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Scheme for splitting quantum information via W states in cavity QED systems*

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Assisted by multipartite entanglement, Quantum information may be split so that the original qubit can be reconstructed if and only if the recipients cooperate. This paper proposes an experimentally feasible scheme for splitting quantum information via W-type entangled states in cavity QED systems, where three-level Rydberg atoms interact with nonresonant cavities. Since W-type states are used as the quantum channel and the cavities are only virtually excited, the scheme is easy to implement and robust against decoherence, and the dependence on the quality factor of the cavities is greatly reduced.

Keywords: splitting quantum information, W-type states, cavity QED
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1. Introduction

In classical cryptography, secret sharing[1] is a procedure for splitting a message into several parts so that only the entire set but not the subset of parts is sufficient to read the message. This procedure can also be implemented by using quantum information techniques, namely shared multipartite entanglement, local quantum operations and classical communication.[2] What is more, secret sharing can further be modified to allow one party Alice to teleport[3] a string of quantum bits (qubits) to two parties Bob and Charlie via shared multipartite entanglement in such a way that only by working together can they obtain the exact qubits.[2,4,5] In this case it is quantum information that is split and teleported, and usually multipartite maximally entangled states, i.e., the Greenberger-Horne-Zeilinger (GHZ) states[6] are used as the quantum channel. Recently, Zheng[7] have described a procedure for quantum information splitting via W-type states. The W states can retain bipartite entanglement when any one of the three qubits is traced out and thus they are much more robust than the GHZ states.[8,9]

Cavity quantum electrodynamics (QED) is considered to be one of the ideal systems for quantum information processing.[10−14] Zheng and Guo[15] have proposed a novel cavity QED scheme for preparing two-atom entangled states and quantum-information processing, where atoms interact with a nonresonant cavity and the cavity is only virtually excited. Osnaghi *et al*[16] have experimentally implemented the scheme and demonstrated that the scheme is insensitive to thermal fields and to photon decay. The scheme has been used to realize teleportation,[15] dense coding,[17] quantum cloning[18] and preparation of multipartite entangled states.[19,20] Recently, it has been used to implement quantum secret sharing via GHZ states.[21] In this paper, we propose a scheme for splitting and reconstructing quantum information via W states in cavity QED systems. Compared with GHZ states, n-partite W states can be obtained straightforwardly and no ancillary atoms and fewer cavities are required, thus our scheme is easier to implement than the schemes using GHZ states. Secondly, our scheme is more robust against decoherence than that in Ref.[21] because a W-type state instead of a GHZ state is used as the quantum channel. Finally, our scheme is based on Zheng–Guo's model, thus can effectively suppress the cavity decay and is experimentally feasible with present technology.

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2. Procedure for splitting quantum information via W states

First we briefly review the procedure presented by Zheng in Ref.[7]. Suppose Alice, Bob and Charlie initially share a tripartite entangled state of W-type

$$\frac{1}{2}\left|0_2 0_3 1_4\right\rangle + \frac{1}{2}\left|0_2 1_3 0_4\right\rangle + \frac{1}{\sqrt{2}}\left|1_2 0_3 0_4\right\rangle . \tag{1}$$

Alice now wants to send the quantum state of qubit 1

$$\left|\psi_1\right\rangle = \alpha\left|0_1\right\rangle + \beta\left|1_1\right\rangle , \tag{2}$$

where $|\alpha|^2 + |\beta|^2 = 1$. The state of the whole system can be written as

$$\begin{aligned}\left|\Psi_{1234}\right\rangle = \alpha & \left(\frac{1}{2}\left|0_1 0_2 0_3 1_4\right\rangle + \frac{1}{2}\left|0_1 0_2 1_3 0_4\right\rangle \right. \\ & \left. + \frac{1}{\sqrt{2}}\left|0_1 1_2 0_3 0_4\right\rangle\right) + \beta\left(\frac{1}{2}\left|1_1 0_2 0_3 1_4\right\rangle \right. \\ & \left. + \frac{1}{2}\left|1_1 0_2 1_3 0_4\right\rangle + \frac{1}{\sqrt{2}}\left|1_1 1_2 0_3 0_4\right\rangle\right) .\end{aligned} \tag{3}$$

To do that, Alice performs a joint measurement on her two particles 1 and 2 with respect to Bell states

$$\begin{aligned}\left|\Phi^{\pm}\right\rangle &= \frac{1}{\sqrt{2}}\left(\left|0_1 0_2\right\rangle \pm \left|1_1 1_2\right\rangle\right) , \\ \left|\Psi^{\pm}\right\rangle &= \frac{1}{\sqrt{2}}\left(\left|0_1 1_2\right\rangle \pm \left|1_1 0_2\right\rangle\right) .\end{aligned} \tag{4}$$

Accordingly, the particles held by Bob and Charlie collapse onto one of the following entangled states

$$\begin{aligned}&\frac{1}{\sqrt{2}}\alpha\left(\left|0_3 1_4\right\rangle + \left|1_3 0_4\right\rangle\right) \pm \beta\left|0_3 0_4\right\rangle , \\ &\alpha\left|0_3 0_4\right\rangle \pm \frac{1}{\sqrt{2}}\beta\left(\left|0_3 1_4\right\rangle + \left|1_3 0_4\right\rangle\right) .\end{aligned} \tag{5}$$

From Eq.(5) it can be seen that the information of the state of particle 1 is encoded in the coefficients of the two components $\frac{1}{\sqrt{2}}\left(\left|0_3 1_4\right\rangle + \left|1_3 0_4\right\rangle\right)$ and $\left|0_3 0_4\right\rangle$. Since the nonclone theorem[22] allows only one particle to be in the original state of particle 1, if Charlie wants to possess the state, Bob must send his particle 3 to Charlie. Then a unitary transformation is performed on the two particles

$$\begin{aligned}\frac{1}{\sqrt{2}}\left(\left|0_3 1_4\right\rangle + \left|1_3 0_4\right\rangle\right) &\rightarrow \left|0_3 1_4\right\rangle , \\ \left|0_3 0_4\right\rangle &\rightarrow \left|0_3 0_4\right\rangle .\end{aligned} \tag{6}$$

Accordingly particle 4 evolves into

$$\alpha\left|1_4\right\rangle \pm \beta\left|0_4\right\rangle , \quad \alpha\left|0_4\right\rangle \pm \beta\left|1_4\right\rangle , \tag{7}$$

with particle 3 left in the state $\left|0_3\right\rangle$. Finally, with the outcome of the joint measurement transmitted to Charlie, he can reconstruct the original state by simply performing Pauli rotations on his particle 4.

3. Implementation in cavity QED systems

Consider that n identical Rydberg atoms simultaneously interact with a single-mode cavity field. The atomic levels and transitions are shown in Fig.1, where

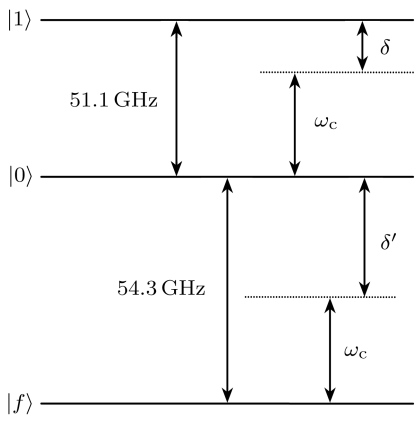


Fig.1. The Rydberg atoms interact with a single-mode cavity field.

the states $\left|1\right\rangle$, $\left|0\right\rangle$, and $\left|f\right\rangle$ represent the three circular levels of the Rydberg atoms with principal quantum numbers 51, 50 and 49, respectively. In the rotating-wave approximation, the interaction Hamiltonian of the system in the interaction picture is (assume $\hbar = 1$)[15,18]

$$H = g\sum_{j=1}^{n}\left(\mathrm{e}^{-\mathrm{i}\delta t}a^{+}s_j^{-} + \mathrm{e}^{\mathrm{i}\delta t}as_j^{+}\right), \tag{8}$$

where $s_j^{-} = \left|0_j\right\rangle\left\langle 1_j\right|$ and $s_j^{+} = \left|1_j\right\rangle\left\langle 0_j\right| (j = 1, 2, \cdots, n)$, a^{+} and a are, respectively, the creation and annihilation operators of the cavity field, g is the atom-cavity coupling strength, and δ is the detuning between the cavity frequency ω_{c} and the atomic $\left|1\right\rangle \leftrightarrow \left|0\right\rangle$ transition frequency. Here we have chosen the detuning $\delta \ll 3.2\,\mathrm{GHz}$, the frequency difference of the transitions $\left|1\right\rangle \leftrightarrow \left|0\right\rangle$ and $\left|f\right\rangle \leftrightarrow \left|0\right\rangle$.

Then the detuning δ' between the cavity frequency ω_c and the $|f\rangle \leftrightarrow |0\rangle$ transition frequency satisfies $\delta' = \delta + 3.2\,\mathrm{GHz} \gg \delta$, and thus the state $|f\rangle$ will not be affected during the atom-cavity interaction. In the large detuning case $\delta \gg g$, there is no energy exchange between the atoms and the cavity. The effective Hamiltonian is given by[15,19]

$$H = \lambda \sum_{i,j=1}^{n} \left(s_j^+ s_i^- a a^+ - s_j^- s_i^+ a^+ a \right), \tag{9}$$

where $\lambda = g^2/\delta$. Suppose that the cavity field is initially in the vacuum state, then the Hamiltonian reduces to[15,19]

$$H = \lambda \left(\sum_{j=1}^{n} |1_j\rangle \langle 1_j| + \sum_{\substack{i,j=1 \\ i \neq j}}^{n} s_j^+ s_i^- \right). \tag{10}$$

The single-qubit unitary operation can be implemented by the resonant interaction of atoms with a classical field. If the classical field is resonant with the $|j\rangle \leftrightarrow |k\rangle$ transition, the evolution operator of the atomic states can be written in the basis $\{|j\rangle,|k\rangle\}$ as[17]

$$U_{jk}(\phi,\varphi) = \begin{pmatrix} \cos\phi & -\mathrm{i}\mathrm{e}^{-\mathrm{i}\varphi}\sin\phi \\ -\mathrm{i}\mathrm{e}^{\mathrm{i}\varphi}\sin\phi & \cos\phi \end{pmatrix}, \tag{11}$$

where $\phi = \Omega_{jk}|\alpha|\,t$, $\alpha = |\alpha|\,\mathrm{e}^{\mathrm{i}\varphi}$ is the complex amplitude of the classical field, and Ω_{jk} is the atom-field coupling constant.

Now we give a detailed description of splitting quantum information via a W-type state in cavity QED systems. Assume that three Rydberg atoms 2, 3 and 4 are initially in the state $|1_2 0_3 0_4\rangle$ and are simultaneously sent into a single-mode cavity with the same velocity. The Hamiltonian in Eq.(10) can be written explicitly as

$$H = \lambda \left[\sum_{j=2,3,4} |1\rangle_{jj} \langle 1| + (s_2^+ s_3^- + s_2^- s_3^+ + s_2^+ s_4^- + s_2^- s_4^+ + s_3^+ s_4^- + s_3^- s_4^+) \right]. \tag{12}$$

So the state evolution of the three atoms can be described by

$$\frac{\mathrm{e}^{-\mathrm{i}3\lambda\tau_1} - 1}{3} \left(|0_2 0_3 1_4\rangle + |0_2 1_3 0_4\rangle \right) + \frac{\mathrm{e}^{-\mathrm{i}3\lambda\tau_1} + 2}{3} |1_2 0_3 0_4\rangle, \tag{13}$$

where τ_1 represents the interaction time. With the choice $\tau_1 = [\pi - \arccos(1/8)]/(3\lambda)$, we obtain the following three-atom state

$$\frac{1}{2} \left(|0_2 0_3 1_4\rangle + |0_2 1_3 0_4\rangle \right) + \frac{1}{\sqrt{2}} \mathrm{e}^{\mathrm{i}\theta} |1_2 0_3 0_4\rangle, \tag{14}$$

where $\theta = -\arctan(11/\sqrt{7})$, and we have discarded the common phase $\arctan(3/\sqrt{7})$. Then we perform on atom 2 the single-qubit rotation $U_{01}\left(\frac{\pi}{2}, -\frac{\theta}{2}\right) U_{01}\left(\frac{\pi}{2}, \pi\right)$ to realize $|0_2\rangle \rightarrow |0_2\rangle$, $|1_2\rangle \rightarrow \mathrm{e}^{-\mathrm{i}\theta}|1_2\rangle$ by the interaction of the atom with classical fields. After that we obtain the required W-type state in Eq.(1).

In Ref.[21], a three-atom GHZ state acts as the quantum channel. To prepare the state, we need an ancillary atom and three cavities. Particles 1 and 2, 3 and 4, initially prepared in appropriate states, are sent through cavities 1 and 2, respectively. Then, after interacting with classical fields, atom 4, together with atom 2, is sent through the third cavity. Finally atom 4 is measured after interacting again with classical fields so that particles 1, 2 and 3 collapse into the required GHZ state. In contrast, the preparation of W-type states is straightforward and requires no ancillary atoms and only one cavity.

In order to perform a joint measurement with respect to Bell states, Alice first performs the operation $U_{f0}\left(\frac{\pi}{2}, \frac{\pi}{2}\right) U_{01}\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ on atom 2, which results in

$$\begin{aligned} |0_2\rangle &\rightarrow \frac{1}{\sqrt{2}} \left(-|f_2\rangle + |1_2\rangle \right), \\ |1_2\rangle &\rightarrow \frac{1}{\sqrt{2}} \left(|f_2\rangle + |1_2\rangle \right). \end{aligned} \tag{15}$$

Then she sends atoms 1 and 2 simultaneously through another single-mode cavity. The state evolution of the two atoms is as follows:

$$
\begin{aligned}
&|0_1 f_2\rangle \to |0_1 f_2\rangle\,,\\
&|0_1 1_2\rangle \to \mathrm{e}^{-\mathrm{i}\lambda\tau_2}\left[\cos(\lambda\tau_2)\,|0_1 1_2\rangle - \mathrm{i}\sin(\lambda\tau_2)\,|1_1 0_2\rangle\right],\\
&|1_1 f_2\rangle \to \mathrm{e}^{-\mathrm{i}\lambda\tau_2}\,|1_1 f_2\rangle\,,\\
&|1_1 1_2\rangle \to \mathrm{e}^{-2\mathrm{i}\lambda\tau_2}\,|1_1 1_2\rangle\,.
\end{aligned}
\tag{16}
$$

We choose the interaction time $\tau_2 = \pi/\lambda$. Finally she performs the operation $U_{01}\left(\frac{\pi}{4},\frac{\pi}{2}\right)$ on atom 1 and $U_{01}\left(\frac{\pi}{4},\frac{\pi}{2}\right)U_{f0}\left(\frac{\pi}{2},\frac{\pi}{2}\right)$ on atom 2, which leads

$$
\begin{aligned}
&|0_1 f_2\rangle \to \frac{1}{2}\left(|0_1 0_2\rangle + |0_1 1_2\rangle + |1_1 0_2\rangle + |1_1 1_2\rangle\right),\\
&|1_1 f_2\rangle \to \frac{1}{2}\left(-|0_1 0_2\rangle - |0_1 1_2\rangle + |1_1 0_2\rangle + |1_1 1_2\rangle\right),\\
&|0_1 1_2\rangle \to \frac{1}{2}\left(-|0_1 0_2\rangle + |0_1 1_2\rangle - |1_1 0_2\rangle + |1_1 1_2\rangle\right),\\
&|1_1 1_2\rangle \to \frac{1}{2}\left(|0_1 0_2\rangle - |0_1 1_2\rangle - |1_1 0_2\rangle + |1_1 1_2\rangle\right).
\end{aligned}
\tag{17}
$$

Therefore we have

$$
\begin{aligned}
&\left|\Phi^+\right\rangle \to -|1_1 0_2\rangle\,, \quad \left|\Phi^-\right\rangle \to -|0_1 0_2\rangle\,,\\
&\left|\Psi^+\right\rangle \to |1_1 1_2\rangle\,, \quad \left|\Psi^-\right\rangle \to |0_1 1_2\rangle\,.
\end{aligned}
\tag{18}
$$

The Bell-state measurement can then be achieved by measuring atoms 1 and 2 separately. As a result, atoms 3 and 4 collapse to one of the states in Eq.(5).

Now Bob and Charlie can cooperate to recover Alice's original state. After receiving atom 3 from Bob, Charlie performs $U_{01}\left(\frac{\pi}{2},\frac{\pi}{4}\right)U_{01}\left(\frac{\pi}{2},\pi\right)$ on atom 3 to realize $|0_3\rangle \to |0_3\rangle$, $|1_3\rangle \to \mathrm{i}\,|1_3\rangle$, then sends atoms 3 and 4 simultaneously through the third single-mode cavity. The state evolution can be described as:

$$
\begin{aligned}
&|0_3 0_4\rangle \to |0_3 0_4\rangle\,,\\
&|0_3 1_4\rangle \to \mathrm{e}^{-\mathrm{i}\lambda\tau_3}\left[\cos(\lambda\tau_3)\,|0_3 1_4\rangle - \mathrm{i}\sin(\lambda\tau_3)\,|1_3 0_4\rangle\right],\\
&|1_3 0_4\rangle \to \mathrm{e}^{-\mathrm{i}\lambda\tau_3}\left[\cos(\lambda\tau_3)\,|1_3 0_4\rangle - \mathrm{i}\sin(\lambda\tau_3)\,|0_3 1_4\rangle\right].
\end{aligned}
\tag{19}
$$

We choose the interaction time $\tau_3 = \pi/(4\lambda)$. After performing the rotation $U_{01}\left(\frac{\pi}{2},\frac{\pi}{8}\right)U_{01}\left(\frac{\pi}{2},\pi\right)$ on atom 4 so that $|0_4\rangle \to |0_4\rangle$, $|1_4\rangle \to \mathrm{e}^{\mathrm{i}\pi/4}\,|1_4\rangle$, Charlie has realized the transformation in Eq.(6), and atom 4 has been in one of the states given by Eq.(7). Following the outcome of the Bell-state measurement from Alice, Charlie performs simple Pauli rotations on atom 4 as follows:

$$
\begin{aligned}
&\left|\Phi^+\right\rangle \to U_{01}\left(\frac{\pi}{2},0\right),\\
&\left|\Phi^-\right\rangle \to U_{01}\left(\frac{\pi}{2},\frac{\pi}{2}\right), \quad \left|\Psi^+\right\rangle \to I,\\
&\left|\Psi^-\right\rangle \to U_{01}\left(\frac{\pi}{2},0\right)U_{01}\left(\frac{\pi}{2},\frac{\pi}{2}\right).
\end{aligned}
\tag{20}
$$

Thus Charlie has reconstructed the original state of atom 1 on atom 4.

4. Discussion and conclusion

We briefly discuss experimental matters. For the Rydberg atoms with principal quantum numbers 49, 50 and 51, the radiative time is about $T_\mathrm{r} = 3\times10^{-2}\,\mathrm{s}$, the efficient decay time of the cavity is about $T_\mathrm{c} = 1\times10^{-3}\,\mathrm{s}$, and the atom-cavity coupling strength is about $g = 2\pi\times25\,\mathrm{kHz}$.[10] With the choice $\delta = 10\,\mathrm{g}$, the total interaction time between the atoms and the cavity field is of the order $3\times10^{-4}\,\mathrm{s}$, which is shorter than T_c. The time required to complete the whole procedure is much shorter than T_r, and the scheme can be realized with the present cavity QED techniques.

In summary, we have proposed an experimental scheme for splitting quantum information via W-type entangled states in cavity QED systems, which is within the reach of current technology. The success probability of the scheme is equal to 1. Since a tripartite W-type entangled state, which can be prepared straightforwardly by one time atom-cavity-field interaction, is used as the quantum channel, our scheme is more robust against decoherence and easier to implement than the schemes using the GHZ states. The cavities are only virtually excited during the passage of the atoms, and thus the dependence on the quality factor of the cavities is greatly reduced.

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