Vol 17 No 2, February 2008
1674-1056/2008/17(02)/0456-06
Chinese Physics B
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Entanglement of a two-qubit anisotropic Heisenberg XYZ chain in nonuniform magnetic fields with intrinsic decoherence*

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(Received 12 June 2007; revised manuscript received 6 July 2007)

Taking the intrinsic decoherence effect into account, this paper investigates the entanglement of a two-qubit anisotropic Heisenberg XYZ model in the presence of nonuniform external magnetic fields by employing the concurrence as entanglement measure. It is found that both the intrinsic decoherence and the anisotropy of the system give a significant suppression to the entanglement. Moreover it finds that the initial state of the system plays an important role in the time evolution of the entanglement, which means that the entanglement of the system is independent of the nonuniformity and uniformity of the magnetic field when the system is in the initial state $|\Psi(0)\rangle = |00\rangle$ and $|\Psi'(0)\rangle = m|01\rangle + n|10\rangle$, respectively.

Keywords: Heisenberg XYZ chain, concurrence, magnetic field, intrinsic decoherence
PACC: 0367, 4250

1. Introduction

Quantum entanglement, as one of the most striking features in quantum mechanics, has been intensively studied for its potential applications in quantum communication and quantum computation such as quantum teleportation,[1,2] superdense coding,[3] and quantum key distribution,[4] etc. A great deal of schemes have been proposed for the generation of the entangled states in various kinds of physical systems including electron spins,[5,6] quantum dots,[7−11] nuclear spins,[12] Cooper pairs,[13] and so on. As a simple but realistic solid-state system, the Heisenberg spin chain seems to be particularly promising among the various realizations of qubits in quantum information processing because of its scalability and flexibility.

Since a magnetic field is a powerful means to control the entanglement in spin systems, much attention has been devoted to examining the entanglement with various quantum Heisenberg chains in the presence of magnetic fields.[14−18] Arnesen *et al* revealed the thermal entanglement of spins in an isotropic Heisenberg chain with an external magnetic field in thermal equilibrium.[19] However, the isotropic interaction of spin chain is only an approximation while in real systems it is usually broken for many reasons: for instance, the spin–orbit coupling may introduce perturbations. As a result, the anisotropic effect on the entanglement of a two-qubit Heisenberg XY chain was discussed,[20,21] and the thermal entanglement with the completely anisotropic Heisenberg XYZ chain in the presence of a uniform magnetic field was also investigated.[22]

On the other hand, from the practical point of views, the decoherence of the entanglement for a quantum system is sometimes inevitable. Therefore, taking decoherence into account is indispensable, and the discussion on the entanglement of Heisenberg chain with intrinsic decoherence is also made by many authors. In Ref.[23], Li and Xu studied the entanglement of a three-qubit Heisenberg XY chain with intrinsic decoherence. It is shown that the nearest neighbour pairwise entanglement rapidly decay due to intrinsic decoherence while next neighbour qubits finally stray in an entangled stationary state in the case without the magnetic impurity. The same authors also investigated the entanglement of a multi-qubit quantum Heisenberg XY chain with phase decoherence, and found that the nearest neighbour pairwise entangle-

*Project supported by the State Key Development Program for Basic Research of China (Grant No 2006CB921604) and the National Natural Science Foundation of China (Grant Nos 60708003, 60578050 and 10434060).
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ment of XY ring with odd number length is more robust against phase decoherence than that with even number length.[24] Recently, He *et al* [25] studied the influence of intrinsic decoherence on quantum teleportation with different initial systems by calculating the average fidelity, and they discovered that one could adjust the magnetic field to reduce the different effects of intrinsic decoherence.

Note that most of the schemes proposed in Heisenberg system are considered in a uniform magnetic field. However, typically, the solid-state heterostructures which realize qubits are inhomogeneous, and magnetic imperfections or impurities are also likely to be present, giving rise to stray magnetic fields. Consequently, studying the entanglement of the qubits in the presence of nonuniform magnetic fields also ought to be necessary. More recently, Hu *et al* [26] discussed the entanglement of an anisotropic Heisenberg XYZ model in a nonuniform magnetic field. It is shown that the introduction of the nonuniformity in the magnetic field reduces the high-concurrence region of the model significantly.

On the basis of above analysis, in this paper we use the concurrence to study the entanglement of a two-qubit anisotropic Heisenberg XYZ chain with intrinsic decoherence in the presence of nonuniform magnetic fields. We find that the anisotropy and the intrinsic decoherence greatly suppress the entanglement of the system. Furthermore, the time evolution of the entanglement varies with different initial states of the system, the uniformity and nonuniform component of the magnetic field play different roles in the time evolution of the entanglement of the system.

2. Theoretic model

We consider the situation that a two-qubit anisotropic Heisenberg XYZ chain is in the presence of nonuniform external magnetic fields. The Hamiltonian for the system can be described as

$$\begin{aligned} H =& J_x\sigma_x \otimes \sigma_x + J_y\sigma_y \otimes \sigma_y + J_z\sigma_z \otimes \sigma_z \\ &+ (B+b)\sigma_z \otimes I + (B-b)I \otimes \sigma_z, \end{aligned} \tag{1}$$

where σ_i $(i = x,\ y,\ z)$ are the Pauli matrices, J_i $(i = x,\ y,\ z)$ are the interaction coefficients which take care of both the ferro- and anti-ferromagnetic coupling between two spin degrees of freedoms, respectively, and the difference between J_x and J_y measures the anisotropy of the system. The external magnetic fields are assumed to be along the z-direction, B describes the uniformity of the field while b measures the degree of the nonuniformity of the field, and I denotes the identity matrix.

In the space expanded by $\{|11\rangle,\ |10\rangle,\ |01\rangle, |00\rangle\}$, the eigenstates $|\Phi_j\rangle$ $(j = 1, 2, 3, 4)$ of the Hamiltonian and the corresponding eigenvalues E_j can be derived as

$$\begin{aligned} |\Phi_1\rangle &= \eta_+ |11\rangle + \epsilon_+ |00\rangle, E_1 = J_z + \delta, \\ |\Phi_2\rangle &= \eta_- |11\rangle + \epsilon_- |00\rangle, E_2 = J_z - \delta, \\ |\Phi_3\rangle &= \nu_+ |10\rangle + \xi_+ |01\rangle, E_3 = \chi - J_z, \\ |\Phi_4\rangle &= \nu_- |10\rangle + \xi_- |01\rangle, E_4 = -\chi - J_z, \end{aligned} \tag{2}$$

where the parameters

$$\begin{aligned} \eta_\pm &= \pm \frac{1}{\sqrt{2}} \frac{2B \pm \delta}{\sqrt{\delta^2 \pm 2B\delta}}, \\ \epsilon_\pm &= \pm \frac{1}{\sqrt{2}} \frac{J_x - J_y}{\sqrt{\delta^2 \pm 2B\delta}}, \\ \nu_\pm &= \pm \frac{1}{\sqrt{2}} \frac{\chi \pm 2b}{\sqrt{\chi^2 \pm 2b\chi}}, \\ \xi_\pm &= \frac{1}{\sqrt{2}} \frac{J_x + J_y}{\sqrt{\chi^2 \pm 2b\chi}}, \end{aligned} \tag{3}$$

in which $\delta = \sqrt{4B^2 + (J_x - J_y)^2}$ and $\chi = \sqrt{4b^2 + (J_x + J_y)^2}$. It is easy to find that Eqs.(2) and (3) reduce to the results reported in Ref.[27] when the coupling constants are taken to be isotropic $J_x = J_y = J_z$.

3. Influence of intrinsic decoherence on the entanglement of the model

Taking the intrinsic decoherence into account, we now study the entanglement of a two-qubit anisotropic Heisenberg XYZ model. Generally, the standard way to deal with decoherence is to consider the system of interest as a part of a larger closed system involving the surrounding environment. In Ref.[28], Milburn proposed a simple modification of Schrödinger equation which contains a term responsible for decay of quantum coherence in the energy eigenstate basis based on an assumption that for sufficiently short time steps the system does not evolve continuously under unitary transformation. According to this assumption, the decay is entirely of phase decoherence, and the time evolution of density matrix in Milburn's

model can be expressed as

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t)=\frac{1}{\gamma}\left\{\exp\left[-\mathrm{i}\gamma H\right]\rho(t)\exp\left[\mathrm{i}\gamma H\right]-\rho(t)\right\}, \quad (4)$$

where γ is the phase decoherence rate. Under Markovian approximation, the master equation governing the time evolution for the system can be expanded as

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t}=-\mathrm{i}\left[H,\rho(t)\right]-\frac{\gamma}{2}\left[H,\left[H,\rho(t)\right]\right]. \quad (5)$$

Then the formal solution of Eq.(5) can be expressed as[29−31]

$$\rho(t)=\sum_{k=0}^{\infty}\frac{(\gamma t)^k}{k!}M^k(t)\,\rho(0)\,M^{\dagger k}(t), \quad (6)$$

where $\rho(0)$ is the density operator of the initial system, M^k is defined as

$$M^k=H^k\exp(-\mathrm{i}Ht)\exp\left(-\frac{\gamma t}{2}H^2\right). \quad (7)$$

Inserting the complete relation $\sum_n |\Phi_n\rangle\langle\Phi_n|$ into Eq.(6), we get the time-dependent density matrix

$$\begin{aligned}\rho(t)=&\sum_{mn}\exp\left[-\frac{\gamma t}{2}(E_m-E_n)^2-\mathrm{i}t(E_m-E_n)\right]\\&\times\langle\Phi_m|\rho(0)|\Phi_n\rangle|\Phi_m\rangle\langle\Phi_n|, \quad (8)\end{aligned}$$

where $E_{m,n}$ and $\Phi_{m,n}$ (m, $n=1$, 2, 3, 4) are the eigenvalues and corresponding eigenvectors of the Hamiltonian given in Eq.(1), respectively.

To quantify the value of entanglement between two qubits, we adopt the concurrence C which is defined as[32]

$$C=\max\left\{\lambda_1-\lambda_2-\lambda_3-\lambda_4,0\right\}, \quad (9)$$

where λ_1, λ_2, λ_3, λ_4 are the positive square roots of the eigenvalues in descending order of the matrix

$$R=\rho\left(\sigma_y\otimes\sigma_y\right)\rho^*\left(\sigma_y\otimes\sigma_y\right), \quad (10)$$

in which ρ is the density operator, and the symbol * indicates the complex conjugation. The concurrence can be calculated no matter whether ρ is a pure or mixed state which varied from $C=0$ for a separate state to $C=1.0$ for a maximally entangled state.

(a) Assumed that the qubits 1 and 2 are both initially in the spin-down states, namely, the system is in an unentangled state $|\Psi(0)\rangle=|00\rangle$ at the beginning. By a direct calculation from Eqs.(2), (3) and (8), the time evolution of the density matrix of the system on the standard basis $\{|11\rangle,|10\rangle,|01\rangle,|00\rangle\}$ can be expressed as

$$\begin{aligned}\rho_1(t)=&\alpha|11\rangle\langle11|+\mu|11\rangle\langle00|\\&+\mu^*|00\rangle\langle11|+\beta|00\rangle\langle00|, \quad (11)\end{aligned}$$

where

$$\begin{aligned}\alpha=&1-\frac{1}{2\delta^2}\left(\delta^2-4B^2\right)\left[1-\cos(2\delta t)\exp\left(-2\delta^2\gamma t\right)\right],\\\mu=&\frac{1}{\delta^2}(J_x-J_y)\left\{B-\frac{1}{2}\left[2B\cos(2\delta t)-\mathrm{i}\delta\sin(2\delta t)\right]\exp\left(-2\delta^2\gamma t\right)\right\},\\\beta=&\frac{1}{2\delta^2}(J_x-J_y)^2\left[1-\cos(2\delta t)\exp\left(-2\delta^2\gamma t\right)\right]. \quad (12)\end{aligned}$$

In the above equations, the parameter δ is the same as that in Eq.(3).

From Eqs.(11) and (12) we can clearly see that the time-dependent density operator of the system is independent of the nonuniform magnetic field b. Next, we investigate the influence of anisotropy and intrinsic decoherence on the concurrence of the system. In Fig.1 we depict the concurrence as a function of time t for different anisotropy (by varying the interaction coefficients J_y while setting $J_x=2.0$) with different decoherence rate γ. We can see that the concurrence oscillates and collapses with the evolution of time t, and the increasing anisotropy leads to a more considerable suppression of the oscillation of the concurrence. Comparing Fig.1(a) with Fig.1(b), we find that the intrinsic decoherence of the system also gives an evident suppression on the oscillation, and the concurrence collapses to zero more quickly with the increase of γ.

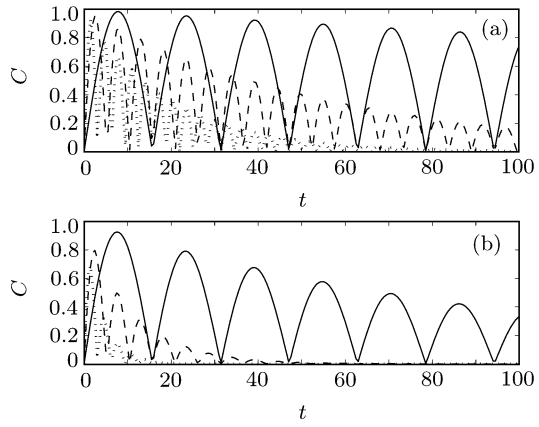


Fig.1. The concurrence as a function of time t for different anisotropy and different decoherence rate, where solid line corresponds to $J_y = 1.9$, dash line to $J_y = 1.7$, and dot line to $J_y = 1.5$. The parameters $B = 0$, $J_x = 2.0$, and the decoherence rate (a) $\gamma = 0.1$; (b) $\gamma = 0.5$.

In what follows, we will give the intuitional description of the influence of intrinsic decoherence on the concurrence in terms of three-dimensional diagram without (for Fig.2(a)) or with (for Fig.2(b)) the uniform magnetic field B. From Fig.2(a), for $B = 0$, it is easy to find that the value of the concurrence also oscillates with time t and decreases with the increase of the decoherence parameter γ. As is shown in Fig.2(b), for $B = 0.1$, the extreme value of the concurrence first decreases, and then quickly approaches to a stable constant value with the increasing γ at a fixed time. This indicates that entanglement can be effectively controlled by external magnetic fields, and the effect of intrinsic decoherence may be weakened by the introduction of the uniform field B.

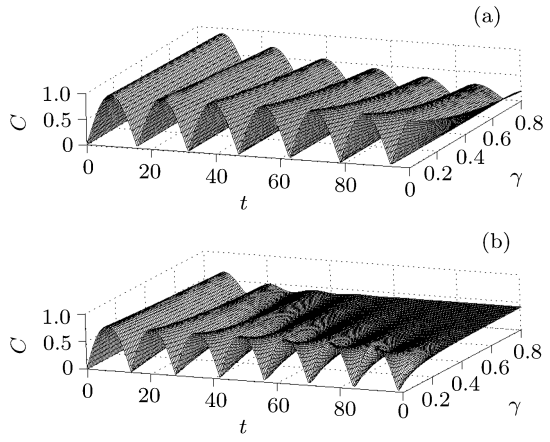


Fig.2. The concurrence as a function of time t and decoherence rate γ with different magnetic fields for $J_x = 2.0$ and $J_y = 1.9$, where (a) $B = 0$; (b) $B = 0.1$.

(b) Assumed that the system is initially in an entangled state $|\Psi'(0)\rangle = m|10\rangle + n|01\rangle$, where m and n are real parameters satisfying $m^2 + n^2 = 1$. Then the time evolution of the density matrix of the system on the standard basis $\{|11\rangle, |10\rangle, |01\rangle, |00\rangle\}$ can be given by

$$\begin{aligned}\rho_2(t) =& \varepsilon |10\rangle\langle 10| + \omega |10\rangle\langle 01| \\ &+ \omega^* |01\rangle\langle 10| + \zeta |01\rangle\langle 01|, \end{aligned} \tag{13}$$

where

$$\begin{aligned}
\varepsilon =& \frac{1}{2\chi^2}\left\{[n(J_x+J_y)+2mb]^2 + m^2\chi^2 - \left[[n(J_x+J_y)+2mb]^2 - m^2\chi^2\right]\cos(2\chi t)\exp\left(-2\chi^2\gamma t\right)\right\}, \\
\omega =& \frac{1}{\chi^2(\chi^2-4b^2)}(J_x+J_y)\{m\chi^2[n(J_x+J_y)+2mb] - b[n(J_x+J_y)+2mb]^2 - m^2\chi^2 b \\
&+ \frac{1}{2}\left[[n(J_x+J_y)+2mb]^2 - m^2\chi^2\right][2b\cos(2\chi t) - \mathrm{i}\chi\sin(2\chi t)]\exp\left(-2\chi^2\gamma t\right)\}, \\
\zeta =& \frac{1}{4\chi^2}(J_x+J_y)^2\{\frac{1}{(\chi+2b)^2}[n(J_x+J_y)+2mb+m\chi]^2 + \frac{1}{(\chi-2b)^2}[n(J_x+J_y)+2mb-m\chi]^2 \\
&+ \frac{2}{(\chi^2-4b^2)}\left[[n(J_x+J_y)+2mb]^2 - m^2\chi^2\right]\cos(2\chi t)\exp\left(-2\chi^2\gamma t\right)\}.
\end{aligned} \tag{14}$$

From Eqs.(13) and (14), one can see that the uniform magnetic field B has no effect on the concurrence of the system, which is different from the case (a) where the initial state of the system is in a separable state. To study the effect of the nonuniform part of the field on the concurrence, in Fig.3 we plot the concurrence versus time t and the field b for different initial systems. From the figure, we find that the concurrence decays monotonically with the increasing b for the case of $|\Psi'(0)\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$, while it first increases sharply, then decreases after a peak value at a given value of b ($b \approx 2.0$) for the $|\Psi'(0)\rangle = |01\rangle$ case. In addition, if there is no nonuniform field ($b = 0$),

the concurrence in Fig.3(a) keeps to unity while that in Fig.3(b) first fluctuates, then decays to zero and there exists a peak-value 0.5 at the time $t \approx 0.2$ during the fluctuation of the concurrence. This trend of concurrence can be explained from Eqs.(13) and (14), when $b = 0$, the coefficients ε, ω, ζ in Eq.(14) for the $|\Psi'(0)\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$ case will respectively become $\varepsilon = \omega = \zeta = 1/2$. In this situation, the effects of both the intrinsic decoherence and anisotropy on the concurrence disappear and the concurrence is time-independent. Moreover, when $b = 0$, the corresponding coefficients for the $|\Psi'(0)\rangle = |01\rangle$ case will respectively be

$$\varepsilon = \frac{1}{2}\{1 - \cos[2(J_x + J_y)t]\exp[-2(J_x + J_y)^2\gamma t]\},$$

$$\omega = -\frac{\mathrm{i}}{2}\sin[2(J_x + J_y)t]\exp[-2(J_x + J_y)^2\gamma t]$$

and

$$\zeta = \frac{1}{2}\{1 + \cos[2(J_x + J_y)t]\exp[-2(J_x + J_y)^2\gamma t]\},$$

which means that the concurrence is dependent on time t. The above analyses show that the nonuniform field b plays different roles in the evolutions of the concurrence of the systems for different initial states.

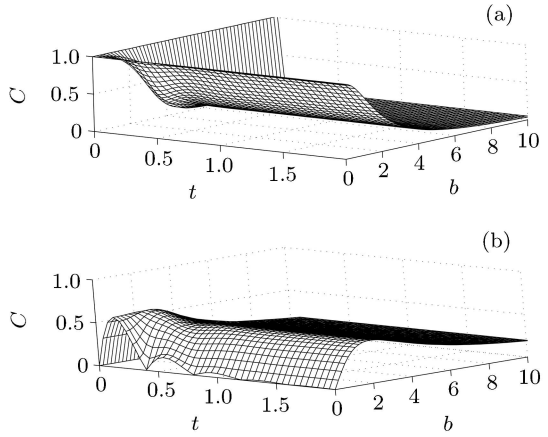


Fig.3. The concurrence as a function of time t and the nonuniform magnetic field b for different initial states, where (a) $|\Psi'(0)\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$; (b) $|\Psi'(0)\rangle = |01\rangle$. The parameters $J_x = 2.0$, $J_y = 1.9$, $\gamma = 0.1$.

The concurrence with intrinsic decoherence as a function of time t is plotted in Fig.4, where the initial state of system and the value of decoherence rate are varied. From the figure, we can observe that the intrinsic decoherence of the system reduces the high-peak region of the short-time oscillation of the concurrence at first, and the concurrence of the systems will remain in a stable value for the long-time case. Furthermore, it is found that no matter what the value of the decoherence rate is, the concurrence of the system with the same initial state will arrive at a steady-going nonzero value in the end. In fact, this phenomenon is observable for the systems in both case (a) and case (b), which can also be seen from Eqs.(11), (12), (13) and (14) that if time t approaches to infinity, the density operators of the systems will be time-independent and the effect of the decoherence rate γ on the concurrence will disappear. In addition, from Fig.4 we can see that after the parameters J_x, J_y and γ are given, the system will finally stay in a higher concurrence during the whole evolution process for the initial larger concurrence of the two qubits, and vice versa.

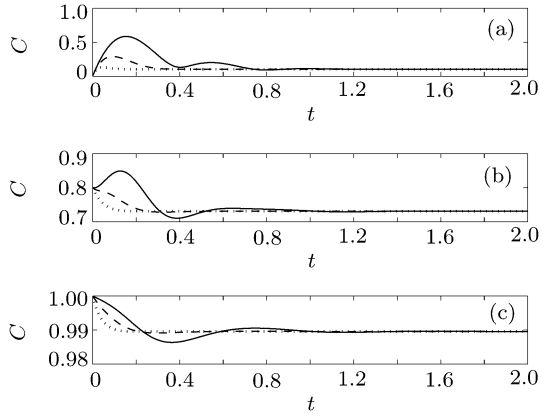


Fig.4. The concurrence as a function of time t for different initial states and different decoherence rate γ, where the solid line corresponds to $\gamma = 0.1$, dash line to $\gamma = 0.3$, dot line to $\gamma = 0.8$, and (a) $|\Psi'(0)\rangle = |01\rangle$; (b) $|\Psi'(0)\rangle = \frac{1}{\sqrt{5}}(|10\rangle + 2|01\rangle)$; (c) $|\Psi'(0)\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$. The other parameters $J_x = 2.0$, $J_y = 1.9$, $b = 0.2$.

4. Conclusions

In summary, we have studied the entanglement of a two-qubit anisotropic Heisenberg XYZ chain in nonuniform external magnetic fields with intrinsic decoherence by calculating the concurrence. Our results show that both the anisotropy of the system and the intrinsic decoherence have impacts on the entanglement. If the system is initially in the separable state $|\Psi(0)\rangle = |00\rangle$, the uniform field B can weaken the destruction of the entanglement suffered by the intrinsic decoherence while the nonuniform field b has no effect on the entanglement. Moreover, if the system is initially in the entangled state $|\Psi'(0)\rangle = m|10\rangle + n|01\rangle$, the nonuniformity of the field plays an important role

in the time evolution of the concurrence while the uniform component of the field has no impact on the entanglement. Besides, it is found that the initial concurrence of the two qubits is of great significance on the time evolution of the entanglement of the system. Under the condition of a fixed set of the parameters J_x, J_y and γ, the larger the concurrence that the initial system contains, the higher the concurrence that the system will retain at last.

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