Vol 17 No 2, February 2008
1674-1056/2008/17(02)/0445-06
Chinese Physics B
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Scheme for probabilistic remotely preparing a multi-particle entangled GHZ state

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(Received 12 June 2007; revised manuscript received 9 July 2007)

This paper presents a scheme for probabilistic remote preparation of a three-particle entangled Greenberger–Horne–Zeilinger (GHZ) state via three-particle orthonormal basis projective measurement, and then directly generalize the scheme to multi-particle case. It is shown that by using N pairs of bipartite non-maximally entangled states as the quantum channel and N-particle orthonormal basis projective measurement, the multi-particle remote preparation can be successfully realized with a certain probability.

Keywords: N-particle entangled GHZ state, probabilistic remote state preparation, N-particle projective measurement, unitary transformation

PACC: 0367, 0365

1. Introduction

Quantum entanglement is one of the most striking features of quantum mechanics. Also multi-particle entanglement is a very important physical resource in quantum information protocols such as quantum teleportation,[1] quantum telecloning,[2] superdense coding,[3] quantum cryptography[4] and so on. Quantum teleportation, proposed by Bennett *et al*,[1] is the process that transmits an unknown two-state particle, or a qubit from a sender (Alice) to a receiver (Bob) via a quantum channel with the help of some classical information. In recent years many theoretical and experimental researches have extensive explored teleportation from all aspects.[5] Considering that an entangled state may be non-maximally entangled, Li *et al* [6] proposed a non-maximally entangled quantum channel, which indicates that teleportation of a single-particle state can be realized with a certain probability. Based on the work of Li *et al*,[6] several schemes of quantum probabilistic teleportation of three-particle and multi-particle states by using the partial entangled states as quantum channels were presented.[7−13]

Recently, Lo,[14] Pati,[15] and Bennett *et al*[16] have presented an interesting new subject, remote state preparation (RSP), in which Alice wishes to help Bob in his laboratory to prepare a quantum state by means of previously shared entanglement and classical communication. In RSP, Alice is assumed to know the state which is to be prepared remotely, while in quantum teleportation, neither Alice nor Bob knows the identity of the teleported state. Some authors[17−23] have investigated various theoretical protocols for generalization of RSP. Shi *et al* [17] have proposed a scheme to remotely prepare an N-particle entangled state by an $(N+1)$-particle maximally entangled state as the quantum channel and via a single-particle von Neumann measurement. Liu *et al* [18] have presented a scheme for preparing remotely a two-particle entangled state via two pairs of entangled particle as the quantum channel and by a two-particle projective measurement. Dai *et al* [23] have proposed a scheme for probabilistic remote preparation of a four-particle entangled state by using two three-particle partial entangled states as the quantum channel and via a two-particle projective measurement. Different from these protocols using a single-particle von Neumann orthogonal measurement or a two-particle projective measurement, recently, Liu *et al* [24] have proposed a scheme for preparing remotely a three-particle(or N-particle) GHZ state via a three-particle(or a N-particle) orthonormal basis measurement. But they considered only the maximally entan-

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gled quantum channel. In the present paper, we propose a scheme for probabilistic remote preparation of a three-particle GHZ state via three-particle orthonormal basis projective measurement. In this scheme, three non-maximally two-particle entangled states are employed as the quantum channel. Then we directly generalize it to the multi-particle case. It is shown that the RSP scheme can be successfully realized with certain probability if a receiver adopts some appropriate unitary transformations.

2. Preparing remotely a three-particle entangled GHZ state by three-pair non-maximally entangled particles

To present our scheme more clearly, let us first begin with probabilistic RSP of a three-particle entangled state. Suppose that the sender Alice wants to help the receiver Bob remotely prepare a tripartite entangled GHZ state $|\phi\rangle = \alpha|000\rangle + \beta|111\rangle$, where α is real and β is a complex number, $|\alpha|^2 + |\beta|^2 = 1$. We assume that Alice knows α and β completely, but Bob does not know them at all. We also suppose that three non-maximally two-particle entangled states shared by Alice and Bob are in the following states, respectively,

$$\begin{aligned}
&|\psi\rangle_{A_1B_1} = a_1|00\rangle_{A_1B_1} + b_1|11\rangle_{A_1B_1},\\
&(|a_1|^2 + |b_1|^2 = 1, |a_1| \le |b_1|),\\
&|\psi\rangle_{A_2B_2} = a_2|00\rangle_{A_2B_2} + b_2|11\rangle_{A_2B_2},\\
&(|a_2|^2 + |b_2|^2 = 1, |a_2| \le |b_2|),\\
&|\psi\rangle_{A_3B_3} = a_3|00\rangle_{A_3B_3} + b_3|11\rangle_{A_3B_3},\\
&(|a_3|^2 + |b_3|^2 = 1, |a_3| \le |b_3|). \qquad (1)
\end{aligned}$$

We assume that the parameters a_i and b_i $(i = 1, 2, 3)$ are real, and still assume Alice is in possession of particles A_1, A_2, and A_3 and Bob is in possession of particles B_1, B_2, and B_3. Since Alice knows exactly the parameters α and β of the state $|\phi\rangle$, she can choose to measure her three particles on whatever basis she wants. Now Alice carries out a three-particle projective measurement on the particle A_1, A_2, and A_3 in a set of mutually orthogonal basis vectors,

$$\begin{aligned}
|\varphi^0\rangle_{A_1A_2A_3} &= \alpha|000\rangle_{A_1A_2A_3} + \beta|111\rangle_{A_1A_2A_3},\\
|\varphi^0_\perp\rangle_{A_1A_2A_3} &= \beta^*|000\rangle_{A_1A_2A_3} - \alpha|111\rangle_{A_1A_2A_3},\\
|\varphi^1\rangle_{A_1A_2A_3} &= \alpha|001\rangle_{A_1A_2A_3} + \beta|110\rangle_{A_1A_2A_3},\\
|\varphi^1_\perp\rangle_{A_1A_2A_3} &= \beta^*|001\rangle_{A_1A_2A_3} - \alpha|110\rangle_{A_1A_2A_3},\\
|\varphi^2\rangle_{A_1A_2A_3} &= \alpha|010\rangle_{A_1A_2A_3} + \beta|101\rangle_{A_1A_2A_3},\\
|\varphi^2_\perp\rangle_{A_1A_2A_3} &= \beta^*|010\rangle_{A_1A_2A_3} - \alpha|101\rangle_{A_1A_2A_3},\\
|\varphi^3\rangle_{A_1A_2A_3} &= \alpha|011\rangle_{A_1A_2A_3} + \beta|100\rangle_{A_1A_2A_3},\\
|\varphi^3_\perp\rangle_{A_1A_2A_3} &= \beta^*|011\rangle_{A_1A_2A_3} - \alpha|100\rangle_{A_1A_2A_3}. \qquad (2)
\end{aligned}$$

The above non-maximally entangled states $\{|\varphi^l\rangle_{A_1A_2A_3}, |\varphi^l_\perp\rangle_{A_1A_2A_3}\}$ $(l = 0, 1, 2, 3)$ form a complete orthonormal basis in a quantum system of eight-dimensional Hilbert space. In this way, the three non-maximally EPR pairs shared previously by Alice and Bob can be rewritten as

$$\begin{aligned}
&|\psi\rangle_{A_1B_1} \otimes |\psi\rangle_{A_1B_1} \otimes |\psi\rangle_{A_1B_1}\\
&= |\varphi^0\rangle_{A_1A_2A_3}(\alpha a_1a_2a_3|000\rangle_{B_1B_2B_3} + \beta^* b_1b_2b_3|111\rangle_{B_1B_2B_3})\\
&\quad + |\varphi^0_\perp\rangle_{A_1A_2A_3}(\beta a_1a_2a_3|000\rangle_{B_1B_2B_3} - \alpha b_1b_2b_3|111\rangle_{B_1B_2B_3})\\
&\quad + |\varphi^1\rangle_{A_1A_2A_3}(\alpha a_1a_2b_3|001\rangle_{B_1B_2B_3} + \beta^* b_1b_2a_3|110\rangle_{B_1B_2B_3})\\
&\quad + |\varphi^1_\perp\rangle_{A_1A_2A_3}(\beta a_1a_2b_3|001\rangle_{B_1B_2B_3} - \alpha b_1b_2a_3|110\rangle_{B_1B_2B_3})\\
&\quad + |\varphi^2\rangle_{A_1A_2A_3}(\alpha a_1b_2a_3|010\rangle_{B_1B_2B_3} + \beta^* b_1a_2b_3|101\rangle_{B_1B_2B_3})\\
&\quad + |\varphi^2_\perp\rangle_{A_1A_2A_3}(\beta a_1b_2a_3|010\rangle_{B_1B_2B_3} - \alpha b_1a_2b_3|101\rangle_{B_1B_2B_3})\\
&\quad + |\varphi^3\rangle_{A_1A_2A_3}(\alpha a_1b_2b_3|011\rangle_{B_1B_2B_3} + \beta^* b_1a_2a_3|100\rangle_{B_1B_2B_3})\\
&\quad + |\varphi^3_\perp\rangle_{A_1A_2A_3}(\beta a_1b_2b_3|011\rangle_{B_1B_2B_3} - \alpha b_1a_2a_3|100\rangle_{B_1B_2B_3}). \qquad (3)
\end{aligned}$$

Now Alice makes a measurement on her particles A_1, A_2, A_3 using the basis vectors $\{|\varphi^l\rangle_{A_1A_2A_3}, |\varphi^l_\perp\rangle_{A_1A_2A_3}\}$. If the result of Alice's measurement on the particles A_1, A_2, A_3 is $|\varphi^0_\perp\rangle_{A_1,A_2,A_3}$, the three-particle B_1, B_2, B_3 at Bob's side will collapse into the state $\beta a_1a_2a_3|000\rangle_{B_1B_2B_3} - \alpha b_1b_2b_3|111\rangle_{B_1B_2B_3}$. Then Alice informs Bob of her measurement result via a classical channel with just three classical bits. According to measurement result by Alice, Bob can make a unitary transformation $-\mathrm{i}\sigma_y$ on the particle B_1 and σ_x on the particle B_2 and B_3, where σ_x and σ_y are the Pauli matrices. So we get the state $\alpha b_1b_2b_3|000\rangle_{B_1B_2B_3} + \beta a_1a_2a_3|111\rangle_{B_1B_2B_3}$. To prepare the initial state $|\phi\rangle$ in a remote place, Bob introduces an auxiliary particle C with the initial state $|0\rangle_C$ and makes a collective unitary transformation on particles B_3 and C under the basis $\{|00\rangle_{B_3C}, |10\rangle_{B_3C}, |01\rangle_{B_3C}, |11\rangle_{B_3C}\}$, an unitary transformation U_1 may take the form

$$U_1 = \begin{pmatrix} a_1a_2a_3/b_1b_2b_3 & 0 & \sqrt{1-(a_1a_2a_3/b_1b_2b_3)^2} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \sqrt{1-(a_1a_2a_3/b_1b_2b_3)^2} & 0 & -a_1a_2a_3/b_1b_2b_3 & 0 \end{pmatrix}, \tag{4}$$

and have

$$\begin{aligned} &U_1(\alpha b_1b_2b_3|000\rangle_{B_1B_2B_3} + \beta a_1a_2a_3|111\rangle_{B_1B_2B_3}) \otimes |0\rangle_C \\ =&a_1a_2a_3(\alpha|000\rangle_{B_1B_2B_3} + \beta|111\rangle_{B_1B_2B_3}) \otimes |0\rangle_C \\ &+ \alpha b_1b_2b_3\sqrt{1-(a_1a_2a_3/b_1b_2b_3)^2}\,|111\rangle_{B_1B_2B_3} \otimes |1\rangle_C . \end{aligned} \tag{5}$$

Then Bob measures the state of the auxiliary particle. If the result is $|1\rangle_C$, the remote preparation of the original state fails; if the result is $|0\rangle_C$, the state of particles B_1, B_2, and B_3 collapses to $a_1a_2a_3(\alpha|000\rangle + \beta|111\rangle)_{B_1B_2B_3}$. The remote preparation of a three-particle entangled state is successfully realized and the success probability is $|a_1a_2a_3|^2$. From Eq.(3), if the results of Alice's measurement are $|\varphi^1_\perp\rangle_{A_1A_2A_3}$, $|\varphi^2_\perp\rangle_{A_1A_2A_3}$ and $|\varphi^3_\perp\rangle_{A_1A_2A_3}$, by the similar method as above, Bob can construct the three-particle entangled state that Alice wishes to prepare remotely with the corresponding probability. According to Eq.(3), we find that if Alice's outcome is $|\varphi^l\rangle_{A_1A_2A_3}(l = 0, 1, 2, 3)$ in the three-particle projective measurement, Bob cannot unitarily convert the state that he gets into the original state $|\phi\rangle$ to be prepared remotely since he has no knowledge of the parameter β^*. However, if α and β are real numbers or the state $|\phi\rangle = (1/\sqrt{2})(|000\rangle + \mathrm{e}^{\mathrm{i}\varphi}|111\rangle)$, Bob can construct the entangled state $|\phi\rangle$ by applying a suitable unitary operation to particles B_1, B_2, and B_3.

3. Preparing remotely a N-particle entangled GHZ state by N-pair non-maximally entangled particles

Now we will generalize the previous scheme for remote preparation of a multiparticle entangled state. Suppose that the sender Alice wishes to help the receiver Bob prepare remotely a N-particle entangled GHZ state

$$|\phi\rangle = \alpha|00\ldots0\rangle + \beta|11\ldots1\rangle , \tag{6}$$

where the parameter α is real and β is a complex number, $|\alpha|^2 + |\beta|^2 = 1$, and which is known completely to Alice but unknown to Bob. We also suppose that Alice and Bob share N-pairs of non-maximally entangled particles, which are given by

$$\begin{aligned} &|\psi\rangle_{A_kB_k} = a_k|00\rangle_{A_kB_k} + b_k|11\rangle_{A_kB_k} , \\ &k = 1, 2, \ldots, N, \end{aligned} \tag{7}$$

where particles $A_1, A_2, \ldots, A_N$ belong to Alice and particles $B_1, B_2, \ldots, B_N$ belong to Bob, and a_k and b_k are real, $|a_k| \leq |b_k|$ and $|a_k|^2 + |b_k|^2 = 1$. To achieve the RSP, different from the single-particle von Neumann measurement given in the scheme[17], Alice carries out a N-particle projective measurement on the particles $A_1, A_2, \ldots, A_N$ in a set of mutually orthogonal basis vectors $\{|\varphi^l\rangle_{A_1A_2\ldots A_N}, \ |\varphi^l_\perp\rangle_{A_1A_2\ldots A_N}\}$, which is given by

$$\begin{aligned}|\varphi^l\rangle_{A_1A_2\ldots A_N} &= \alpha|0\rangle_{A_1} \otimes \prod_{m=2}^{N} |\zeta(m)\rangle_{A_m} \\ &\quad + \beta|1\rangle_{A_1} \otimes \prod_{m=2}^{N} |\overline{\zeta}(m)\rangle_{A_m}, \\ |\varphi^l_\perp\rangle_{A_1A_2\ldots A_N} &= \beta^*|0\rangle_{A_1} \otimes \prod_{m=2}^{N} |\zeta(m)\rangle_{A_m} \\ &\quad - \alpha|1\rangle_{A_1} \otimes \prod_{m=2}^{N} |\overline{\zeta}(m)\rangle_{A_m}, \quad (8)\end{aligned}$$

where $|\zeta(m)\rangle_{A_m}$ and $|\overline{\zeta}(m)\rangle_{A_m}$ stand for orthogonal states of the m-th particle, the symbol $\zeta(m)$ is a binary variable $\in \{0,1\}$ with its complement $\overline{\zeta}(m) = 1 - \zeta(m)$. We have defined a decimal numeral l as[25] $l = (\zeta(2)\zeta(3)\ldots\zeta(N))_2$, which means

$$\begin{aligned}l &= \sum_{j=2}^{N} \zeta(j)2^{N-j} \\ &= \zeta(2)\cdot 2^{N-2} + \zeta(3)\cdot 2^{N-3} + \cdots \\ &\quad + \zeta(N)\cdot 2^0. \quad (9)\end{aligned}$$

Thus, an unique representation of l is established with a set of binary number $\zeta(2), \zeta(3), \ldots, \zeta(N)$, obviously $l = 0, 1, \ldots, 2^{N-1} - 1$. Then, the N pairs of non-maximally entangled particles at this moment can be represented as

$$\begin{aligned}|\Psi\rangle &= |\psi\rangle_{A_1B_1} \otimes |\psi\rangle_{A_2B_2} \otimes \cdots \otimes |\psi\rangle_{A_NB_N} \\ &= (a_1|00\rangle + b_1|11\rangle)_{A_1B_1} \otimes (a_2|00\rangle + b_2|11\rangle)_{A_2B_2} \otimes \cdots \otimes (a_N|00\rangle + b_N|11\rangle)_{A_NB_N} \\ &= \sum_{l=0}^{2^{N-1}-1} [|\varphi^l\rangle_{A_1A_2\ldots A_N}(\alpha a_1 \otimes \prod_{m=2}^{N} P_{\zeta(m)}|0\rangle_{B_1} \otimes \prod_{m=2}^{N} |\zeta(m)\rangle_{B_m} + \beta^* b_1 \otimes \prod_{m=2}^{N} P_{\overline{\zeta}(m)}|1\rangle_{B_1} \otimes \prod_{m=2}^{N} |\overline{\zeta}(m)\rangle_{B_m}) \\ &\quad + |\varphi^l_\perp\rangle_{A_1A_2\ldots A_N}(\beta a_1 \otimes \prod_{m=2}^{N} P_{\zeta(m)}|0\rangle_{B_1} \otimes \prod_{m=2}^{N} |\zeta(m)\rangle_{B_m} - \alpha b_1 \otimes \prod_{m=2}^{N} P_{\overline{\zeta}(m)}|1\rangle_{B_1} \otimes \prod_{m=2}^{N} |\overline{\zeta}(m)\rangle_{B_m})], \quad (10)\end{aligned}$$

where we defined:

$$P_{\zeta(m)} = \begin{cases} a_m, & \text{if } \zeta(m) = 0, \\ b_m, & \text{if } \zeta(m) = 1, \end{cases}$$

$$P_{\overline{\zeta}(m)} = \begin{cases} b_m, & \text{if } \overline{\zeta}(m) = 1, \\ a_m, & \text{if } \overline{\zeta}(m) = 0. \end{cases} \quad (11)$$

Now Alice makes a measurement on her N particles $A_1, A_2, \ldots, A_N$ by using the basis vectors $\{|\varphi^l\rangle_{A_1A_2\ldots A_N}, |\varphi^l_\perp\rangle_{A_1A_2\ldots A_N}\}$. Alice sends the measurement result to Bob through a classical channel with N classical bits, In general, if the result of Alice's measurement on the particles $A_1, A_2, \ldots, A_N$ is $|\varphi^l_\perp\rangle_{A_1A_2\ldots A_N}$, the N-particle $B_1, B_2, \ldots, B_N$ at Bob's side will be in an entangled state

$$\beta a_1 \otimes \prod_{m=2}^{N} P_{\zeta(m)}|0\rangle_{B_1} \otimes \prod_{m=2}^{N} |\zeta(m)\rangle_{B_m}$$

$$-\alpha b_1 \otimes \prod_{m=2}^{N} P_{\overline{\zeta}(m)}|1\rangle_{B_1} \otimes \prod_{m=2}^{N} |\overline{\zeta}(m)\rangle_{B_m}. \quad (12)$$

Now, Bob can make an unitary transformation $-\mathrm{i}\sigma_y$ on the particle B_1 and $|0\rangle\langle\overline{\zeta}(m)| + |1\rangle\langle\zeta(m)|$ on the particle B_m, where $m = 2, 3, \ldots, N$. Then, Eq.(12) will be transformed into the state

$$\begin{aligned}&A \otimes \beta|11\ldots1\rangle_{B_1B_2\ldots B_N} \\ &\quad + B \otimes \alpha|00\ldots0\rangle_{B_1B_2\ldots B_N}, \quad (13)\end{aligned}$$

where we defined

$$A = a_1 \otimes \prod_{m=2}^{N} P_{\zeta(m)}, \quad = b_1 \otimes \prod_{m=2}^{N} P_{\overline{\zeta}(m)}.$$

There are two case regarding the relationship between A and B. One case is $A \leq B$. To prepare the initial state $|\phi\rangle$, Bob introduces an auxiliary particle C with the initial state $|0\rangle_C$ and performs a

collective unitary transformation U_2 under the basis $\{|00\rangle_{B_NC}, |10\rangle_{B_NC}, |01\rangle_{B_NC}, |11\rangle_{B_NC}\}$

$$U_2 = \begin{pmatrix} \frac{A}{B} & 0 & \sqrt{1-\left(\frac{A}{B}\right)^2} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ \sqrt{1-\left(\frac{A}{B}\right)^2} & 0 & -\frac{A}{B} & 0 \end{pmatrix}, \quad (14)$$

which turns the state of the particles $B_1, B_2, \ldots, B_N$ and C into

$$\begin{aligned} & U_2(B\alpha|00\ldots0\rangle_{B_1B_2\ldots B_N} \\ & +A\beta|11\ldots1\rangle_{B_1B_2\ldots B_N})\otimes|0\rangle_C \\ & = A(\alpha|00\ldots0\rangle_{B_1B_2\ldots B_N}+\beta|11\ldots1\rangle_{B_1B_2\ldots B_N})\otimes|0\rangle_C \\ & +B\sqrt{1-(\frac{A}{B})^2}\alpha|11\ldots1\rangle_{B_1B_2\ldots B_N}\otimes|1\rangle_C. \end{aligned} \quad (15)$$

Now Bob will measure the state of the auxiliary particle. If the result is $|1\rangle_C$, the remote preparation of the original state fails. If the result is $|0\rangle_C$, the state of particles $B_1, B_2, \ldots, B_N$ become $A(\alpha|00\ldots0\rangle + \beta|11\ldots1\rangle)_{B_1B_2\ldots B_N}$, this means that remote preparation of N-particle entangled state can be achieved with the success probability $|A|^2$.

The other is $A \geq B$. Under the basis $\{|10\rangle_{B_NC}, |00\rangle_{B_NC}, |11\rangle_{B_NC}, |01\rangle_{B_NC}\}$, the unitary transformational U_3 may take the form

$$U_3 = \begin{pmatrix} \frac{B}{A} & 0 & \sqrt{1-\left(\frac{B}{A}\right)^2} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ \sqrt{1-\left(\frac{B}{A}\right)^2} & 0 & -\frac{B}{A} & 0 \end{pmatrix}. \quad (16)$$

In the same way,

$$\begin{aligned} & U_3(B\alpha|00\ldots0\rangle_{B_1B_2\ldots B_N} \\ & +A\beta|11\ldots1\rangle_{B_1B_2\ldots B_N})\otimes|0\rangle_C \\ & = B(\alpha|00\ldots0\rangle_{B_1B_2\ldots B_N}+\beta|11\ldots1\rangle_{B_1B_2\ldots B_N})\otimes|0\rangle_C \\ & +A\sqrt{1-\left(\frac{B}{A}\right)^2}\beta|00\ldots0\rangle_{B_1B_2\ldots B_N}\otimes|1\rangle_C. \end{aligned} \quad (17)$$

From Eq.(17), it is evident that Bob can construct the N-particle entangled state that Alice wishes to prepare remotely with the success probability being $|B|^2$.

Based on Eq.(10),we find that if Alice's outcome is $|\varphi^l\rangle_{A_1A_2\ldots A_N}$ in the N-particle projective measurement, Bob will get the state

$$\begin{aligned} & \alpha a_1\otimes\prod_{m=2}^{N}P_{\zeta(m)}|0\rangle_{B_1}\otimes\prod_{m=2}^{N}|\zeta(m)\rangle_{B_m} \\ & +\beta^* b_1\otimes\prod_{m=2}^{N}P_{\overline{\zeta}(m)}|1\rangle_{B_1}\otimes\prod_{m=2}^{N}|\overline{\zeta}(m)\rangle_{B_m}. \end{aligned} \quad (18)$$

Since Bob has no knowledge of Eq.(18), he cannot unitarily convert them into the original state $|\phi\rangle$. However, if α and β are real numbers or the state $|\phi\rangle = \frac{1}{\sqrt{2}}(|00\ldots0\rangle_{12\ldots N} + \mathrm{e}^{\mathrm{i}\varphi}|11\ldots1\rangle_{12\ldots N})$, and if Alice's measurement result is $|\varphi^l\rangle_{A_1A_2\ldots A_N}$, Bob can construct the entangled state $|\phi\rangle$ by means of an unitary operation with N classical bits from Alice. Hence, for some specially chosen N-particle two-dimensional entangled states, RSP scheme can be achieved with a certain probability by using N classical bits and a N-particle projective measurement.

4. Conclusion

In conclusion, we have proposed a scheme for probabilistic RSP of a multi-particle quantum state. To present the scheme more clearly, we first investigate a scheme of RSP of a three-particle GHZ state by using three two-particle partial entangled state as quantum channel. Then we directly generalize the scheme to N particles case. It shows that, for the N bipartite nonmaximally entangled states shared by Alice and Bob as quantum channel, the probabilistic RSP of a N-particle GHZ state can be successfully realized with the N-particle projective measurement and appropriate unitary transformation.

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