorem 1 Hyperchaotic system (6) is a minimum phase system.

Proof Choose a storage function

$$V(z,y) = W(z) + \frac{1}{2} y^{\mathrm{T}} y, \tag{10}$$

where

$$W(z) = \frac{1}{2} z^{\mathrm{T}} z = \frac{1}{2}(z_1^2 + z_2^2) \tag{11}$$

is the Lyapunov function of $f_0(z)$, and $W(0) = 0$.

The zero dynamics of system (8) describes the internal dynamics, which is consistent with external constraint $y = 0$, i.e. $\dot{z} = f_0(z)$, then we have

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} W(z) =& \frac{\partial W(z)}{\partial z} f_0(z) \\ =& [z_1, z_2][-az_1, -bz_2]^{\mathrm{T}} \\ =& -az_1^2 - bz_2^2 \le 0. \end{aligned} \tag{12}$$

Thus, it can be concluded that $W(z)$ is the Lyapunov function of $f_0(z)$ and that $f_0(z)$ is globally asymptotically stable. That is to say, the zero dynamics of hyperchaotic Chen system is Lyapunov stable, meanwhile, $L_{\mathrm{g}} h(0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is nonsingular, so system (6) is in a minimum phase[13,17] and can be equivalent to a passive system.

Theorem 2 Hyperchaotic system (6) can be equivalent to a passive system and globally asymptotically stabilized by the following state feedback controllers

$$\begin{aligned} u =& a^{-1}(z,y)[-b(z,y) - \left(\frac{\partial W}{\partial z} \cdot p(z,y)\right)^{\mathrm{T}} \\ & - \alpha y + v], \end{aligned} \tag{13}$$

where $\alpha > 0$ is a positive constant, $u = (u_1, u_2)^{\mathrm{T}}$ is the control vector and $v = (v_1, v_2)^{\mathrm{T}}$ is an external signal vector.

Proof From Theorem 1, hyperchaotic dynamical system (6) is a minimum phase system,

$$\frac{\partial W(z)}{\partial z} f_0(z) \le 0,$$

then the derivative of $V(z,y)$ along the trajectory of system (8) is

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} V(z,y) =& \frac{\partial W(z)}{\partial z} f_0(z) + \frac{\partial W(z)}{\partial z} p(z,y)y + \dot{y}^{\mathrm{T}} y \\ \le& \frac{\partial W(z)}{\partial z} p(z,y)y + \dot{y}^{\mathrm{T}} y \\ =& \frac{\partial W(z)}{\partial z} p(z,y)y \\ & + (b(z,y) + a(z,y)u)^{\mathrm{T}} y. \end{aligned} \tag{14}$$

Substitute Eq.(13) into Eq.(14), we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t} V(z,y) \le -\alpha y^{\mathrm{T}} y + v^{\mathrm{T}} y.$$

Integrating the above inequality yields

$$\begin{aligned} & V(z,y) - V(z_0, y_0) \\ \le& \int_0^t v^{\mathrm{T}}(\tau) y(\tau) \mathrm{d}\tau - \int_0^t \alpha y^{\mathrm{T}}(\tau) y(\tau) \mathrm{d}\tau. \end{aligned} \tag{15}$$

Let $\mu = V(z_0, y_0)$ and notice that $V(z,y) \ge 0$, then

$$\int_0^t v^{\mathrm{T}}(\tau) y(\tau) \mathrm{d}\tau + \mu \ge \int_0^t \alpha y^{\mathrm{T}}(\tau) y(\tau) \mathrm{d}\tau, \tag{16}$$

which satisfies passive definition (3).

If an external signal vector $v = 0$ (zero-input state response), from inequality (15) we have $V(z,y) \le V(z_0, y_0)$. Because $W(z)$ is radially unbounded, it follows from function (10) that $V(z,y)$ is also radially

unbounded, so that a closed-loop system is a bounded state stable for $[z^{\mathrm{T}}, y^{\mathrm{T}}]^{\mathrm{T}}$. This means that we can use simple passive controllers (13) to regulate system (6) to the origin.

In the following section, we will study the synchronization of two identical hyperchaotic Chen systems.

Suppose that the drive system is

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1) + x_4,\\ \dot{x}_2 &= dx_1 - x_1x_3 + cx_2,\\ \dot{x}_3 &= x_1x_2 - bx_3,\\ \dot{x}_4 &= x_2x_3 + rx_4,\end{aligned} \tag{17}$$

and the response system is

$$\begin{aligned}\dot{y}_1 &= a(y_2 - y_1) + y_4,\\ \dot{y}_2 &= dy_1 - y_1y_3 + cy_2 + u_1,\\ \dot{y}_3 &= y_1y_2 - by_3,\\ \dot{y}_4 &= y_2y_3 + ry_4 + u_2,\end{aligned} \tag{18}$$

and let the error states be

$$e_i(t) = y_i(t) - x_i(t), \quad (i = 1, 2, 3, 4), \tag{19}$$

the from Eqs.(17)–(18) we will have the error dynamics as

$$\begin{aligned}\dot{e}_1(t) &= a(e_2 - e_1) + e_4,\\ \dot{e}_2(t) &= de_1 - x_1e_3 - x_3e_1 - e_1e_3 + ce_2 + u_1,\\ \dot{e}_3(t) &= -be_3 + x_1e_2 + x_2e_1 + e_1e_2,\\ \dot{e}_4(t) &= re_4 + x_2y_3 + x_3e_2 + e_2e_3 + u_2.\end{aligned} \tag{20}$$

Introducing

$$z \triangleq \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} e_1 \\ e_3 \end{pmatrix},$$

$$w \triangleq \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} e_2 \\ e_4 \end{pmatrix},$$

system (20) can be rewritten as the normal form

$$\begin{aligned}\dot{z}_1 &= -az_1 + aw_1 + w_2,\\ \dot{z}_2 &= -bz_2 + x_2z_1 + x_1w_1 + z_1w_1,\\ \dot{w}_1 &= dz_1 - x_1z_2 - x_3z_1 - z_1z_2 + cw_1 + u_1,\\ \dot{w}_2 &= x_2z_2 + x_3w_1 + z_2w_1 + rw_2 + u_2,\end{aligned} \tag{21}$$

and we have

$$\begin{aligned}\dot{z} &= f_0(z) + p(z, w)w,\\ \dot{w} &= b(z, w) + a(z, w)u,\end{aligned} \tag{22}$$

where

$$\begin{aligned}f_0(z) &= [-az_1, -bz_2 + x_2z_1]^{\mathrm{T}},\\ p(z, w) &= \begin{pmatrix} a & 1 \\ x_1 + z_1 & 0 \end{pmatrix},\\ b(z, w) &= [dz_1 - x_1z_2 - x_3z_1 - z_1z_2 + cw_1,\\ &\qquad x_2z_2 + x_3w_1 + z_2w_1 + rw_2]^{\mathrm{T}},\\ a(z, w) &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.\end{aligned} \tag{23}$$

Theorem 3 Error dynamical system (20) is a minimum phase system.

Proof Choose a storage function

$$V(z, y) = W(z) + \frac{1}{2}w^{\mathrm{T}}w, \tag{24}$$

where

$$W(z) = \frac{M^2}{4ab}z_1^2 + \frac{1}{2}z_2^2 \tag{25}$$

is the Lyapunov function of $f_0(z)$ and $W(0) = 0$, M is a bound of $x_2(t)$, i.e. $|x_2(t)| \le M$.

The zero dynamics of system (20) is consistent with external constraint $w = 0$, i.e. $\dot{z} = f_0(z)$, and we have

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}W(z) =& \frac{\partial W(z)}{\partial z}f_0(z)\\ =& \frac{M^2}{2ab}z_1\dot{z}_1 + z_2\dot{z}_2\\ =& -\frac{M^2}{2b}z_1^2 - bz_2^2 + x_2z_1z_2\\ =& -b\left(z_2 - \frac{x_2}{2b}z_1\right)^2 + \frac{x_2^2}{4b}z_1^2 - \frac{M^2}{2b}z_1^2\\ \le& -b\left(z_2 - \frac{x_2}{2b}z_1\right)^2 - \frac{M^2}{4b}z_1^2\\ \le& 0.\end{aligned} \tag{26}$$

So $f_0(z)$ is globally asymptotically stable. At the same time, $L_{\mathrm{g}}h(0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is nonsingular, therefore, error dynamical system (20) has a minimum phase,[13,17] and it can be equivalent to a passive system.

Theorem 4 Error dynamical system (20) can be equivalent to a passive system and globally asymptotically stabilized by the following state feedback controller:

$$\begin{aligned}u =& a^{-1}(z, w)\Big[-b(z, w) - \left(\frac{\partial W}{\partial z} \cdot p(z, w)\right)^{\mathrm{T}}\\ & - \alpha w + v\Big],\end{aligned} \tag{27}$$

where $\alpha > 0$ is a positive constant, $u = (u_1, u_2)^{\mathrm{T}}$ is the control vector and $v = (v_1, v_2)^{\mathrm{T}}$ is an external signal that is connected with the reference input.

This theorem can be proved in a similar way as shown in Theorem 2, so its proof is omitted here.

When external signal vector $v = 0$, dynamical error system (20) will be asymptotically stabilized at a zero equilibrium point, that is, hyperchaotic Chen systems (17) and (18) will be synchronized with each other.

4. Numerical simulations

In this section, in order to verify and demonstrate the effectiveness of the theoretical method, the fourth-order Runge-Kutta method is used to solve the differential equations in steps of 0.001. In the following numerical simulations, the control inputs are switched on at t=5s. The constants are set to be $\alpha = 1$, $v_1 = 0$, and $v_2 = 0$ to guarantee that Eqs.(6) and (20) are globally asymptotically stable at the origin.

According to theorem 2, with the controllers

$$u_1 = -(d + a)x_1 - (c + \alpha)x_3,$$
$$u_2 = -x_1 - x_2x_3 - (r + \alpha)x_4,$$

the controlled state variables of system (6) will converge to zero, and the simulation results with initial vales $x_1(0) = 1.0$, $x_2(0) = 2.0$, $x_3(0) = 1.0$, and $x_4(0) = 4.0$ are displayed in Fig.2. By theorem 4, with $M = 25$, under the control of

$$u_1 = -\left(d + \frac{M^2}{2b}\right)e_1 + e_1x_3 - (c + \alpha)e_2,$$
$$u_2 = -x_2e_3 - x_3e_2 - e_2e_3 - \frac{M^2}{2ab}e_1 - (r + \alpha)e_4,$$

systems (17) and (18) will be synchronized with each other. The synchronization errors between them are shown in Fig.3, with the initial conditions $x_1(0) = 3.0$, $x_2(0) = -4.0$, $x_3(0) = 2.0$, $x_4(0) = 5.0$ and $y_1(0) = 4.0$, $y_2(0) = -5.0$, $y_3(0) = 1.0$, $y_4(0) = 2.0$.

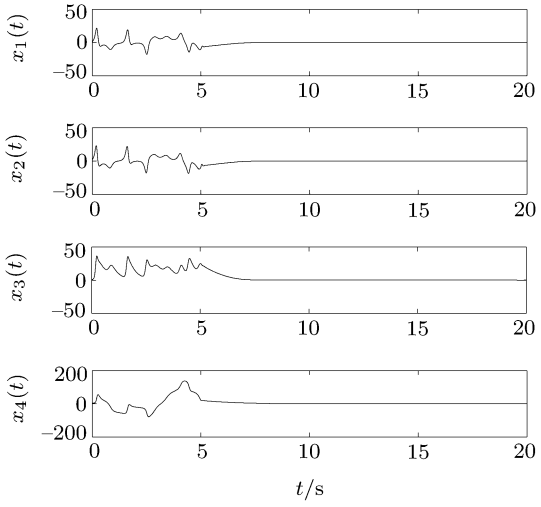


Fig.2. Controlled state variables of system (6).

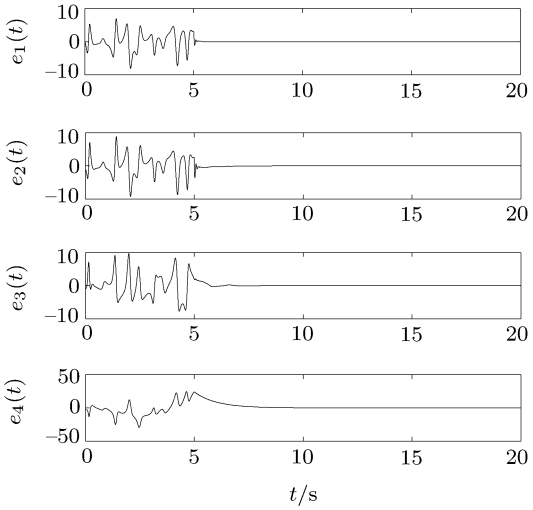


Fig.3. Synchronization errors between systems (17) and (18).

5. Conclusion

In this paper, we have investigated the passive control and synchronization of hyperchaotic Chen system.[14] Based on the passive system theory, novel passive controllers are proposed to realize the global asymptotical stability of the hyperchaotic Chen system and the error dynamical system. Finally, numerical simulations are provided to verify the theoretical analysis. Similarly, the work can be done for the passive control and synchronization of hyperchaotic Lü system, hyperchaotic Lorenz system and hyperchaotic Rössler system.

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