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# Scheme for teleportation of unknown states of trapped ion*

Chen Mei-Feng(陈美锋)† and Ma Song-She(马宋设)

*Department of Electronic Science and Applied Physics, Fuzhou University, Fuzhou* 350002, *China*

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A scheme is presented for teleporting an unknown state in a trapped ion system. The scheme only requires a single laser beam. It allows the trap to be in any state with a few phonons, e.g. a thermal motion. Furthermore, it works in the regime, where the Rabi frequency of the laser is on the order of the trap frequency. Thus, the teleportation speed is greatly increased, which is important for decreasing the decoherence effect. This idea can also be used to teleport an unknown ionic entangled state.

**Keywords:** teleportation, trapped ion, Rabi frequency
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## 1. Introduction

Entanglement is one of the striking features of quantum mechanics. Entanglement is not only fundamental for demonstrating quantum nonlocality, but also useful in quantum information theory, such as quantum cryptographic conference[1] and quantum secret sharing.[2] In recent years, with the development of quantum information, much attention has been paid to entanglement.[3−5] Multi-photon entanglement has been observed.[6] Entangled states for three or more massive particles have been experimentally realized in microwave cavity quantum electrodynamics (QED)[7] and trapped ion systems.[8]

Quantum teleportation is one of the most important applications of quantum entanglement. In quantum teleportation process, unknown quantum information can be transmitted from a sender to a receiver without the transmission of the carrier of the quantum information.[9] In the original scheme for quantum teleportation, the sender Alice and the receiver Bob share a maximally entangled state quantum channel. To realize the teleportation, Alice must operate a joint measurement on the particle that carries the unknown quantum information and one of the entangled particles she possesses. Then she will inform Bob her measurement result. Finally, Bob will operate a single particle transformation on the particle he possesses to transform the state into the unknown state to be teleported based on the sender's measurement result.

Since the first quantum teleporting protocol was suggested, there has been rapid progress in quantum teleportation. Experimental realizations of quantum teleportation have been reported in which optical systems[10] and nuclear magnetic resonance[11] were used. Recently, Riebe *et al* and Barrett *et al* have experimentally implemented the teleportation of the states of $Ca^+$ [12] and $Be^+$ [13] respectively. The two schemes describe not only the first experimental realizations of teleportation with atoms, but also the techniques of manipulating and transforming quantum states in an ion-trap system. This will attract much more attention for quantum information processing in the field of cavity QED and trapped ion system. Many theoretical schemes have been proposed for teleporting unknown states in cavity QED system and trapped ion system.[14−16] Zheng [14] presented a simple scheme to teleport atomic states within cavities with a success probability of 1/2. The scheme is insensitive to the cavity field states and cavity decay. Ye *et al*[15] also presented a scheme to teleport an unknown atomic state in cavity QED with a success probability of 1/2, and they generalized their scheme to teleport an unknown atomic entangled state. Yang *et al*[16] teleported an unknown atomic state in cavity QED

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†E-mail: meifchen@126.com

with a success probability of 1. In the teleportation processes the atoms interact with a single-mode non-resonant cavity with the assistance of a strong classical driving field.

Teleportation with trapped ions has an important advantage: trapped ion is a long-live quantum system. Solano *et al*[17] have made a proposal for teleporting an unknown ionic state between ions kept in two well-separated traps by using a two-ion entangled state as the quantum channel. Recently, Zheng *et al*[18] proposed a scheme to teleport an unknown ionic state by entanglement swapping. More recently, Zheng *et al*[19] proposed another entanglement swapping scheme to teleport an ionic entangled state. These schemes all require that the ions are illuminated by two classical homogeneous lasers simultaneously. They work in the weak-excitation regime, in which the Rabi frequency of the lasers should be much smaller than the trap frequency.

In this paper, we propose a scheme for teleporting an unknown state in a trapped ion system. The scheme only requires a single laser beam. It allows the trap to be in any state with a few phonons, e.g. a thermal motion. Furthermore, it works in the regime, where the Rabi frequency of the laser is on the order of the trap frequency. Thus, the teleportation speed is greatly increased, which is important for decreasing the decoherence effect. We also use our scheme to teleport an unknown ionic entangled state.

## 2. Teleportation of an unknown state of trapped ion

We consider two two-level ions in a linear trap. We drive simultaneously the two ions with a laser beam tuned to the ion transition. In the rotating-wave approximation, the Hamiltonian for this system is given by (assuming $\eta = 1$)[20,21]

$$H = \nu a^{+}a + \omega_0 \sum_{j=1,2} \sigma_{zj} + \frac{\Omega}{2} \sum_{j=1,2} \{\mathrm{e}^{-\mathrm{i}[\omega_0 t - \eta(a+a^{+})]}\sigma_j^{+} + \mathrm{H.c.}\}\,, \quad (1)$$

where $\sigma_{zj} = (|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|)/2$ , $\sigma_j^{+} = |e_j\rangle\langle g_j|$ and $\sigma_j^{-} = |g_j\rangle\langle e_j|$, with $|e_j\rangle$ and $|g_j\rangle$ being the excited and ground states of the $j$th ion, $a^{+}$ and $a$ are the creation and annihilation operators for the centre-of-mass mode with vibrational frequency being $\nu$, $\omega_0$ is the transition frequency for the two-level ion, and $\Omega$ is the Rabi frequency of the laser field. $\eta = k/\sqrt{2\nu M}$ is the Lamb–Dicke parameter with $k$ being the wave vector along the trap axis and $M$ being the mass of the ion collection. Consider the behaviour of the trapped ions in the Lamb-Dick regime (i.e. $\eta\sqrt{\bar{n}+1} \ll 1$), with $\bar{n}$ being the phonon number of the centre-of-mass mode. In the interaction picture the interaction Hamiltonian is given by

$$H_i = \frac{\Omega}{2} \sum_{j=1,2} \sigma_j^{+}[1 + \mathrm{i}\eta(a^{+}\mathrm{e}^{\mathrm{i}\nu t} + \mathrm{H.c.})] + \mathrm{H.c.}\,. \quad (2)$$

Define the new atomic basis as

$$|+_j\rangle = \frac{1}{\sqrt{2}}(|e_j\rangle + |g_j\rangle),$$
$$|-_j\rangle = \frac{1}{\sqrt{2}}(|e_j\rangle - |g_j\rangle). \quad (3)$$

Then $H_i$ can be rewritten as

$$H_i = \Omega \sum_{j=1,2} \left[S_{z,j} + \frac{1}{2}(S_j^{-} - S_j^{+})\mathrm{i}\eta(a^{+}\mathrm{e}^{\mathrm{i}\nu t} + \mathrm{H.c.})\right], \quad (4)$$

where $S_{z,j} = (|+_j\rangle\langle+_j| - |-_j\rangle\langle-_j|)/2$, $S_j^{+} = |+_j\rangle\langle-_j|$ and $S_j^{-} = |-_j\rangle\langle+_j|$.

The time evolution of this system is determined by Schrödinger's equation

$$\mathrm{i}\frac{\mathrm{d}|\psi(t)\rangle}{\mathrm{d}t} = H_i|\psi(t)\rangle\,. \quad (5)$$

Perform the unitary transformation

$$|\psi(t)\rangle = \mathrm{e}^{-\mathrm{i}H_0 t}|\psi'(t)\rangle\,, \quad (6)$$

with

$$H_0 = \Omega \sum_{j=1,2} S_{z,j}\,. \quad (7)$$

Then we obtain

$$\mathrm{i}\frac{\mathrm{d}|\psi'(t)\rangle}{\mathrm{d}t} = H_i'|\psi'(t)\rangle\,, \quad (8)$$

where

$$H_i' = \mathrm{i}\eta\frac{\Omega}{2} \sum_{j=1,2} (S_j^{-}\mathrm{e}^{-\mathrm{i}\Omega t} - S_j^{+}\mathrm{e}^{\mathrm{i}\Omega t}) \times (a^{+}\mathrm{e}^{\mathrm{i}\nu t} + \mathrm{H.c.}). \quad (9)$$

Assuming $\nu + \Omega \gg \nu - \Omega = \delta$, $H_i'$ reduces to

$$H_i' = \mathrm{i}\eta\frac{\Omega}{2}\sum_{j=1,2}(S_j^- a^+ \mathrm{e}^{\mathrm{i}\delta t} - S_j^+ a\mathrm{e}^{-\mathrm{i}\delta t})\,. \tag{10}$$

In the case $\delta \gg \eta(\Omega/2)\sqrt{\bar{n}+1}$, there is no energy exchange between the external and internal degrees of freedom. The energy conserving transitions run between $|+_j\,-_k\,n\rangle$ and $|-_j\,+_k\,n\rangle$. The effective Rabi frequency $\lambda$ for the transitions between these states, mediated by $|-_j\,-_k\,n+1\rangle$ and $|+_j\,+_k\,n-1\rangle$, is given by

$$\begin{aligned}\lambda =& \frac{\langle +_j\,-_k\,n|\,H_i'\,|-_j\,-_k\,n+1\rangle\,\langle -_j\,-_k\,n+1|\,H_i'\,|-_j\,+_k\,n\rangle}{-\delta}\\ &+\frac{\langle +_j\,-_k\,n|\,H_i'\,|+_j\,+_k\,n-1\rangle\,\langle +_j\,+_k\,n-1|\,H_i'\,|-_j\,+_k\,n\rangle}{\delta}\\ =& \frac{(\eta\Omega)^2}{4\delta}.\end{aligned} \tag{11}$$

Since the two transition paths interfere destructively, the Rabi frequency is independent of the phonon-number. In addition, the Stark shifts for the states $|+_j n\rangle$ and $|-_j n\rangle$ are as follows respectively:

$$\begin{aligned}&\frac{\langle +_j n|\,H_i'\,|-_j n+1\rangle\,\langle -_j n+1|\,H_i'\,|+_j n\rangle}{-\delta}\\ =&\ \lambda(n+1),\end{aligned} \tag{12}$$

$$\begin{aligned}&\frac{\langle -_j n|\,H_i'\,|+_j n-1\rangle\,\langle +_j n-1|\,H_i'\,|-_j n\rangle}{\delta}\\ =&\ -\lambda n.\end{aligned} \tag{13}$$

Then the effective Hamiltonian can be written as

$$\begin{aligned}H_\mathrm{e} =& \lambda\sum_{j=1,2}(|+_j\rangle\langle +_j|\,aa^+ - |-_j\rangle\langle -_j|\,a^+a)\\ &+\lambda(S_1^+S_2^- + S_1^-S_2^+).\end{aligned} \tag{14}$$

The first and second terms describe the phonon-number dependent Stark shifts, the third and the fourth terms describe the coupling between the first and the second ions. If define the number of excited atoms as

$$N_\mathrm{e} = \sum_{j=1}^{2}|+_j\rangle\langle +_j|\,, \tag{15}$$

then $[H_\mathrm{e}, a^+a] = 0$ and $[H_\mathrm{e}, N_\mathrm{e}] = 0$, so the number of phonons and the number of excited ions remain constant during the interaction between the ions and the laser. Using Eqs.(6), (7) and (14), we can obtain the following evolutions:

$$\begin{aligned}|+-\rangle_{12} \to\ & \mathrm{e}^{-\mathrm{i}\lambda t}[\cos(\lambda t)\,|+-\rangle_{12}\\ &-\mathrm{i}\sin(\lambda t)\,|-+\rangle_{12}],\end{aligned} \tag{16}$$

$$\begin{aligned}|-+\rangle_{12} \to\ & \mathrm{e}^{-\mathrm{i}\lambda t}[\cos(\lambda t)\,|-+\rangle_{12}\\ &-\mathrm{i}\sin(\lambda t)\,|+-\rangle_{12}].\end{aligned} \tag{17}$$

Note that the above evolutions are all independent of the phonon number, thus the scheme allows the trap to be in any state with a few phonons, e.g. a thermal motion.

Now we discuss how to teleport an unknown ionic state. Assume that the unknown state to be teleported is as follows:

$$|\psi\rangle_1 = \alpha\,|+\rangle_1 + \beta\,|-\rangle_1\,, \tag{18}$$

where $\alpha$ and $\beta$ are unknown coefficients, $|\alpha|^2 + |\beta|^2 = 1$.

In the first step, we prepare a quantum channel to teleport the unknown state. Assume two ions are initially in the state $|+-\rangle_{23}$ and the vibrational mode is in the thermal state. The two ions interact simultaneously with a laser for a time $t$. With the choice of $\lambda t = \pi/4$, we obtain the maximally two-ion entangled state

$$|\psi\rangle_{23} = \frac{1}{\sqrt{2}}(|+-\rangle_{23} - \mathrm{i}\,|-+\rangle_{23}), \tag{19}$$

where we have discarded the common phase factor. Equation (19) is the quantum channel we shall use to teleport the unknown state. The ion 2 belongs to Alice, the ion 3 belongs to Bob. Then the state of the whole system is

$$\begin{aligned}&|\psi\rangle_{123}\\ =&\ \frac{1}{\sqrt{2}}(|+-\rangle_{23} - \mathrm{i}\,|-+\rangle_{23})(\alpha\,|+\rangle_1 + \beta\,|-\rangle_1)\\ =&\ \frac{1}{2}\left[\left|\varphi^+\right\rangle(\alpha\,|+\rangle_3 + \beta\,|-\rangle_3) + \left|\varphi^-\right\rangle(\alpha\,|+\rangle_3\right.\\ &-\beta|-\rangle_3) + \left|\phi^+\right\rangle(\alpha\,|-\rangle_3 - \beta\,|+\rangle_3) + \left|\phi^-\right\rangle\\ &\left.\times(\alpha\,|-\rangle_3 + \beta\,|+\rangle_3)\right],\end{aligned} \tag{20}$$

where $|\varphi^{\pm}\rangle$ and $|\phi^{\pm}\rangle$ are the Bell states

$$|\varphi^{\pm}\rangle = \frac{1}{\sqrt{2}}(-\mathrm{i}\,|+-\rangle_{12} \pm |-+\rangle_{12}), \tag{21}$$

$$|\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|++\rangle_{12} \pm \mathrm{i}\,|--\rangle_{12}). \tag{22}$$

In the second step, Alice makes a measurement of the above Bell states. Alice confines the ion 1 and ion 2 in a linear trap. She excites simultaneously the ion 1 and ion 2 with a laser as described above. By selecting the interaction time to satisfy $\lambda t = \pi/4$ and discarding the common phase factor, the Bell states $|\varphi^{\pm}\rangle$ evolve into

$$|\varphi^{+}\rangle \to -\mathrm{i}\,|+-\rangle_{12}\,, \tag{23}$$

$$|\varphi^{-}\rangle \to -\,|-+\rangle_{12}\,. \tag{24}$$

On the other hand, $|\phi^{\pm}\rangle$ involve two terms $|++\rangle_{12}$ and $|--\rangle_{12}$, which do not undergo transition but impart shifts during the interaction. Then Alice detects the ion 1 and ion 2 separately. If Alice obtains an outcome $|+-\rangle_{12}$, she can tell Bob that ion 3 has been prepared in the initial state of ion 1. If Alice obtains $|-+\rangle_{12}$, she tells Bob to perform a transformation on ion 3. If Alice obtains the outcome $|++\rangle_{12}$ or $|--\rangle_{12}$, the scheme fails. Therefore, the present scheme is a probabilistic one with a successful probability of 0.5.

## 3. Teleportation of an unknown entangled state of trapped ion

The scheme can also be used to teleport an ionic entangled state. Assume Alice has an entangled ion pair, which consists of ion 1 and ion 2. She wants to teleport the unknown state $|\psi\rangle_{12}$ of the pair to Bob. The state $|\psi\rangle_{12}$ may be expressed as

$$|\psi\rangle_{12} = \alpha\,|-+\rangle_{12} + \beta\,|+-\rangle_{12}\,, \tag{25}$$

where $\alpha$ and $\beta$ are the unknown coefficients, $|\alpha|^2 + |\beta|^2 = 1$. The ion 3, ion 4 and ion 5 have been previously prepared in the maximally entangled GHZ state as the quantum channel.

$$|\psi\rangle_{345} \to \frac{1}{\sqrt{2}}(|+-+\rangle_{345} - \mathrm{i}\,|-++\rangle_{345}), \tag{26}$$

where the ion 4 and ion 5 belong to Bob and the ion 3 belongs to Alice. Then the initial state of the whole system is given by

$$\begin{aligned}|\psi\rangle_{12345} &= \frac{1}{\sqrt{2}}(|+-+\rangle_{345} - \mathrm{i}\,|-++\rangle_{345})(\alpha\,|-+\rangle_{12} + \beta\,|+-\rangle_{12}) \\ &= \frac{1}{2}[|\varphi^{+}\rangle\,(\alpha\,|-+\rangle_{45} - \beta\,|+-\rangle_{45}) + |\varphi^{-}\rangle\,(\alpha\,|-+\rangle_{45} + \beta\,|+-\rangle_{45}) \\ &\quad + |\phi^{+}\rangle\,(\beta\,|-+\rangle_{45} - \alpha\,|+-\rangle_{45}) + |\phi^{-}\rangle\,(\beta\,|-+\rangle_{45} + \alpha\,|+-\rangle)_{45}]\,,\end{aligned} \tag{27}$$

where $|\varphi^{\pm}\rangle$ and $|\phi^{\pm}\rangle$ are the Bell states

$$|\varphi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|-++\rangle_{123} \pm \mathrm{i}\,|+--\rangle_{123}), \tag{28}$$

$$|\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|+-+\rangle_{123} \pm \mathrm{i}\,|-+-\rangle_{123})\ . \tag{29}$$

To realize the teleportation, Alice confines the ion 1, ion 2 and ion 3 in a linear trap. Firstly, she excites simultaneously the ion 2 and ion 3 with a laser as described above. By selecting the interaction time to satisfy $\lambda t = \pi/4$ and discarding the common phase factor, the Bell states $|\phi^{\pm}\rangle$ evolve into

$$\begin{aligned}|\phi^{+}\rangle \to \frac{1}{\sqrt{2}}&(|+-+\rangle_{123} - \mathrm{i}\,|++-\rangle_{123} \\ &+\mathrm{i}\,|-+-\rangle_{123} + |--+\rangle_{123})\,,\end{aligned} \tag{30}$$

$$|\phi^{-}\rangle \to \frac{1}{\sqrt{2}}(|+-+\rangle_{123} - \mathrm{i}\,|++-\rangle_{123}$$

$$-\mathrm{i}\,|-+-\rangle_{123} - |--+\rangle_{123}). \tag{31}$$

On the other hand, $|\varphi^{\pm}\rangle$ involve two terms $|++\rangle_{23}$ and $|--\rangle_{23}$, which do not undergo transition but impart shifts during the interaction. Then Alice drives ion 1 with a laser tuned to ion transition.[22] After choosing the interaction time appropriately, the ion undergoes transition as follows:

$$|+\rangle_1 \to \frac{1}{\sqrt{2}}(|+\rangle_1 + |-\rangle_1), \tag{32}$$

$$|-\rangle_1 \to \frac{1}{\sqrt{2}}(|+\rangle_1 - |-\rangle_1\,)\,. \tag{33}$$

Finally, Alice obtains the evolutions

$$|\phi^{+}\rangle \to \frac{1}{\sqrt{2}}(|+-+\rangle_{123} - \mathrm{i}\,|-+-\rangle_{123}), \tag{34}$$

$$|\phi^{-}\rangle \to \frac{1}{\sqrt{2}}(|--+\rangle_{123} - \mathrm{i}\,|++-\rangle_{123}), \tag{35}$$

$$|\varphi^+\rangle \rightarrow \frac{1}{\sqrt{2}}(|+++\rangle_{123} - |-++\rangle_{123} + \mathrm{i}|+--\rangle_{123} - \mathrm{i}|---\rangle_{123}), \quad (36)$$

$$|\varphi^-\rangle \rightarrow \frac{1}{\sqrt{2}}(|+++\rangle_{123} - |-++\rangle_{123} - \mathrm{i}|+--\rangle_{123} + \mathrm{i}|---\rangle_{123}). \quad (37)$$

We can see that Alice cannot distinguish $|\varphi^+\rangle$ from $|\varphi^-\rangle$.

Now Alice measures the ion 1, ion 2 and ion 3 and sends her measurement result to Bob by a classical channel. If she detects her ions in the state $|+-+\rangle_{123}$ or $|-++\rangle_{123}$ or $|--+\rangle_{123}$ or $|++-\rangle_{123}$, the initial unknown state can be teleported to Bob after Bob makes a standard rotation on his ions 1 and 2. If Alice obtains another measurement result, the scheme fails. So the successful probability of teleporting an unknown entangled state is also 0.5.

# 4. Summary

In summary, we have presented a scheme for teleporting an unknown state and an unknown entangled state in a trapped ion system. The successful probability of the scheme is 0.5, which is smaller then the previous schemes.[17−19] However, our scheme has its advantages. Firstly, it only requires a single laser beam. Secondly, it allows the trap to be in any state with a few phonons, e.g. a thermal motion. Thirdly, it works in the regime, where the Rabi frequency of the laser is on the order of the trap frequency. Thus, the teleportation speed is greatly increased, which is important for decreasing the decoherence effect. We believe that our scheme can be realized in practice with the current technique.[12,13]

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