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Factors influencing entanglement of two excitons in a coupled exciton–photon system*

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The relation between the excitonic purity and the concurrence in a system of two coupled large semiconduction quantum dots mediated by a single-mode cavity field is investigated by using linear entropy theory. The results show the difference in describing two modes of excitonic entanglement between linear entropy and concurrence. The relation between nonclassical property of cavity field and the entanglement degree of excitons is also discussed. The results show that two modes of exciton can reach maximal entanglement when the cavity exhibits an antibunching effect.

Keywords: quantum dot, exciton, linear entropy, antibunching effect
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1. Introduction

Entanglement is a resource used in many quantum information protocols including quantum teleportation, quantum cryptography and quantum communication. Over the last decade, scientists have developed the abstract theory and also made efforts to find some physical systems to prepare quantum entanglement. A representative list includes the trapped ions,[1] the liquid-state nuclear magnetic resonance[2] and solid-state material.[3]

In recent years, with the development of nanotechnology, semiconductor nanostructures, especially quantum dot structures, are viewed as one of the most promising candidates for the realization of quantum computation and the quantum information processing. Sanders *et al*[4] have discussed the scalable solid-state quantum computer based on electric dipole transitions within coupled single-electron dots. Imamoğlu *et al*[5] proposed a scheme which realized controllable interactions between two distant quantum dot spins by combining the cavity quantum electrodynamics (QED) with electronic spin degrees in quantum dots. Entanglement of the exciton states in a single quantum dot or in a quantum dot molecule has been also demonstrated experimentally.[6] Reference [7] has investigated how to prepare maximally entangled Bell and Greenberger–Horne–Zeillinger states in the system of optically driven coupled quantum dots. There have still existed many schemes of generating entanglement of the exciton states.[8−10]

The references cited above focus mainly on the entanglement of the excitonic orthogonal states. Not long ago, Liu *et al*[11] investigated how to prepare entangled nonorthogonal states, for example, the entangled coherent states and presented the dynamics of the entangled excitonic coherent states in the system of two large semiconductor quantum dots coupled by a single-mode field. In their paper, the conditions for the preparation of maximally entangled excitonic states are derived when the effective radius R of each quantum dot is much large than Bohr-radius $a_{\rm B}$ of the bulk-exciton. The interaction between two quantum dots is mediated only by the single-mode cavity field. Inspired by their paper, we study the relation between the concurrence and the linear entropy for the prepared state. We also discuss the effect of nonclassical properties of radiation field on excitonic entanglement.

The rest of this paper is organized as follows. In Section 2, we describe the model of two quantum dots interacting with a single-mode cavity field and make a comparison between the concurrence and linear entropy. In Section 3, we discuss the effect of nonclassical properties of cavity field on the entanglement of

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excitons, and finally the conclusion of this study is presented in Section 4.

2. Comparison between concurrence and the linear entropy

We first briefly recall Liu's work. Two quantum dots are coupled to a single mode of the electromagnetic field inside a high-Q cavity. Assume that a few electron are excited from valence band to a conduction valence, then the exciton number in the ground state for each quantum dot will be so low that the mean distance between two excitons is much larger than extension of the exciton. Under this condition, the exciton can be described by a boson operator. The distance between the dots is assumed to be large, so the direct interaction between any two excitons can be safely omitted. The ground energies of the excitons in each dot are assumed to be the same. There is a resonant interaction between the cavity field and the excitons. Then from Ref.[11] the Hamiltonian for this system in the rotating wave approximation can be written as

$$\begin{aligned} H_0 =&\hbar\omega\hat{a}^\dagger\hat{a} + \hbar\omega\sum_{m=1}^{2}\hat{b}_m^\dagger\hat{b}_m \\ &+ \hbar\sum_{m=1}^{2} g_m(\hat{b}_m^\dagger\hat{a} + \hat{a}^\dagger\hat{b}_m), \end{aligned} \tag{1}$$

where $\hat{a}^\dagger$ and $\hat{a}$ are the operators of the cavity field with frequency ω; $\hat{b}_m^\dagger$ and $\hat{b}_m$ denote the exciton operators with the same frequency ω as that of the cavity field; m=1 and m=2 represent the first dot and the second dot respectively; g_1 and g_2 refer to the coupling constants between quantum dot 1 and cavity field and between quantum dot 2 and cavity field respectively.

Reference [11] points out if the cavity field is initially prepared in a superposition of two distinct coherent states $|\alpha_1\rangle$ and $|\alpha_2\rangle$ in the normalized form of

$$|\psi(0)\rangle = C|\alpha_1\rangle + D|\alpha_2\rangle, \tag{2}$$

after a unitary evolution one can obtain entangled coherent exciton states with a reduced density matrix as

$$\begin{aligned} \rho(t) =&[C^*D\mathcal{N}(|\alpha_2 v_1\rangle\langle\alpha_1 v_1|)_1 \otimes (|\alpha_2 v_2\rangle\langle\alpha_1 v_2|)_2 + \text{h.c.}] \\ &+ |C|^2(|\alpha_1 v_1\rangle\langle\alpha_1 v_1|)_1 \otimes (|\alpha_1 v_2\rangle\langle\alpha_1 v_2|)_2 \\ &+ |D|^2(|\alpha_2 v_1\rangle\langle\alpha_2 v_1|)_1 \otimes (|\alpha_2 v_2\rangle\langle\alpha_2 v_2|)_2 \end{aligned} \tag{3}$$

with

$$v_1(t) = -\mathrm{i}\frac{g_1}{g}\sin(gt)\mathrm{e}^{-\mathrm{i}\omega t}, \tag{4}$$

and

$$v_2(t) = -\mathrm{i}\frac{g_2}{g}\sin(gt)\mathrm{e}^{-\mathrm{i}\omega t}, \tag{5}$$

where h.c. denotes complex conjugate; $\mathcal{N}$ is the inner product of two coherent states $|\alpha_2 u\rangle$ and $|\alpha_1 u\rangle$; $u(t) = \cos(gt)\mathrm{e}^{-\mathrm{i}\omega t}$;

$$\mathcal{N} = \exp\left[-\cos^2(gt)\left(\frac{1}{2}|\alpha_1|^2 + \frac{1}{2}|\alpha_2|^2 - \alpha_1^*\alpha_2\right)\right].$$

Reference [11] investigates the concurrence in order to determine whether maximally entangled coherence excitonic states can be generated. It is well known that the linear entropy is also a good tool to measure the purity of quantum states.[12] The linear entropy S_L is defined as [13]

$$S_\mathrm{L}(\rho) = \frac{N}{N-1}[1 - \mathrm{Tr}(\rho^2)], \tag{6}$$

of which the value ranges from 0 (for pure state) to 1 (for a maximally mixed state). N above denotes the dimension of the quantum system. For $C^2 \otimes C^2$ systems, the linear entropy can be written explicitly as

$$S_\mathrm{L}(\rho) = \frac{4}{3}[1 - \mathrm{Tr}(\rho^2)]. \tag{7}$$

Next we investigate the evolution of linear entropy between two mode excitons. From Ref.[11], for the cavity field initially prepared in an odd coherent state, namely the state vector of

$$|\mathrm{odd}\rangle = N_-(|\alpha\rangle - |-\alpha\rangle) \tag{8}$$

with normalization constant $N_- = (2 - 2e^{-2|\alpha|^2})^{-1/2}$, we can calculate the linear entropy $S_\mathrm{L}(\rho)$ of the prepared state as

$$\begin{aligned} S_\mathrm{oL}(\rho) =&\frac{4}{3}\left[\frac{1}{2} + \frac{\mathrm{e}^{-2|\alpha|^2}}{(1-\mathrm{e}^{-2|\alpha|^2})^2}\right. \\ &\left. - \frac{\mathrm{e}^{-4\sin^2(gt)|\alpha|^2} + \mathrm{e}^{-4\cos^2(gt)|\alpha|^2}}{2(1-\mathrm{e}^{-2|\alpha|^2})^2}\right]. \end{aligned} \tag{9}$$

Equation (9) shows the evolution of linear entropy for two subsystems of the exciton. The linear entropy attains the ideal case in which it is a periodic function of gt, and it becomes zero at the time when the condition $gt = n\dfrac{\pi}{2}$ is satisfied for any intensity $|\alpha|^2$ of the coherent cavity field. In comparison with concurrence C_0 obtained in Ref.[11], the period of S_oL is twice as large as that of C_0, and during one period oscillation of C_0 the exciton subsystem arrives at the pure states twice, one arrival at the time when C_0 reaches its minimum (the two exciton are in a separable state) and another arrival at the time when C_0

reaches its maximum (the two excitons are in a maximally entangled state). Figure 1 shows the numerical calculations, which indicate the difference in describing two modes of excitonic entanglement between the linear entropy and the concurrence. The linear entropy can reflect the entanglement between two subsystems to some degree. However, it cannot exactly present the entanglement degree between two subsystems. It mainly characterizes the purity between two subsystems. If the cavity field is initially prepared in an even coherent state, when the coherent intensity $|\alpha|$ of the cavity field tends to infinity (for example , $|\alpha| > 2$), we can obtain the same result as above.

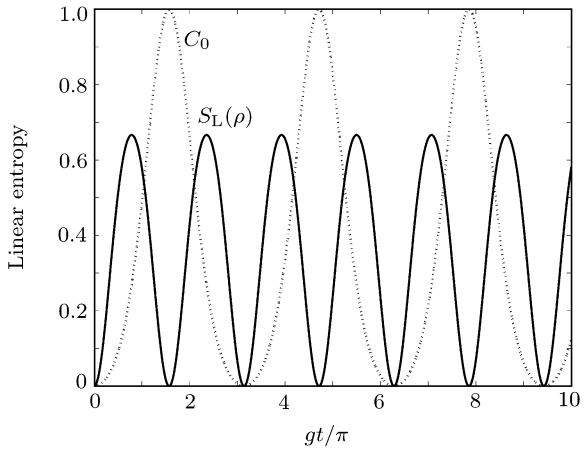


Fig.1. Time evolution of linear entropy $S_{\rm oL}$ as a function of gt/π. The solid line and the dashed line correspond to $S_{\rm oL}$ and C_0, respectively.

3. The influence of nonclassical property of cavity field on excitonic entanglement

Reference [11] indicates that when the cavity field is initially in an odd coherent state, maximally entangled coherent states of excitons can be obtained. But when the cavity is initially in an even coherent state, maximally entangled coherent states of excitons cannot be obtained except for the fact that coherent intensity $|\alpha|$ of the cavity field tends to infinity. We will give a reasonable explanation to the problem. For the reason, we study an antibunching effect of cavity field by making use of Mandel's Q parameter

$$Q = \frac{\langle (a^+a)^2\rangle - (\langle a^+a\rangle)^2 - \langle a^+a\rangle}{\langle a^+a\rangle} = \frac{\langle a^{+2}a^2\rangle}{\langle a^+a\rangle} - \langle a^+a\rangle. \quad (10)$$

Factor Q reflects the degree of field deviation from classical field. $Q = 0$ means that the cavity field is in a classical state and photon distribution function corresponds to Poisson distribution. $Q > 0$ implies that photons present the bunching effect corresponding to super-Poisson distribution. $Q < 0$ refers to the fact that photons exhibit an antibunching effect corresponding to sub-Poisson distribution, which is one of the nonclassical features of cavity field.

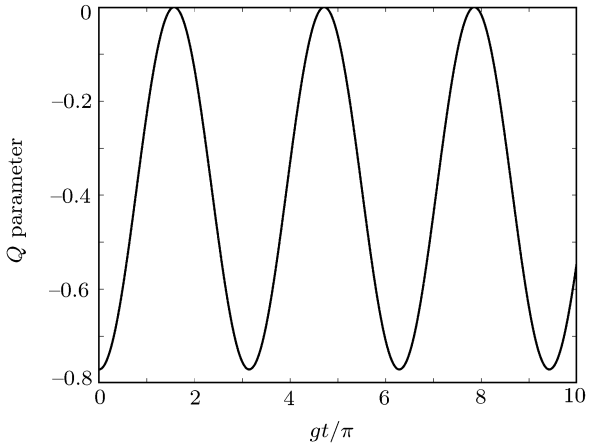


Fig.2. Q versus t for the cavity field being in an odd coherent state.

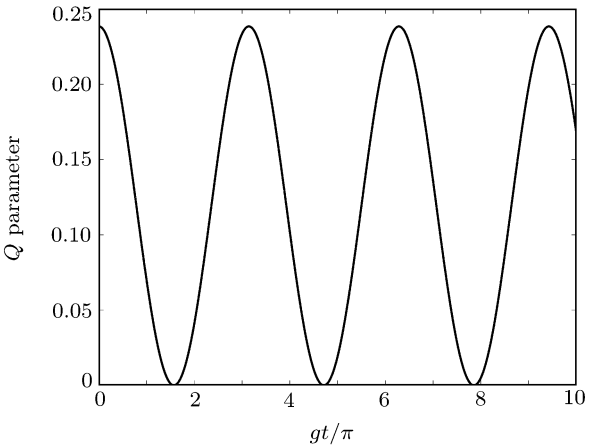


Fig.3. Q versus t for the cavity field being in an even coherent state.

In order to find out the Q factor, we trace over the modes of excitons in the density matrix of the system given by Eq.(9) in Ref[11]. Thus we obtain a reduced density matrix of cavity field as follows:

$$\rho(t) = (C^* D\mathcal{N}|\alpha_2 u\rangle\langle\alpha_1 u| + \text{h.c.}) + |C|^2|\alpha_1 u\rangle\langle\alpha_1 u| + |D|^2|\alpha_2 u\rangle\langle\alpha_2 u|. \quad (11)$$

Utilizing Eqs.(10) and (11), we can obtain the Q factor as follows: when the cavity field is in an odd coherent

state,

$$Q_0 = |\alpha|^2 \cos^2(gt)\left(1 - \frac{1 + \mathrm{e}^{-2|\alpha|^2}}{1 - \mathrm{e}^{-2|\alpha|^2}}\right); \qquad (12)$$

when the cavity field is in an even coherent state,

$$Q_\mathrm{e} = |\alpha|^2 \cos^2(gt)\left(1 - \frac{1 - \mathrm{e}^{-2|\alpha|^2}}{1 + \mathrm{e}^{-2|\alpha|^2}}\right). \qquad (13)$$

From expressions (12) and (13), the antibunching effect exists when the cavity field is initially in an odd coherent state. However, in an even coherent state, cavity field does not display any nonclassical property. Figures 2 and 3 show the evolutions of the Q factor of the cavity field in an odd coherent state and even coherent state, respectively. Figures 4 and 5 show a comparison between the concurrence and Q parameter. It is apparent that maximally entangled coherent states of excitons can be obtained when the nonclassical property of sub-Poissonian statistics occurs on condition that cavity field is in an odd coherent state, whereas when the nonclassical property of sub-Poissonian statistics does not occur in case of the cavity field being in an even coherent state, maximally entangled coherent states of excitons cannot be generated. In fact, numerical results show that only when the coherent intensity $|\alpha| \geq 2$ can the concurrence approximately reach its maximum. It predicts that nonclassical property of the cavity field will influence entanglement degree between excitons.

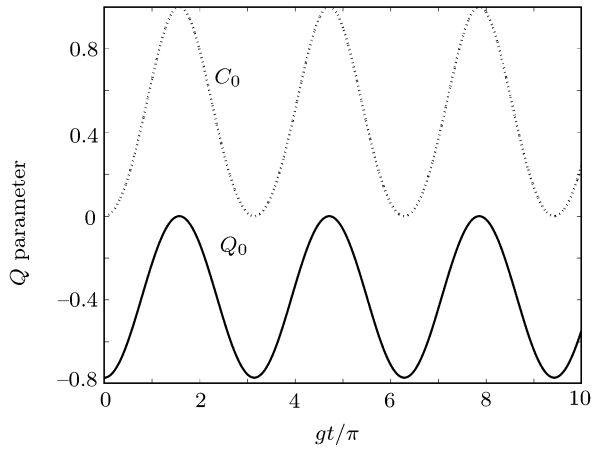


Fig.4. The time evolutions of Q parameter (solid curve) and the concurrence C_0 (dashed curve) as a function of gt/π with $|\alpha| = 1$.

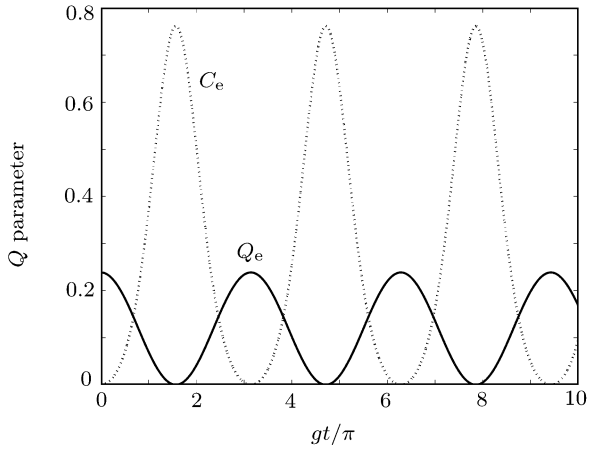


Fig.5. The time evolutions of Q parameter (solid curve) and the concurrence C_e (dashed curve) as a function of gt/π with $|\alpha| = 1$.

4. Concluding remarks

The relation between the linear entropy and concurrence for entangled excitonic coherent states in a system of two coupled quantum dots by a cavity field is investigated. The result implies that the linear entropy is also a periodic function of gt, and its period is twice as large as that of C_0, namely when the value of the linear entropy is minimum, two excitons are in a separable state one time (C_0 minimum) and they are in a maximally entangled state another time (C_0 maximum). It indicates the difference in describing two modes of excitonic entanglement between linear entropy and concurrence. The linear entropy can reflect the entanglement between two subsystems to some degree. However, it cannot exactly present the entanglement degree between two subsystems. It mainly characterizes the purity between two subsystems.

We have also discussed the relation between the nonclassical property of cavity field and the entanglement degree of excitons. When the cavity exhibits an antibunching effect, two modes of exciton can reach their maximum entanglement.

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