Vol 17 No 2, February 2008
1674-1056/2008/17(02)/0431-04

Chinese Physics B

© 2008 Chin. Phys. Soc.
and IOP Publishing Ltd

Scheme for implementing quantum dense coding with four-particle decoherence-free states in an ion trap*

Zheng Xiao-Juan(郑小娟)[a)c)†], Cao Shuai(曹 帅)[b)c)],
Fang Mao-Fa(方卯发)[c)], and Liao Xiang-Ping(廖湘萍)[d)]

[a)] *School of Physics Science and Technology, Central South University, Changsha* 410083, *China*
[b)] *College of Sciences, South China Agricultural University, Guangzhou* 510642, *China*
[c)] *College of Physics and Information Science, Hunan Normal University, Changsha* 410081, *China*
[d)] *Department of Physics, Hunan Industrial University, Zhuzhou* 412000, *China*

(Received 27 June 2007; revised manuscript received 29 June 2007)

This paper proposes an experimentally feasible scheme for implementing quantum dense coding of trapped-ion system in decoherence-free states. As the phase changes due to time evolution of components with different eigenenergies of quantum superposition are completely frozen, quantum dense coding based on this model would be perfect. The scheme is insensitive to heating of vibrational mode and Bell states can be exactly distinguished via detecting the ionic state.

Keywords: entanglement, quantum dense coding, trapped ions,decoherence-free state
PACC: 0365, 4250

Dense coding, originally introduced by Bennett and Wiesner,[1] is the surprising utilization of entanglement[2−4] to enhance the capacity of a quantum communication channel. In the original protocol,[1] with the assistance of a maximally entangled qubit pair, Alice can send faithfully 2 bits of classical information to Bob by sending a single qubit. The original dense coding protocol has been generalized by other authors in various directions. Dense coding has been investigated theoretically in the context of continuous variables.[5−8] It has also been considered for more than two entangled bits[9−16] or entangled degrees of freedom.[17,18]

An experimental demonstration of the original dense coding[1] has been first made by using photonic polarization entanglement.[19] The protocol was also simulated in nuclear magnetic resonance by using temporal averaging.[20] Dense coding has been investigated experimentally[21,22] in the context of continuous variables.

With advances in ion cooling and trapping, the trapped-ion system has been considered as one of the most possible candidates for engineering quantum entanglement. This is due to the fact that cold and localized ions are not only the important resource of entanglement, but also well suited for storing quantum information in long-lived internal states. Particularly, Riebe *et al*[23] and Barrett *et al*[24] have implemented quantum teleportation of atomic qubits by using Ca^+ ions in a linear Paul trap and using Be^+ ions in a segmented ion trap, respectively. It shows that the techniques of manipulating and transforming quantum states have made a striking progress in trapped-ion system. Thus dense coding using trapped-ion system attracts much attention. Two-particle quantum dense coding has been experimentally demonstrated by using trapped-ion system.[25]

Decoherences have been main obstacle in achievements of quantum information processing, while dephasing is one of the most important decoherences. To keep decoherence away from qubits in quantum-information processing, various proposals[26−28] have been put forward to resist the attack of decoherence by using decoherence-free states (DFS). An experiment[29] with trapped ions that encodes qubits in DFS has demonstrated that the simplest case of DFS, i.e., two-qubit DFS can resist collective dephasing and it has clearly showed the effectiveness of DFS in information storage. In this paper, we present a feasible scheme of quantum dense coding of trapped ions in DFS. This scheme is insensitive to heating of vibrational mode. More importantly, four Bell-state

*Project supported by the Important Program of Hunan Provincial Education Department (Grant No 06A038), Department of Education of Hunan Province (Grant No 06C080), and Hunan Provincial Natural Science Foundation, China (Grant No 06JJ4003).
†Corresponding author. E-mail: xiaojuan_11@sohu.com

measurements can be exactly discriminated only by one step with the probability of success 1.

We consider that N two-level ions are confined in a linear trap. All ions are simultaneously illuminated by two classical homogeneous lasers of frequencies $\omega_0+\nu+\delta$ and $\omega_0-\nu-\delta$, where ω_0 is the frequency of the transition $|e\rangle \rightarrow |g\rangle$ and ν is the frequency of the collective vibrational mode. We assume that the lasers have the same Rabi frequency Ω, the same relative phase $\phi=\frac{\pi}{2}$ and the same Lamb-Dicke parameter η. Suppose $\delta \ll \nu$, and thus we can neglect other vibrational modes. In these cases the Hamiltonian of the system can be described as follows[30,31]

$$\hat{H} = \omega_0 \sum_{j=1}^{N} \hat{S}_{z,j} + \nu \hat{a}^{+}\hat{a} + \Bigg\{ -\mathrm{i}\Omega \sum_{j=1}^{N} \hat{S}_j^{+} \mathrm{e}^{\mathrm{i}\eta(\hat{a}^{+}+\hat{a})} \times [\mathrm{e}^{-\mathrm{i}(\omega_0+\nu+\delta)t} + \mathrm{e}^{-\mathrm{i}(\omega_0-\nu-\delta)t}] + \mathrm{H.c.} \Bigg\}, \quad (1)$$

where $\hat{a}^{+}(\hat{a})$ denotes the creation (annihilation) operator for the centre-of-mass mode of the collective motion of the ions, $\hat{S}_{z,j}=\frac{1}{2}(|e\rangle_j\langle e| - |g\rangle_j\langle g|)$, $\hat{S}_j^{+} = |e\rangle_j\langle g|$, $\hat{S}_j^{-} = |g\rangle_j\langle e|$ with $|e\rangle_j$, $|g\rangle_j$ are the excited and ground states of jth ion, respectively. We consider the resolved sideband regime, where the vibrational frequency ν is much larger than other characteristic frequencies of the problem. In this case we discard the rapidly oscillating terms and obtain the Hamiltonian in the interaction picture,

$$\hat{H} = -\mathrm{i}\Omega \mathrm{e}^{-\frac{\eta^2}{2}} \sum_{j=1}^{N} \hat{S}_j^{+} \sum_{l=0}^{\infty} \frac{(\mathrm{i}\eta)^{2l+1}}{l!(l+1)!} \times [\hat{a}^{+(l+1)}\hat{a}^{l}\mathrm{e}^{-\mathrm{i}\delta t} + \hat{a}^{+l}\hat{a}^{l+1}\mathrm{e}^{\mathrm{i}\delta t}] + \mathrm{H.c.} \quad (2)$$

In the Lamb-Dicke regime, the Hamiltonian of Eq.(2) can be approximated by the expansion to the first order in η,

$$\hat{H} = \Omega\eta \sum_{j=1}^{N} (\hat{S}_j^{+} + \hat{S}_j^{-})(\hat{a}^{+}\mathrm{e}^{-\mathrm{i}\delta t} + \hat{a}\mathrm{e}^{\mathrm{i}\delta t}). \quad (3)$$

In the case $\delta \gg \eta\Omega$, there is no energy exchange between the ionic system and the vibrational mode. So the effective Hamiltonian is given by[32]

$$H_{\mathrm{e}} = \lambda \Bigg[\frac{1}{2} \sum_{j=1}^{N} (|e\rangle_j\langle e| + |g\rangle_j\langle g|) + \sum_{j,k=1}^{N} (S_j^{+}S_k^{+} + S_j^{+}S_k^{-} + \mathrm{H.c.}) \Bigg], \quad j \neq k, \quad (4)$$

where $\lambda = \frac{2\eta^2\Omega^2}{\delta}$. From the form of the effective Hamiltonian, we conclude that the interaction Hamiltonian is independent of the phonon number of the vibrational mode. So the vibrational mode may be in any state, for example, a thermal state. The evolution operator for Hamiltonian $\hat{H}_{\mathrm{e}}$ can be written as

$$U(t) = \mathrm{e}^{-\mathrm{i}\hat{H}_{\mathrm{e}}t}. \quad (5)$$

Assume four ions are initially in the state $|egeg\rangle$, at the interaction time $t=\frac{\pi}{4\lambda}$, the state of the ions system evolves into

$$|egeg\rangle_{1234} \longrightarrow \frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\frac{\pi}{4}}(|egeg\rangle_{1234} + \mathrm{i}|gege\rangle_{1234}). \quad (6)$$

Suppose that ions 3 and 4 belong to Alice and ions 1 and 2 belong to Bob. Suppose $|eg\rangle = |\uparrow\rangle$ and $|ge\rangle = |\downarrow\rangle$. To realize the quantum dense coding, Alice performs on her ions 3 and 4 one of the four local operations$\{I_3\otimes I_4, \sigma_{3x}\otimes\sigma_{4x}, \sigma_{3z}\otimes I_4, \sigma_{3x}\otimes \mathrm{i}\sigma_{4y}\}$, (where I is the identity operator and σ_i are three Pauli operators), which will, respectively, transform the state of four ions into one of the following four states:

$$|\phi^{+}\rangle_{BA} = \frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\frac{\pi}{4}}(|\uparrow\uparrow\rangle_{BA} + \mathrm{i}|\downarrow\downarrow\rangle_{BA}), \quad (7)$$

$$|\psi^{+}\rangle_{BA} = \frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\frac{\pi}{4}}(|\uparrow\downarrow\rangle_{BA} + \mathrm{i}|\downarrow\uparrow\rangle_{BA}), \quad (8)$$

$$|\phi^{-}\rangle_{BA} = \frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\frac{\pi}{4}}(|\uparrow\uparrow\rangle_{BA} - \mathrm{i}|\downarrow\downarrow\rangle_{BA}), \quad (9)$$

$$|\psi^{-}\rangle_{BA} = \frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\frac{\pi}{4}}(|\uparrow\downarrow\rangle_{BA} - \mathrm{i}|\downarrow\uparrow\rangle_{BA}), \quad (10)$$

where the subscript A denotes Alice and the subscript B denotes Bob.

Then Alice sends her ions 3 and 4 to Bob. In this case Bob possesses four ions 1, 2, 3 and 4. Now what Bob wants to do is knowing how to discriminate the four states.

Bob let ions 1, 2, 3 and 4 be simultaneously illuminated by two classical homogeneous lasers of frequencies $\omega_0+\nu+\delta$ and $\omega_0-\nu-\delta$. The system evolution is described by Eq.(5). If we choose the interaction time t to satisfy the condition $t=\frac{\pi}{4\lambda}$, the above quantum states evolve into

$$|\phi^{+}\rangle_{BA} \longrightarrow |\downarrow\downarrow\rangle_{BA}, \quad (11)$$

$$|\psi^{+}\rangle_{BA} \longrightarrow |\downarrow\uparrow\rangle_{BA}, \quad (12)$$

$$|\phi^{-}\rangle_{BA} \longrightarrow -\mathrm{i}|\uparrow\uparrow\rangle_{BA}, \quad (13)$$

$$|\psi^{-}\rangle_{BA} \longrightarrow -\mathrm{i}|\uparrow\downarrow\rangle_{BA}. \quad (14)$$

Hence, the Bell-state measurement can be achieved by detecting ions 1, 2, 3 and 4 separately. With the outcome of the measurement on Bob's four ions, Bob can distinguish Alice's operation on ions 3 and 4. So he receives two bits of classical information from Alice. That is, the dense coding is achieved.

We now give a brief discussion on the experimental feasibility. With a vibrational frequency $\frac{\nu}{2\pi} = 200\,\mathrm{kHz}$, we choose $\delta = 0.1\nu$, $\eta = 0.10$, $\Omega = 0.1\nu$. Therefore, the conditions $\delta \ll \nu$, $\nu \gg \Omega$ and $\nu \gg \eta\Omega$ can be satisfied. The next eigenfrequency, corresponding to the stretch mode, is $\nu_r = \sqrt{3}\nu$. Then the detuning between the Rabi frequency and this vibrational frequency is $|\delta_r| = |\Omega - \nu_r| \approx 1.632\nu \gg \delta$. Thus it is reasonable to neglect stretch modes. In addition, because during the interaction between trapped ions and the lasers, the heating of vibrational mode has been eliminated. Thus we only need to consider the radiative time of the ions. We consider calcium ions $^{40}Ca^{+}$ with the 'ground' (computational) state $|g = S_{1/2}\rangle$ and the 'excited' (computational) state $|e = D_{5/2}\rangle$. In Refs.[33, 34], the lifetime of the ion on the metastable $D_{5/2}$ level is 1.045 s. At present the most precise experiment[35] shows that the lifetime of the ion on the metastable $D_{5/2}$ level is 1.1689 s. While the interaction time of ion-laser is of the order 10^{-3} s, which is much shorter than the spontaneous radiative time. Thus the losses due to the spontaneous emission can be neglected. Of course, in our scheme if Bob detects the states of the ions pair(1,2) and pair(3,4), he must distinguish the ions. Fortunately, Riebe *et al* [23] have been solved this problem. Riebe *et al*. have developed a technique to address any specified target ion by using tightly focused laser beams and to 'hide' the remaining ions from the target ion's fluorescence by changing their internal states so that they are insensitive to the fluorescent light. So the scheme is feasible in the current technology.

The number of ions used for quantum dense coding in our scheme is twice that in former scheme,[25] which is a challenge to the present ion-trap technique unavailable to confine large numbers of ions. Moreover, with the increase of the number of ions, the spatial separation of trapped ions decreases, which makes their spatial resolution more and more difficult.[36] So to solve the problem, we have to use multiple traps. One of the promising schemes in this respect was proposed by Kielpinski *et al*,[37] (see Figure 1) in which each ion-trap quantum computer consists of two regions. One of them is called the memory region, which stores thousands of ions and acts as a quantum register. The other is called the interaction region, in which quantum logic operations take place. In our scheme, the interaction of laser-ion takes place in the interaction region, after the interaction, the ions are moved adiabatically to the memory region. With the transfer of states between different ion pairs, quantum dense coding is finished on two separate ion pairs.

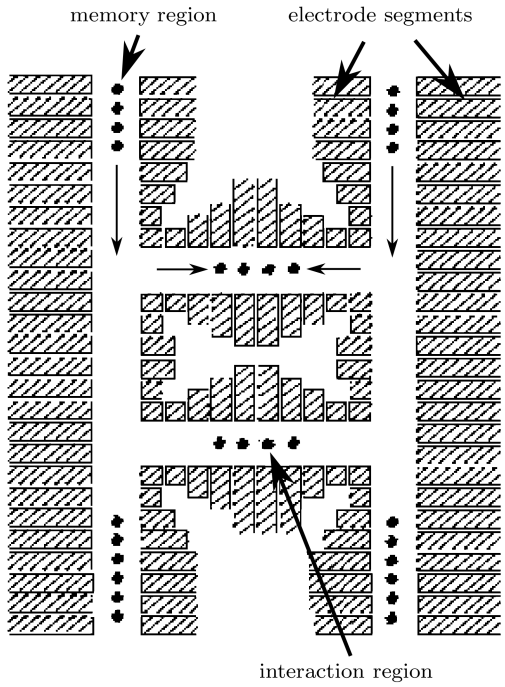


Fig.1. Diagram of the quantum charge-coupled device (QCCD). Ions are stored in the memory region and moved to the interaction region for logic operations. Thin arrows show transport and confinement along the local trap axis.

In summary, we have proposed a simple scheme to realize quantum dense coding by using four trapped ions in DFS. Compared with various former schemes in Refs.[1, 12, 16, 25, 38], the distinct character of our scheme is that quantum dense coding would be made more perfectly, without any possible phase change and to the largest extent, excluding the detrimental effect of decoherence, because it corresponds to DFS. In addition, as the vibrational state is only virtually excited, our scheme is insensitive to heating of vibrational mode. More importantly, we can make a complete measurement of Bell states with our scheme. Four states codified by Alice can be completely discriminated only via the ion–laser interaction. The success probability of this scheme is equal to 1.

References

[1] Bennett C H and Wiesner S J 1992 *Phys. Rev. Lett.* **69** 2881

[2] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777

[3] Wu Y, Payne M G, Hagley E W and Deng L 2004 *Phys. Rev.* A **69** 063803

[4] Wu Y and Yang X 1997 *Phys. Rev. Lett.* **78** 3086
Wu Y and Deng L 2004 *Optics Letters* **29** 1144

[5] Ban M 1999 *J. Opt. B: Quantum Semiclassical Opt.* **1** L9

[6] Ralph T C and Huntington E H 2002 *Phys. Rev.* A **66** 042321

[7] Braunstein S L and Kimble H J 2000 *Phys. Rev.* A **61** 042302

[8] Zhang J, Xie C and Peng K 2002 *Phys. Rev.* A **66** 032318

[9] Hao J C, Li C F and Guo G C 2001 *Phys. Rev.* A **63** 054301

[10] Liu X S, Long G L, Tong D M and Li F 2002 *Phys. Rev.* A **65** 022304

[11] Bruss D, DAriano G M, Lewenstein M, Macchiavello C, Sen(De) A and Sen U 2004 *Phys. Rev. Lett.* **93** 210501

[12] Tan J and Fang M F 2006 *Chin. Phys.* **15** 1895

[13] Bose S, Vedral V and Knight P L 1998 *Phys. Rev.* A **57** 822

[14] Lee H J, Ahn D and Hwang S W 2002 *Phys. Rev.* A **66** 024304

[15] Wójcik A and Grudka A 2003 *Phys. Rev.* A **68** 016301
Akhavan O and Rezakhani A T 2003 *Phys. Rev.* A **68** 016302

[16] Cheng W W, Huang Y X, Liu T K and Li H 2007 *Chin. Phys.* **16** 38

[17] Shimizu K, Imoto N and Mukai T 1999 *Phys. Rev.* A **59** 1092

[18] Feng Y, Duan R y and Ji Z F 2006 *Phys. Rev.* A **74** 012310

[19] Mattle K, Weinfurter H, Kwiat P G and Zeilinger A 1996 *Phys. Rev. Lett.* **76** 4656

[20] Fang X M, Zhu X W, Feng M, Mao X and Du F 2000 *Phys. Rev.* A **61** 022307

[21] Li X, Pan Q, Jing J, Zhang J, Xie C and Peng K 2002 *Phys. Rev. Lett.* **88** 047904

[22] Mizuno J, Wakui K, Furusawa A and Sasaki M Preprint quant-ph/0402040

[23] Riebe M, Häffner H, Roos C F, Hänsel W, Benhelm J, Lancaster G P T, Körber T W, Becher C, Schmidt-Kaler F, James D F V and Blatt R 2004 *Nature* **429** 734

[24] Barrett M D, Chiaverini J, Schaetz T, Britton J, Itano W M, Jost J D, Knill E, Langer C, Leibfried D, Ozeri R and Wineland D J 2004 *Nature* **429** 737

[25] Schaetz T, Barrett M D, Leibfried D, Chiaverini J, Britton J, Itano W M, Jost J D, Langer C and Wineland D J 2004 *Phys. Rev. Lett.* **93** 040505

[26] Viola L, Knill E and Laflamme R 2001 *J. Phys.* A **34** 7067

[27] Lidar D A, Bacon D and Whaley K B 1999 *Phys. Rev. Lett.* **82** 4556
Kempe J, Bacon D, Lidar D A and Whaley K B 2001 *Phys. Rev.* A **63** 042307
Lidar D A and Wu L A Preprint quant-ph/0109021

[28] Feng M and Wang X 2002 *Phys. Rev.* A **65** 044304

[29] Kielpinski D, Meyer V, Rowe M A, Sackett C A, Itano W M, Monroe C and Wineland D J 2001 *Science* **291** 1013

[30] Sørensen A and Mølmer K 1999 *Phys. Rev. Lett.* **82** 1971

[31] Solano E, de Matos Filho R L and Zagury N 1999 *Phys. Rev.* A **59** R2539

[32] Zheng S B 2003 *Phys. Rev.* A **68** 035801

[33] Roos C F, Riebe M, Haffner H, Hansel W, Benhelm J, Lancaster G P T, Becher C, Schmidt-Kaler F and Blatt R 2004 *Science* **304** 1478

[34] James D F V 1998 *Appl. Phys.* B **66** 181

[35] Kreuter A, Becher C, Lancaster G P T, Mundt A B, Russo C, H. Häffner, Roos C, Hänsel W, Schmidt-Kaler F and Blatt R 2005 *Phys. Rev.* A **71** 032504

[36] Meyrath T P and James D F V 1998 *Phys. Lett.* A **240** 37

[37] Kielpinski D, Monroe C and Wineland D J 2002 *Nature* **417** 709

[38] Ye L and Guo G C 2005 *Phys. Rev.* A **71** 034304