Vol 17 No 2, February 2008
1674-1056/2008/17(02)/0440-05

Chinese Physics B

© 2008 Chin. Phys. Soc.
and IOP Publishing Ltd

Relation between quantum NOT gate speed and asymmetry of the potential of RF-SQUID*

Ji Ying-Hua(嵇英华)[a)b)†], Cai Shi-Hua(蔡十华)[a], and Hu Ju-Ju(胡菊菊)[a)]

[a)] *Department of Physics, Jiangxi Normal University, Nanchang* 330022, *China*

[b)] *Key Laboratory of Optoelectronic & Telecommunication of Jiangxi, Nanchang,* 330022, *China*

(Received 26 February 2007; revised manuscript received 29 May 2007)

This paper investigates the relationship between the speed of a quantum not gate and the asymmetry of the potential in an interactive system formed by a two-level RF-SQUID qubit and a classical microwave pulse. The RF-SQUID is characterized by an asymmetric double well potential which gives rise to diagonal matrix elements that describe the interaction of the SQUID with the microwave pulse. And the diagonal matrix elements account for the interaction of the microwave pulse with the SQUID. The results indicate that, when the angular frequency of the microwave field is chosen as near resonate with the transition $|0\rangle \leftrightarrow |1\rangle$, i.e. $\omega_1 - \omega_0 \approx \omega_{\rm m}$, (1) the gate speed is decided by three factors, the Rabi frequency, the difference of the diagonal matrix elements between the two levels, and the angular frequency of the applied microwave pulse $\omega_{\rm m}$; (2) the gate speed descends when the asymmetry of the potential is considered.

Keywords: gate speed, asymmetry, microwave pulse, RF-SQUID
PACC: 0365, 7335

1. Introduction

Superconducting devices such as Cooper pair boxes, Josephson junctions, and superconducting quantum interference devices (SQUIDs) have attracted much attention in the quantum information community.[1−4] Because they are relatively easy to scale up and have been demonstrated to have relatively long decoherence time,[5−7] they have been considered as promising candidates for physical implementation of quantum computing. It is known that the building blocks of quantum computers are single-qubit logic gates and two-qubit gates.[8] One particular scheme is based on magnetic flux states in SQUIDs.[9−14] In some of these schemes, the SQUID quantum bit, or qubit,[13−15] is based on a two-level system. The conventional configuration of SQUID qubit utilizes the lowest level in each of the double well (denoted as $|0\rangle$ and $|1\rangle$ in Fig.1) as the computational basis. Unitary transformations required to accomplish one-qubit rotations, such as the Hadamard and NOT gates, are implemented by controlling the pulse area of the microwaves with frequency tuned to the level separation.[16−19]

However, compared with other qubit candidates (such as trapped ions, nuclear spins, and cavity QED),[20−22] decoherence presents a much more formidable challenge to superconducting qubits. The operation time $\tau_{\rm op}$ should be far less than the decoherence time $\tau_{\rm d}$ to get correct quantum operation. Since coupling between solid-state qubits and the environment resulting in decoherence are inevitable, it is very important to increase the gate speed when the decoherence time is certain in engineering design. The gate operation number per decoherence time, $n_{\rm op} = \tau_{\rm d}/\tau_{\rm op}$, provides a good measure for the merit of physical qubits.[23] The previous results indicate that, in ideal resonance, without considering the asymmetry of the potential of radio frequency (RF)-SQUID, the gate speed depends on the Rabi frequency Ω for NOT gate.[24] In fact, the RF-SQUID has an asymmetric double well potential for the need of quantum manipulation, which is similar to a two-level (dipolar) molecule. Just as the interaction between dipolar molecule and laser field, the asymmetry of the molecular potential has magnitude effects on the dynamics of the system,[25,26] the dynamics of the system (accurate manipulation of qubit is needed) also necessarily

*Project supported by the Science and Technology Research Foundation of the Education Department of Jiangxi Province, China (Grant No 200592).
†E-mail: jyh2006@jxnu.edu.cn

depends on the difference of the interaction parameters arising from the diagonal matrix elements. In this paper we investigate the relationship between quantum NOT gate speed and the asymmetry and detuning of the potential in an interactive system formed by a two-level RF-SQUID qubit and a classical microwave pulse.

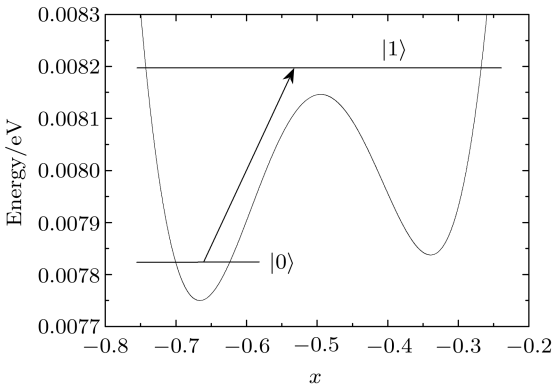


Fig.1. Potential energy and first two energy levels of SQUID with $L = 100\,\mathrm{pH}$, $C = 40\,\mathrm{fF}$ and $I_\mathrm{C} = 3.95\mu\mathrm{A}$ ($\omega = 5\times10^{11}\,\mathrm{rad/s}$, $\gamma = 1.2$). The static flux bias is $x' = -0.501$.

2. General equations

Our model consists of an RF-SQUID which interacts with a microwave field.[27,28] The RF-SQUID is made up of a superconducting ring interrupted by a Josephson tunnel junction. The system's generalized coordinate is the total magnetic flux Φ through the ring, which is subjected to the potential

$$U(\Phi) = \frac{(\Phi - \Phi_x)^2}{2L} - E_\mathrm{J}\cos\left(2\pi\frac{\Phi}{\Phi_0}\right).$$

Here, L is the ring inductance, Φ_x is an externally applied magnetic flux to the SQUID, $E_\mathrm{J} = I_\mathrm{C}\Phi_0/2\pi$ is the maximum value of the Josephson energy with I_C being the critical current of the junction, and $\Phi_0 = h/2e$ is the flux quantum. The behaviour of the Cooper electron pair in the superconductor Josephson junction is just like that of the boson, so in Ref.[29] the Josephson junction's Hamiltonian is expressed by a Bose operator and projected onto the entangled state representation. In the present paper, the SQUID is quantized in coordinate-momentum space.[30] The SQUID Hamiltonian is given by

$$H_0 = \frac{q^2}{2C} + U(\Phi),$$

where $q = -\mathrm{i}\hbar\partial/\partial\Phi$ is the displacement charge on the junction capacitance C. Using the second Josephson relation, we identify the total magnetic flux Φ associated with the Josephson junction as the superconducting phase difference φ across the junctions. The flux Φ and the charge q are canonically conjugate operators satisfying the commutation relation $[\Phi, q] = \mathrm{i}\hbar$. When performing calculations, it is convenient to define a new coordinate $x = \Phi/\Phi_0$ so that the Hamiltonian can be written as

$$H_0 = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m\omega^2(x - x')^2 - \frac{1}{4\pi^2}m\omega^2\gamma\cos(2\pi x), \quad (1)$$

where $m = C\Phi_0^2$, $\omega^2 = 1/LC$, $\gamma = 2\pi LI_\mathrm{C}/\Phi_0$ and $x' = \Phi_x/\Phi_0$. Note that the RF-SQUID's energy level structure can be controlled by adjusting γ and x'. For $x' = \Phi_x/\Phi_0 = 1/2$, the potential has symmetric double wells. Increasing (decreasing) x' slightly from 1/2 tilts the potential curve to the right (left). Then the energy difference between left and right increasing with x' provides an easy way to control the potential and interwell level separations. And the total number of levels in the two wells increases with γ. We describe a realistic SQUID system and use the same parameters as those in the work of Zhou *et al*, i.e., $L = 100\,\mathrm{pH}$, $C = 40\,\mathrm{fF}$ and $I_\mathrm{C} = 3.95\,\mu\mathrm{A}$, leading to $\omega = 5\times10^{11}$ rad/s, $\gamma = 1.2$.[31] We also take $x' = -0.501$. Then the form of the potential for these parameters is asymmetric and is shown in Fig.1, which is obtained by solving the time-independent Schrodinger equation and using the Hamiltonian H_0 with the above parameters. The values of the six lowest energies of the system were obtained in Ref.[32].

The interaction between the SQUID and microwave pulse, which is considered as linearly polarized electromagnetic field with their magnetic field perpendicular to the plane of the SQUID ring, is described by the time-dependent potential

$$V_\mathrm{int}^\mathrm{f}(x,t) = m\omega^2(x - x')V_\mathrm{m}f(t)\cos(\omega_\mathrm{m}t). \quad (2)$$

Here V_m is the field amplitude (in units of Φ_x/Φ_0), $f(t)$ is the dimensionless pulse envelope and ω_m is the field angular frequency. In the case of a rectangular microwave pulse of duration T, $f(t) = 1$ for $0 \le t \le T$ and zero elsewhere. So, the interaction between the SQUID and microwave pulse can be written as

$$V_\mathrm{int}(x,t) = m\omega^2(x - x')V_\mathrm{m}\cos(\omega_\mathrm{m}t). \quad (3)$$

Then, the system's Hamiltonian is

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m\omega^2\left[(x-x')^2 - \frac{1}{2\pi^2}\beta\cos(2\pi x) + 2(x-x')V_{\rm m}\cos(\omega_{\rm m}t)\right]. \quad (4)$$

The system is governed by the time-dependent Schrödinger equation

$$\mathrm{i}\hbar\frac{\partial}{\partial t}|\Psi\rangle = H|\Psi\rangle. \quad (5)$$

If the angular frequency of the microwave field is chosen to near resonate with the transition $|0\rangle \leftrightarrow |1\rangle$, the dynamics of the system is governed by the interaction of the two lower states of the SQUID with the microwave pulse. This is usually referred to as the two-level system approximation. The state vector of the system is expanded in terms of the eigenvectors of the H_0 as $|\Psi(t)\rangle = a_0(t)|0\rangle + a_1(t)|1\rangle$. Substituting the Hamiltonian (4) and the state vector into the time-dependent Schrodinger equation we obtain a system of ordinary coupled differential equation for the expansion amplitudes $a_j(t)$:

$$\mathrm{i}\hbar\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} a_0(t) \\ a_1(t)\end{pmatrix} = \begin{pmatrix} \hbar\omega_0 & 0 \\ 0 & \hbar\omega_1\end{pmatrix}\begin{pmatrix} a_0(t) \\ a_1(t)\end{pmatrix} + \hbar\cos(\omega_{\rm m}t+\delta) \times \begin{pmatrix} \Omega_{00} & \Omega_{01} \\ \Omega_{10} & \Omega_{11}\end{pmatrix}\begin{pmatrix} a_0(t) \\ a_1(t)\end{pmatrix}, \quad (6)$$

where

$$\Omega_{01} = \Omega_{10} = V_{\rm m}m\omega^2 x_{01}/\hbar, \quad (7)$$

$$\Omega_{00} = V_{\rm m}m\omega^2(x_{00}-x')/\hbar, \quad (8)$$

$$\Omega_{11} = V_{\rm m}m\omega^2(x_{11}-x')/\hbar, \quad (9)$$

and

$$x_{jk} = \langle j|\, x\,|k\rangle, \qquad j,k = 0,1. \quad (10)$$

In order to identify the resonances in Eq.(6), and to incorporate the effects of the diagonal dipole matrix elements into the rotating-wave approximation, it is convenient to transform the a representation to the interaction representation b defined by

$$b_j(t) = a_j(t)\exp\left[\mathrm{i}\omega_j t + \mathrm{i}\Omega_{jj}\int_0^t \mathrm{d}t'\cos(\omega_{\rm m}t')\right], \quad (11)$$

$j = 0, 1$. From Eq.(6) we obtain

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}b_0(t) = \Omega_{01}\cos(\omega_{\rm m}t) \times \exp\left[-\mathrm{i}(\omega_1-\omega_0)t - \mathrm{i}\frac{\beta}{\omega_{\rm m}}\sin(\omega_{\rm m}t')\right], \quad (12)$$

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}b_1(t) = \Omega_{01}\cos(\omega_{\rm m}t) \times \exp\left[\mathrm{i}(\omega_1-\omega_0)t + \mathrm{i}\frac{\beta}{\omega_{\rm m}}\sin(\omega_{\rm m}t)\right], \quad (13)$$

where

$$\beta = \Omega_{11} - \Omega_{00}. \quad (14)$$

Using the relation $\exp(\mathrm{i}z\sin\theta) = \sum_{n=-\infty}^{\infty} J_n(z)\exp(\mathrm{i}n\theta)$, where $J_n(\bullet)$ is the nth order ordinary Bessel function, we can write Eqs.(12) and (13) as

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}b_0(t) = \frac{\Omega_{01}}{2}\sum_{n=-\infty}^{\infty} J_n\left(\frac{\beta}{\omega_{\rm m}}\right)\exp\left[-\mathrm{i}(\omega_1-\omega_0)t - \mathrm{i}(n-1)\omega_{\rm m}t\right]b_1(t) + \frac{\Omega_{01}}{2}\sum_{n=-\infty}^{\infty} J_n\left(\frac{\beta}{\omega_{\rm m}}\right) \times \exp\left[-\mathrm{i}(\omega_1-\omega_0)t - \mathrm{i}\,(n+1)\omega_{\rm m}t\right]b_1(t), \quad (15)$$

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}b_1(t) = \frac{\Omega_{01}}{2}\sum_{n=-\infty}^{\infty} J_n\left(\frac{\beta}{\omega_{\rm m}}\right)\exp\left[\mathrm{i}(\omega_1-\omega_0)t + \mathrm{i}(n+1)\omega_{\rm m}t\right]b_0(t) + \frac{\Omega_{01}}{2}\sum_{n=-\infty}^{\infty} J_n\left(\frac{\beta}{\omega_{\rm m}}\right) \times \exp\left[\mathrm{i}(\omega_1-\omega_0)t + \mathrm{i}(\,n+1)\omega_{\rm m}t\right]b_0(t), \quad (16)$$

As the angular frequency of the microwave field is chosen to be near the resonant state with the transition $|0\rangle \leftrightarrow |1\rangle$, i.e., $\omega_1 - \omega_0 \approx \omega_{\rm m}$, then in the realistic limit that $\omega_{\rm m} \gg \Omega_{01}$ only the term with $n = 0$ is sig-

nificant in the first (second) sum and the term with $n = -2$ or $n = 0$ is significant in the second (first) sum of Eqs.(15) and (16). The rest terms in the sums are highly off-resonant and can be omitted. Under this approximation, Eqs.(15) and (16) become

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}b_0(t) = g\exp(-\mathrm{i}\Delta t)b_1(t), \tag{17}$$

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}b_1(t) = g\exp(\mathrm{i}\Delta t)b_0(t), \tag{18}$$

where

$$\Delta = \omega_1 - \omega_0 - \omega_\mathrm{m}, \tag{19}$$

$$g = \frac{\Omega_{01}}{2}\left[J_0\left(\frac{\beta}{\omega_m}\right) + J_2\left(\frac{\beta}{\omega_\mathrm{m}}\right)\right]. \tag{20}$$

Eq.(18) can be rewritten as $\mathrm{i}b_0(t) = -g^{-1}\exp(-\mathrm{i}\,\Delta t)\dot{b}_1(t)$ and substituted into Eq.(17) after first order differentiation with respect to t, then we have

$$\frac{\mathrm{d}^2b_1(t)}{\mathrm{d}t^2} - \mathrm{i}\Delta\frac{\mathrm{d}b_1(t)}{\mathrm{d}t} + g^2b_1(t) = 0. \tag{21}$$

The solution is

$$b_0(t) = -\frac{1}{g}\exp\left(-\mathrm{i}\frac{\Delta}{2}t\right)\left[A\left(\frac{\Delta}{2}+G\right)\exp(\mathrm{i}Gt) + B\left(\frac{\Delta}{2}-G\right)\exp(-\mathrm{i}Gt)\right], \tag{22}$$

where

$$4G^2 = \Delta^2 + 4g^2.$$

Substituting Eq.(22) into Eq.(17), we obtain

$$b_1(t) = \exp\left(\mathrm{i}\frac{\Delta}{2}t\right)\left[A\exp(\mathrm{i}Gt) + B\exp(-\mathrm{i}Gt)\right]. \tag{23}$$

where A and B are undetermined constants decided by initial conditions:

$$A = -\frac{g}{2G}b_0(0) - \frac{\Delta - 2G}{4G}b_1(0), \tag{24}$$

$$B = \frac{g}{2G}b_0(0) + \frac{\Delta + 2G}{4G}b_1(0). \tag{25}$$

If the system is initially in state $|0\rangle$, then $a_0(0) = 1$, $a_1(0) = 0$, which means $b_0(0) = 1$, $b_1(0) = 0$. From Eqs.(22)–(23), we have

$$b_0(t) = \exp\left(-\mathrm{i}\frac{\Delta}{2}t\right)\left[\cos(Gt) + \mathrm{i}\frac{\Delta}{2G}\sin(Gt)\right], \tag{26}$$

$$b_1(t) = -\mathrm{i}\frac{g}{G}\exp\left(\mathrm{i}\frac{\Delta}{2}t\right)\sin(Gt). \tag{27}$$

The probability for the system to be in state $|0\rangle$ or $|1\rangle$ are

$$P_0(t) = \cos^2(Gt) + \frac{\Delta^2}{4G^2}\sin^2(Gt), \tag{28}$$

$$P_1(t) = \frac{g^2}{G^2}\sin^2(Gt). \tag{29}$$

Obviously $P_0(t) + P_1(t) = 1$. If the system is initially in state $|1\rangle$, from Eqs.(22)–(25), we have

$$b_0'(t) = -\mathrm{i}\frac{g}{G}\exp\left(-\mathrm{i}\frac{\Delta}{2}t\right)\sin(Gt),$$

$$b_1'(t) = \exp\left(\mathrm{i}\frac{\Delta}{2}t\right)\left[\cos(Gt) - \mathrm{i}\frac{\Delta}{2G}\sin(Gt)\right].$$

The probability for the system to be in state $|0\rangle$ or $|1\rangle$ are

$$P_0'(t) = \frac{g^2}{G^2}\sin^2(Gt),$$

$$P_1'(t) = \cos^2(Gt) + \frac{\Delta^2}{4G^2}\sin^2(Gt),$$

$P_0'(t) + P_1'(t) = 1$. Note that $4G^2 = \Delta^2 + 4g^2$, the probability depends on Δ and g. From Eq.(20), we know that the parameter g is the interaction parameter between the microwave field and the RF-SQUID that depends on three factors, i.e., the Rabi frequency Ω_{01}, the difference of the interaction parameters arising from the diagonal matrix elements β, and the angular frequency of the applied microwave pulse ω_m. From Eqs.(28) and (29), we know that for the existence of Δ, the system is in state $|1\rangle$ at t with the probability of $P_1(t) = g^2\sin^2(Gt)/G^2$, which oscillates with $2G$ and varies between $0 - g^2/G^2$. Then, the gate speed of quantum NOT gate is defined as the time of the probability of completing one time from the minimum to the maximum. Namely, when the microwave pulse duration is chosen as

$$T_\mathrm{int} = \frac{\pi}{\sqrt{\Delta^2 + \Omega_{01}^2\left[J_0\left(\frac{\beta}{\omega_\mathrm{m}}\right) + J_2\left(\frac{\beta}{\omega_\mathrm{m}}\right)\right]^2}}, \tag{30}$$

the quantum gate completes a NOT operation (rotation). There is no question that detuning difference easily results in computation error for the multilevel structure of quantum system. So the detuning difference should be kept zero ($\Delta = 0$) in quantum manipulation, which can be completely realized by existing technology.

We are particularly interested in the case of exact resonance of the microwave field with the $|0\rangle \leftrightarrow |1\rangle$

transition, i.e., $\omega_1 - \omega_0 = \omega_{\rm m}$ ($\Delta = 0$). From Eqs.(22) and (23), we obtain

$$b_0(t) = \cos(gt)b_0(0) - {\rm i}\sin(gt)b_1(0), \quad (31)$$

$$b_1(t) = -{\rm i}\sin(gt)b_0(0) + \cos(gt)b_1(0). \quad (32)$$

The probability for the system in state $|0\rangle$ or $|1\rangle$ are

$$P_0(t) = \cos^2\left\{\frac{\Omega_{01}}{2}\left[J_0\left(\frac{\beta}{\omega_{\rm m}}\right) + J_2\left(\frac{\beta}{\omega_{\rm m}}\right)\right]\right\},$$

$$P_1(t) = \sin^2\left\{\frac{\Omega_{01}}{2}\left[J_0\left(\frac{\beta}{\omega_{\rm m}}\right) + J_2\left(\frac{\beta}{\omega_{\rm m}}\right)\right]\right\}.$$

If the microwave pulse duration is chosen as

$$T'_{\rm int} = \frac{\pi}{\Omega_{01}\left[J_0\left(\frac{\beta}{\omega_{\rm m}}\right) + J_2\left(\frac{\beta}{\omega_{\rm m}}\right)\right]},$$

then it is clear that when the difference of diagonal matrix elements are neglected, i.e. $\beta = \Omega_{11} - \Omega_{00} \approx 0$, the speed of Not gate which depends on the Rabi frequency Ω_{01} decreases.

3. Conclusion

The quantum NOT gate speed formula for the interaction of a two-level RF-SQUID qubit with a classical microwave pulse has been derived. Our model is: the SQUID system is described by an asymmetric double well potential and a small detuning difference exists between the external microwave frequency and the transition frequency of a two-level RF-SQUID qubit. Therefore, the investigation results are more significant for the generalization. The results indicate that, in resonance, the gate speed depends not only on the Rabi frequency, but also on the difference of the interaction parameters arising from the diagonal matrix elements and the angular frequency of the applied microwave pulse $\omega_{\rm m}$. Since the two-level model in this paper is useful for its correctness, it provides a basis for a multi-level system interacting with a pulsed laser.

References

[1] Chiorescu I, Bertet P, Semba K, Nakamura Y, Harmans C J P M and Mooij J E 2004 *Nature* (London) **431** 159

[2] Yu Y, Han S, Chu X, Chu S I, Wang Z 2002 *Science* **296** 889

[3] Chiorescu I, Nakamura Y, Harmans C J P M and Mooij J E 2003 *Science* **299** 1869

[4] van der C H, Wal, ter A C J, Haar, Wilhelm F K, Schouten R N, Harmans C J P M, Orlando T P, Lloyd S and Mooij J E 2000 *Science* **290** 773

[5] Han S, Rouse R and Lukens J E 1996 *Phys. Rev. Lett.* **76** 3404

[6] Wang Z W and Xiao J L 2007 *Acta Phys. Sin.* **56** 678 (in Chinese)

[7] Majer J B, Paauw F G, ter Haar A C J, Harmans C J P M and Mooij J E 2005 *Phys. Rev. Lett.* **94** 090501

[8] Sleator T and Weinfurter H 1995 *Phys. Rev. Lett.* **74** 4087

[9] Amin M H S, Smirnov A Yu and van der Brink A M 2003 *Phys. Rev.* B **67** 100508®

[10] Yang C P and Han S 2004 *Phys. Lett.* A **321** 273

[11] Kis Z and Paspalakis E 2004 *Phys. Rev.* B **69** 024510

[12] Schuster D I, Wallraff A, Blais A, Frunzio L, Huang R S, Majer J, Girvin S M and Schoelkopf R J 2005 *Phys. Rev. Lett.* **94** 123602

[13] Hu X N and Li X Q 2006 *Acta Phys. Sin.* **55** 3259 (in Chinese)

[14] Zhu J M and Wang S J 2006 *Acta Phys. Sin.* **55** 5018 (in Chinese)

[15] Migliore R and Messina A 2004 J. Opt. B: *Quantum Semiclassical Opt.* **6** S136

[16] Chiarello F 2000 *Phys. Lett.* A **277** 189

[17] Averin D V, Friedman J R and Lukens J E 2000 *Phys. Rev.* B **62** 11802

[18] Everitt M J, Stiffell P, Clark T D, Vourdas A, Prance H and Prance R J 2001 *Phys. Rev.* B **63** 144530

[19] Everitt M J, Clark T D, Stiffell P, Prance H and Prance R J 2001 *Phys. Rev.* B **64** 184517

[20] Cirac J I and Zoller P 1995 *Phys. Rev. Lett.* **74** 4091

[21] Xiang S H and Song K H 2005 *Acta Phys. Sin.* **54** 1190 (in Chinese)

[22] Pellizzari T, Gardiner S A, Cirac J I and Zoller P 1995 *Phys. Rev. Lett.* **75** 3788

[23] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information*, 1st edn (Cambridge: Cambridge University Press) Chap 7

[24] Blais A, Huang R S, Wallraff A, Girvin S M and Schoelkopf R J 2004 *Phys. Rev.* A **69** 062320

[25] Kondo A E and Meath W J 1997 *Phys. Rev.* A **92** 805

[26] Brown A, Meath W J and Tran P 2000 *Phys. Rev.* A **63** 013403

[27] Spiller T P, Clark T D, Prance R J and Widom A 1992 *In Progress in Low Temperature Physics*, ed. D. F. Brewer (Amsterdam: North-Holland), Vol. XIII, p 219

[28] Vourdas A 1994 *Phys. Rev.* B **49** 12040

[29] Fan H Y, Wang J S, Tang X B and Liang B L 2007 *Chinese Journal of Quantum Electronics* **24** 168

[30] Yang C P and Han S Y 2004 *Phys. Lett.* A **321** 273

[31] Zhou Z Y, Chu S I and Han S 2002 *Phys. Rev.* B **66** 054527

[32] Paspalakis E and Kylstra N J 2004 *J. Mod. Opt.* **20** 1679