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Transitions in a genotype selection model driven by coloured noises*

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This paper investigates a genotype selection model subjected to both a multiplicative coloured noise and an additive coloured noise with different correlation time τ_1 and τ_2 by means of the numerical technique. By directly simulating the Langevin Equation, the following results are obtained. (1) The multiplicative coloured noise dominates, however, the effect of the additive coloured noise is not neglected in the practical gene selection process. The selection rate μ decides that the selection is propitious to gene A haploid or gene B haploid. (2) The additive coloured noise intensity α and the correlation time τ_2 play opposite roles. It is noted that α and τ_2 can not separate the single peak, while α can make the peak disappear and τ_2 can make the peak be sharp. (3) The multiplicative coloured noise intensity D and the correlation time τ_1 can induce phase transition, at the same time they play opposite roles and the reentrance phenomenon appears. In this case, it is easy to select one type haploid from the group with increasing D and decreasing τ_1.

Keywords: genotype selection model, coloured noise, stationary probability distribution
PACC: 0540

1. Introduction

The study of the dynamical systems subjected to a noise perturbation has been an interesting and recurrent theme in physics systems,[1−14] biology systems,[15−22] laser systems[22−29] etc. Traditionally, the stochastic dynamics systems are investigated by means of the approximate Fokker–Planck Equation from theory, at the same time numerical simulation is often used as a yardstick for theoretical approaches. It is known that the stochastic dynamics systems is more complex, and the dynamical characters of some stochastic systems can not be directly described by an analytic expression. However, it is emphasized that the numerical simulation can not only describe these characters directly, but also lose little information. Therefore, numerical–simulation algorithms have been developed for Gaussian white noise and coloured noise[30−34] and widely applied to many stochastic dynamical systems. It has been a strong tool for the treatment of complex stochastic dynamical systems.

On the other hand, in previous study, the Gaussian white noise source is generally considered in some systems and the coloured noise source is rarely considered for mathematic difficulties. However, more realistic models of physical system require considering noise source with finite correlation time (i.e., coloured noise). For example, in order to describe the static and dynamical properties of dye laser, the usual model incudes not only the standard external white noise, but also an external coloured noise.[11]

The genotype selection model has been widely studied.[15−18,35] In previous works, only the multiplicative coloured noise is considered, however, it is unpractical that the additive coloured noise is neglected in the real genotype selection process. Castro *et al*[15] only considered the multiplicative coloured noise and showed that the genotype selection model undergoes a purely noise induced transition, from a monostable regime to a bistable, when it is submitted to a white noise or colour noise source, and that for large values of the correlation time, the system undergoes a new transition to a monostable state, indicating a reentrance phenomenon. Ai *et al*[18] investigated the multiplicative white noise, additive white

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noise and the correlation between them, and showed that the correlation noise breaks the balance of gene selection and induces the phase transition. But, in their works, the finite correlation time is not taken into account. Therefore, it is very necessary that investigating the genotype selection model driven by an additive coloured noise and a multiplicative coloured noise is more practical. However, when coloured noise source is taken account into the genotype selection model, the approximate method, such as the unified coloured noise approximation, which requires that the parameters are in certain range, must be used in order to get the steady probability distribution from the approximate Fokker-Planck Equation. Moreover, the analytical expression is very complex and hardly to treat for mathematical difficulties. So, the numerical technique is used in this paper.

In the present paper, the steady probability distribution (SPD) of the genotype selection model subjected to an additive coloured noise and a multiplicative coloured noise is analysed by directly simulating the Langevin Equation. The paper is organized as follows. In Section 2 the genotype selection model is simply introduced. The SPD of this model is analysed in detail in Section 3. In Section 4, some obtained results are summarized.

2. The genotype selection model

Consider the simplest gene selection dynamics equation, which is introduced in Refs.[18,35] in detail. It is shown as

$$\dot{x} = \beta - x + \mu x(1-x), \tag{1}$$

where β is the mass of genotype B and μ is the selection rate, $\mu = 1$ means that the selection is completely propitious to gene A haploid, and $\mu = -1$ means that the selection is completely propitious to gene B haploid. $x(t)$ is the ratio of gene A number to the total.

It is known that all physical parameters are subjected to random perturbations that may have internal or external origin. So we should not consider only deterministic quantities. Here, we consider that some environmental external factors disturb the gene selection rate of the haploid, it is likely to affect both β and μ in the form of additive and multiplicative noises. In other words, the external fluctuations affect the parameter μ which fluctuates around a mean value, thus generating a multiplicative noise and at the same time environmental fluctuations perturb the dynamics directly and give rise to an additive noise. Thus, Eq.(1) becomes a Langevin equation

$$\dot{x} = \beta - x + \mu x(1-x) + x(1-x)\epsilon(t) + \Gamma(t), \tag{2}$$

where $\epsilon(t)$ and $\Gamma(t)$ are Gaussian coloured noise (the Ornstein–Uhlenbeck noise), which are characterized by their mean and variance.

$$\langle \epsilon(t) \rangle = \langle \Gamma(t) \rangle = 0, \tag{3}$$

$$\langle \epsilon(t)\epsilon(t') \rangle = \frac{D}{\tau_1} \exp\left[-\frac{|t-t'|}{\tau_1} \right], \tag{4}$$

$$\langle \Gamma(t)\Gamma(t') \rangle = \frac{\alpha}{\tau_2} \exp\left[-\frac{|t-t'|}{\tau_2} \right]. \tag{5}$$

Here τ_1 and D are the correlation time and intensity of the multiplicative coloured noise, respectively. α and τ_2 are the intensity and the noise correlation time of additive coloured noise, respectively.

3. The SPD

Making use of the expression (Eq.(2)), we can analyse the effects of μ, α, D, τ_1 and τ_2 on SPD of the genotype selection model by means of the numerical simulation, and the simulation technique is given by Ref.[31] in detail. We set β=0.5 in the all following figures.

3.1. Additive coloured noise

We only consider the effect of the additive coloured noise on the genotype selection model in this subsection. Namely,

$$\dot{x} = \beta - x + \mu x(1-x) + \Gamma(t), \tag{6}$$

Where $\Gamma(t)$ is described by Eq.(3) and Eq.(5).

After numerical calculation, the results are shown in Figs.1–2. The time course of $x(t)$ and the SPD are plotted by directly simulating the stochastic differential equation(6) for different μ in Fig.1, respectively. Figure 1(a) shows that the peak of the curve is biased to the left when μ gets a negative value ($\mu = -1$), that is, the selection is unfair and propitious to gene B haploid. We can also see the same result from the time course of $x(t)$, the value of x fluctuates in the value of $x = 0.3$, which corresponds to the peak position. Figure 1(b) shows a symmetrical structure when $\mu = 0$. In this case, the gene A haploid (x) and the gene B haploid ($1 - x$) have the same probability distribution. Figure 1(c) shows that the peak of the curve is biased to the right when $\mu = 1$, namely, the selection is propitious to gene A haploid.

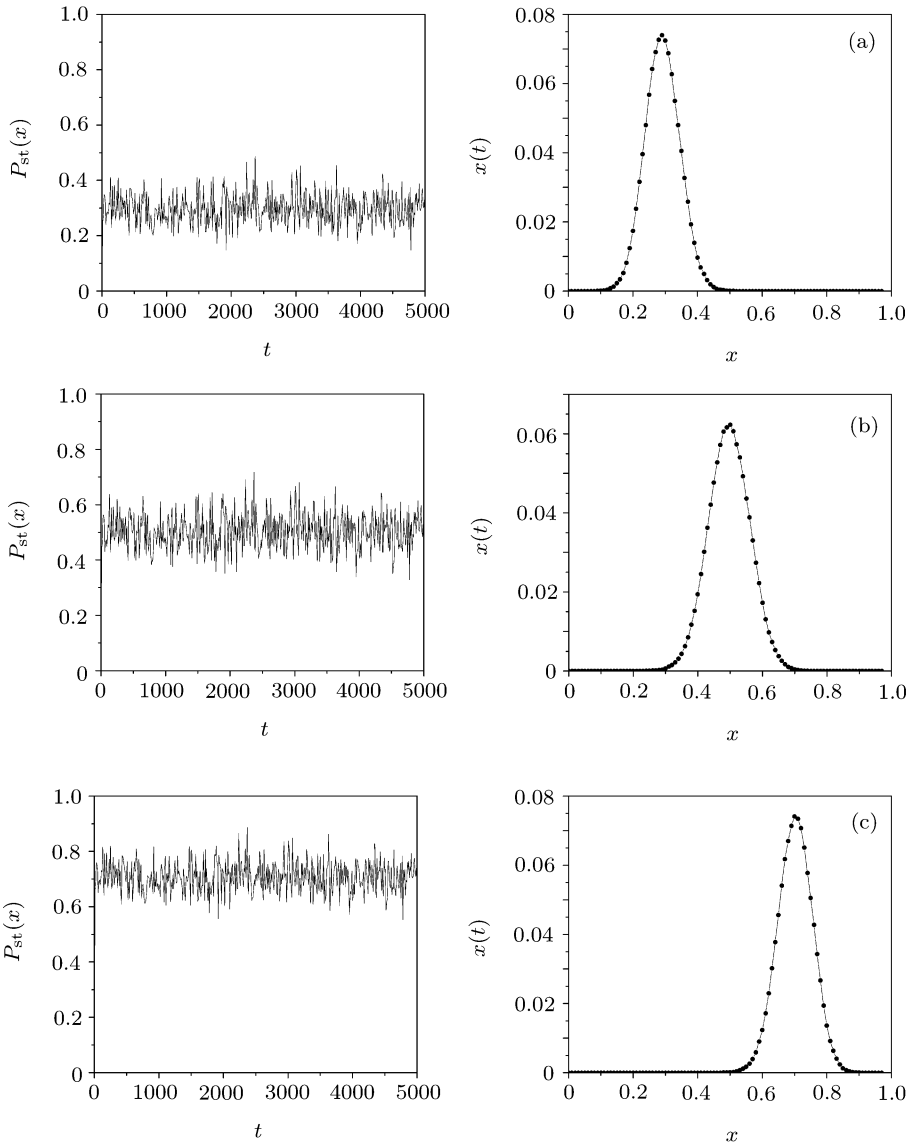


Fig.1. Sample paths and probability distribution of $x(t)$ for different selection rate μ, when additive coloured noise only exists. (a) μ=-1, (b) μ=0, (c) μ=1. The other parameter values are α=8.0, τ_2=1.0.

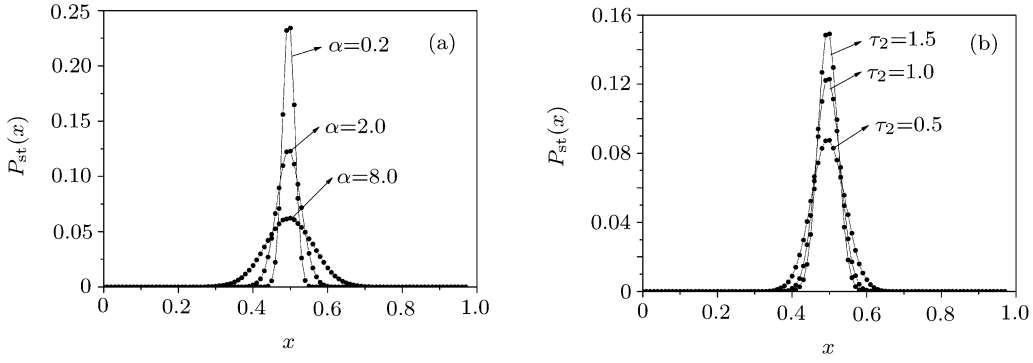


Fig.2. The SPD vs x with μ=0, when additive coloured noise only exists. (a) for different α=0.5, 2.0, 8.0 with τ_2=1.0. (b) for different τ_2=0.5, 1.0, 1.5 with α=2.0.

In Fig.2, we study the effect of additive coloured noise intensity α and the correlation time τ_2 on SPD by numerical. For studying the effect of the coloured noise on the system easily we adopt the symmetrical case (the selection rate $\mu = 0$) for the following discussion. The SPD as the function of x is plotted for different α when τ_2 is fixed in Fig.2(a). It is clearly that the peak of the SPD becomes narrow and grows in height and the position of the peak is not affected with α is decreasing. It means that α can not separate the single peak while it can make the peak disappear as a diffuse term. In Fig.2(b) the SPD as the function of x is plotted for different τ_2 when α is fixed. The peak of the SPD becomes broad and drops in height and the position of the peak is not affected as τ_2 is decreasing. That is, τ_2 can not also separate the single peak and only make the peak be sharp as a diffuse term. It is evident that α and τ_2 play different roles in a genotype selection process.

3.2. Multiplicative coloured noise

In this subsection, the effect of the multiplicative coloured noise on the genotype selection model is considered. The Langevin equation is given,

$$\dot{x} = \beta - x + \mu x(1-x) + x(1-x)\epsilon(t), \qquad (7)$$

where $\epsilon(t)$ is described by Eq.(3) and Eq.(4).

The results are shown in Figs.3–4 by directly simulating the stochastic differential equation(7). Figure 3 shows the effect of μ on the SPD. The structure of

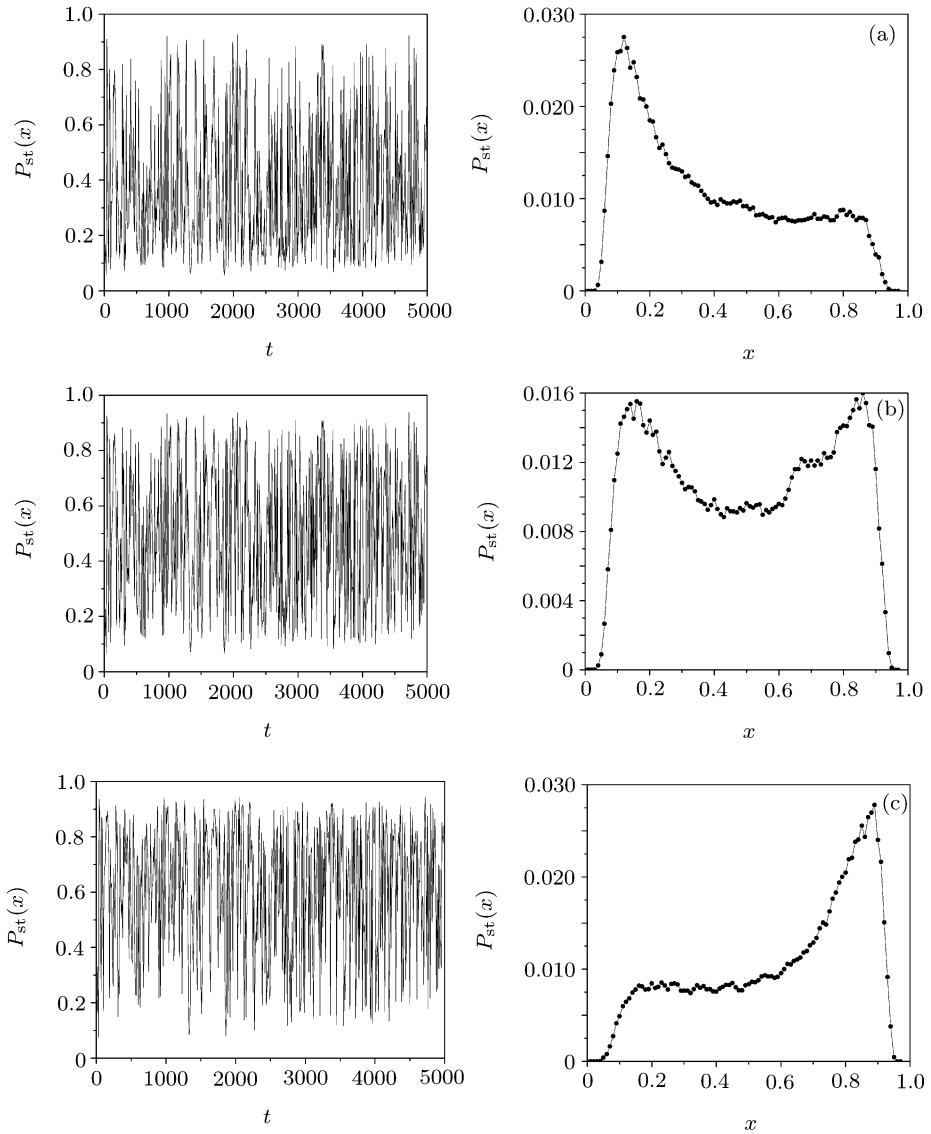


Fig.3. Sample paths and probability distribution of $x(t)$ for different μ, when multiplicative coloured noise only exists. (a) μ=-1, (b) μ=0, (c) μ=1. The other parameter values are D=8.0, τ_1=1.0.

the SPD is destroyed when μ increases from -1 to 1. For $\mu = -1$, the peak is biased to the left(Fig.3(a)); For $\mu = 0$, the peak is symmetrical(Fig.3(b)); For $\mu = 1$, the peak is biased to the right(Fig.3(c)). We can also get the same results from the time evolution of $x(t)$. These results show that the effect of μ on the genotype selection process is the same as that in the case of additive coloured noise.

The effect of D and τ_1 on the SPD is plotted in Fig.4. We can find that phase transition appears with D and τ_1 are changed. When D increases, the peak changes from an unimodal structure to a bimodal structure in Fig.4(a). Nevertheless, the peak changes from a bimodal structure to an unimodal structure when τ_1 increases in Fig.4(b). Therefore, D and τ_1 play different roles. It is a reentrance phenomenon, which has been shown by Castro *et al* in Ref.[15]. Namely, for a small value of D and a large value of τ_1 the curve shows a single peak region which indicates that environmental selection gives the same chance to both gene A haploid and gene B haploid and it is not easy for us to select one type haploid from the group. As the value of D increases and the value of τ_1 decreases, the single peak region vanishes and it evolves a double peak region. In the case, the environmental selection gives an absolute big chance to one of the two (gene A or gene B). So that, it is easy to select one type haploid from the group since the other type haploid number can be neglected with regard to the selected type number.

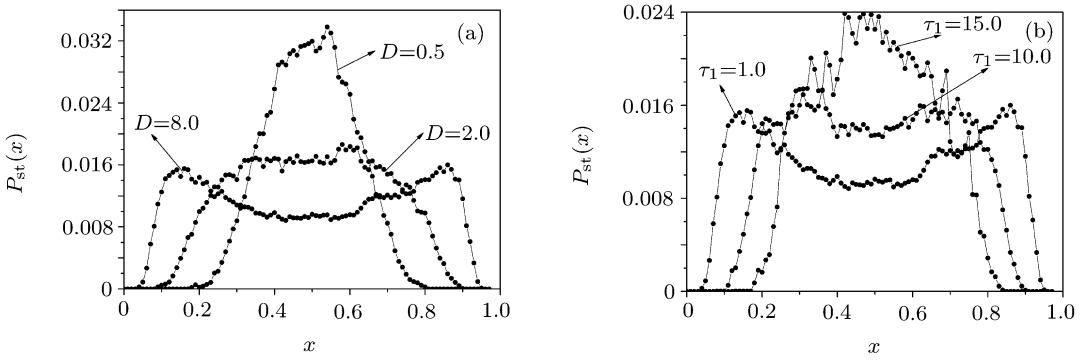


Fig.4. The SPD vs x with μ=0, when multiplicative coloured noise only exists: (a) for different D=0.5, 2.0, 8.0 with τ_1=1.0; (b) for different τ_1=0.5, 1.0, 1.5 with D=8.0.

3.3. Additive coloured noise and Multiplicative coloured noise

In previous works, only the multiplicative coloured noise is considered in the gene selection model. However, the effect of the additive coloured noise is not neglected based on the results of the subsection 3.1, therefore, the genotype model subjected to the two coloured noises is considered in the subsection. In the case, the Langevin Equation is given by Eq.(2) and Eqs.(3)–(5). Some interesting results are got from Figs.5–9 by directly simulating Eq.(2).

The SPD as the function of x is plotted in Fig.5 for different μ, when the other parameters are fixed. It is shown that the peak of the SPD shifts from the left to the right when μ is changed from negative to positive.

For simplification of discussion, we make $\mu = 0$. The effect of multiplicative coloured noise intensity D on the SPD is plotted in Fig.6 when the other parameters are fixed. It is shown that the phase transition appears when D increases, at the same time, in Fig.7, the same phase transition phenomenon appears when τ_1 decreases. Therefore, the structure of the SPD can be destroyed by τ_1 and D. Note that τ_1 and D play opposite roles in the gene selection process like the results in subsection 3.2 and there is almost no distinction when only multiplicative colour noise exists.

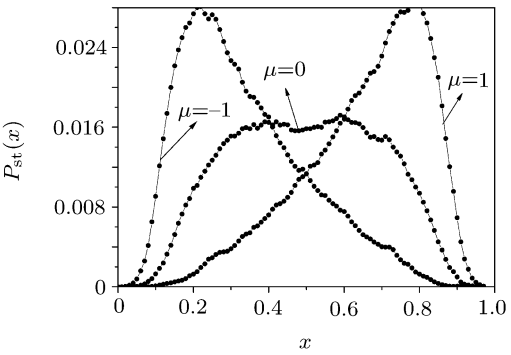


Fig.5. The SPD vs x for different μ, when the two coloured noises exist. (a) μ=-1, (b) μ=0, (c) μ=1. The other parameters are D=2.0, τ_1=1.0, τ_2=1.0 α=8.0.

The SPD as the function of x is plotted in Fig.8 for different α, when the other parameters are fixed. When α increases, the peak of the SPD drops in height, but the changes is very weak. Comparing Fig.8 with Fig.2(a), we find that there is obvious distinctions in the height of the peak of the SPD.

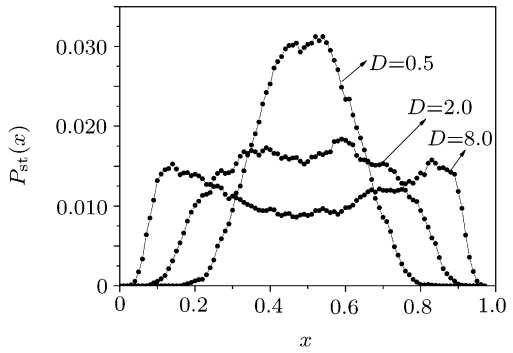


Fig.6. The SPD vs x for different D=0.5, 2.0, 8.0, when the two coloured noises exit. The other parameters are τ_1=1.0, τ_2=1.0 α=2.0.

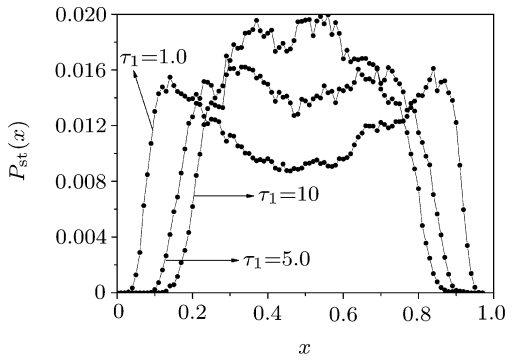


Fig.7. The SPD vs x for different τ_1=1.0, 5.0, 10.0, when the two coloured noises exit. The other parameters are D=8.0, τ_2=1.0 and α=1.0.

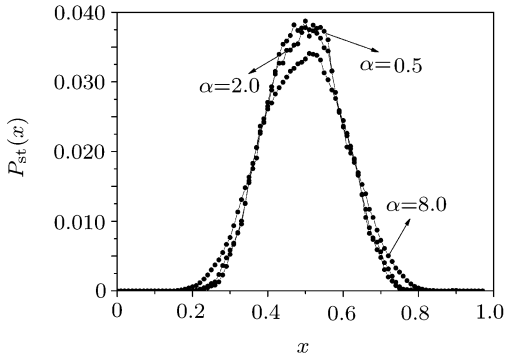


Fig.8. The SPD vs x for different α=0.5, 2.0, 8.0 with D=0.5, τ_1=2.0 τ_2=1.0 when the two coloured noises exit.

The SPD as the function of x is plotted in Fig.9 for different τ_2, when the other parameters are fixed. There is no distinctions for different values of τ_2, namely, τ_2 can not almost affect the gene selection when two coloured noise exists. So, the multiplicative coloured noise dominates and the additive coloured noise is not evident in the gene selection process.

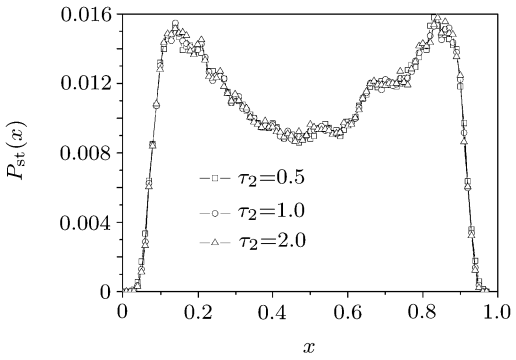


Fig.9. The SPD vs x for different τ_2=0.5, 1.0, 2.0, when the two coloured noises exist with D=8, τ_1=1.0 α=1.0.

4. Conclusions

In summary, the effects of the coloured noise on the genotype selection model are investigated by directly simulating the Langevin Equation. Some interesting results are obtained.

In the gene selection process, the multiplicative coloured noise dominates, however, the effect of additive coloured noise is not evident. But the effect of the additive coloured noise is not neglected in the real gene selection process. The selection rate μ decides that the selection is propitious to gene A haploid or gene B haploid. The additive coloured noise intensity α and the correlation time τ_2 play opposite roles. It is noted that α and τ_2 can not separate the single peak, while α can make the peak disappear and τ_2 can make the peak be sharp. On the other hand, the multiplicative coloured noise intensity D and the correlation time τ_1 can induce phase transition, but they play opposite roles and the reentrance phenomenon appears. In this case, it is easy to select one type haploid from the group with increasing D and decreasing τ_1.

Appendix A Methods of numerical simulations

We consider the general one variable problem,

$$\dot{x} = f(x) + g(x)\epsilon(t) + \Gamma(t), \qquad \text{(A1)}$$

$$\dot{\epsilon} = -\frac{\epsilon}{\tau_1} + \frac{1}{\tau_1} g_w, \tag{A2}$$

$$\dot{\Gamma} = -\frac{\Gamma}{\tau_2} + \frac{1}{\tau_2} h_w, \tag{A3}$$

in which x is the variable of interest, $f(x)$ is a nonlinear function, and $\epsilon(t)$, $\Gamma(t)$ are the exponentially correlated coloured noises(the Ornstein–Uhlenbeck noise). The characters of the noises are given by Eqs.(3)–(5). g_w and h_w are the Gaussian white noises, which can be used by Box–Mueller algorithm to realize. The Euler version of the integration of Eqs.(A1–A3) is given by

$$x|_{t+\triangle t} = x + (f(x) + g(x)\epsilon + \Gamma)\triangle t, \tag{A4}$$

$$\epsilon|_{t+\triangle t} = \epsilon - \triangle t \frac{\epsilon}{\tau_1} + \frac{g_w}{\tau_1}, \tag{A5}$$

$$\Gamma|_{t+\triangle t} = \Gamma - \triangle t \frac{\Gamma}{\tau_2} + \frac{h_w}{\tau_2}. \tag{A6}$$

in which $g_w = [-4D\triangle t \ln a]^{1/2} \cos(2\pi b)$ and $h_w = [-4\alpha\triangle t \ln c]^{1/2} \cos(2\pi d)$ are the Gaussian white noise, and

$$\epsilon = \Big[-\frac{2D}{\tau_1} \triangle t \ln e \Big]^{1/2} \cos(2\pi f)$$

and

$$\Gamma = \Big[-\frac{2\alpha}{\tau_2} \triangle t \ln g \Big]^{1/2} \cos(2\pi h),$$

where a, b, c, d, e, f, g and h are all independent random numbers. Repeating this process, averaging over many such realizations of algorithm we realize the average over the stationary probability distribution.

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