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Generation of unconventional geometric phase gates in ion trap-optical cavity system by squeezed operators*

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Based on squeezed operators this paper has implemented an ideal unconventional geometric quantum gate (GQG) in ion trap-optical cavity system by radiating the trapped ions with the cavity field of frequency ω_c and an external laser field of frequency ω_L. It can ensure that the gate time is shorter than the coherence time for qubits and the decay time of the optical cavity by appropriately tuning the ionic transition frequency ω_0, the frequencies of the cavity mode ω_c and the vibrational mode ν. It has also realized the unconventional GQG under the influence of the cavity decay based on the squeezed-like operators and found that the present scheme works well for the smaller cavity decay by investigating the corresponding fidelity and success probability.

Keywords: quantum phase gates, squeezed operators, trapped ions
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1. Introduction

Quantum information represents the latest evolvement of the present quantum physics and one of the most promising aspects in the age of information. Quantum computers host quantum logic gates to perform calculations and are expected to be able to process huge amounts of information so that they can rapidly solve problems which take a long time even for supercomputers. Because of this, much attention has been concentrated on quantum logic gates.[1−9] All the quantum logic gates can be created by two kinds of methods, the dynamical evolution[2−5,10−12] and geometric operation.[13−16] A comparison of the geometric gates with the dynamic gates shows that the former have a higher fidelity and are robust against dephasing.[17] The conventional geometric quantum gates,[13,16,18] obtained by driving the qubits through adiabatic cyclic evolution,[19] require some additional operations to cancel the effects produced by dynamical phases. Whereas the unconventional geometric quantum gates(GQG), obtained by displacing a harmonic oscillator along a closed path related to the state of the qubits,[19] not only possess all the advantages of the conventional geometric gates including the property to resist parameter fluctuations but also are independent of the initial states of the system. Therefore, the unconventional GQG has been considered as a promising choice. So far most of the unconventional GOG are created based on the displacement operators $\hat{D}(\alpha)$[19−23] most of the unconventional GQG are created. For example, Zheng[19,23] proposed two schemes on two-qubit quantum logic gates for atoms in cavity QED and for trapped ions in thermal motion via geometric phase shifts using displacement operators, respectively. Then Chen *et al*[20] proposed a two-qubit unconventional GQG for atoms in cavity QED using displacement operators. Different from the scheme[19] that used the large-detuning interaction of the two atoms with the cavity, Chen's scheme[20] is based on the resonant coupling of the atoms to the classical field. However, recently several proposals[24−26] demonstrated the possibility that a highly squeezed operator $\hat{S}(\xi)$ can finish the same job as the displacement operator $\hat{D}(\alpha)$, i.e., realize the unconventional GQG in certain systems. For example, Chen *et al*[24] created a two-qubit unconventional GQG using squeezed operators $\hat{S}(\xi)$ in the case of the atoms and cavity mode, assisted by a classical field, undergoing two-photon interaction. Joshi and Xiao[25]

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realized a cavity-QED based two-qubit unconventional GQG by squeezed operators $\hat{S}(\xi)$ in the case of bichromatic field modes interacting with the atoms. Applying squeezed operators, Zhu *et al*[26] implemented an unconventional GQG in a system that includes the two-photon interaction between two atoms and an initial vacuum cavity field, assisted by a resonant strong driving classical field. Then a question arises: whether it is possible to break out the limited models of the two-photon interaction or bichromatic field modes interaction in the atom–cavity system to realize the unconventional GQG based on the squeezed operators in other systems. Based on the promising model,[11] in which quantum logic gates with the trapped ions interacting with external laser and quantized cavity field via dynamical evolution were implemented, we implement an unconventional GQG in the ion trap-optical cavity system via geometric phase shift by using squeezed operators.

The outline of the paper is as follows: In Section 2, we introduce the generalized unconventional geometric phase based on squeezed operators. In Section 3, we describe a model in which the trapped ions inside a high finesse cavity couple to the cavity field and the external laser field. In Sections 4 and 5, the unconventional GQG are obtained in ideal cavity and in the cavity with decay, respectively. Discussion and summary appear in Section 6.

2. Generalized unconventional geometric phase based on squeezed operators

As a generalization of the standard unconventional geometric phase shift due to a displacement operator along an arbitrary path in phase space,[27,28] the two-mode squeezed operator can be expressed as[25]

$$S(\xi) = \mathrm{e}^{\xi a^\dagger b^\dagger - \xi^* ab}, \tag{1}$$

where $a^\dagger(a)$ and $b^\dagger(b)$ are the creation(annihilation) operators, respectively, for the cavity mode and the vibrational mode. $\xi = r\mathrm{e}^{\mathrm{i}\theta}$ is a squeezing parameter in the squeezing parameter phase space. When $\Delta\xi_1$ and $\Delta\xi_2$ are approaching zero, we obtain the following relation for the squeezed operators approximately:

$$\begin{aligned} &S(\Delta\xi_2)S(\Delta\xi_1) \\ \simeq &S(\Delta\xi_2+\Delta\xi_1) \\ &\times \exp\left[\mathrm{i}\ \mathrm{Im}(\Delta\xi_2\Delta\xi_1^*)(a^\dagger a + b^\dagger b + 1)\right]. \end{aligned} \tag{2}$$

Consider a path in parametric space consisting of N short straight displacements $\Delta\xi_j$, the total operator is

$$\begin{aligned} S_\mathrm{t} = S(\Delta\xi_N)\cdots S(\Delta\xi_1) \simeq S\left(\sum_{j=1}^{N}\Delta\xi_j\right) \\ \times\exp\left[\mathrm{i}\ \mathrm{Im}\left(\sum_{j=2}^{N}\Delta\xi_j\sum_{k=1}^{j-1}\Delta\xi_k^*\right)(a^\dagger a+b^\dagger b+1)\right]. \end{aligned} \tag{3}$$

If the cavity mode and the vibrational mode are initial in the vacuum state, the term $(a^\dagger a + b^\dagger b + 1)$ has no contribution on the evolution of the system. When $N\to\infty$, the operator S_t is

$$S_\mathrm{t} \simeq S\left(\int_\gamma \mathrm{d}\xi\right)\mathrm{e}^{\mathrm{i}\Theta}, \tag{4}$$

where

$$\Theta \simeq \mathrm{Im}\left(\int_\gamma \xi^*\mathrm{d}\xi\right). \tag{5}$$

For a closed path

$$S_\mathrm{t} \simeq S(0)\mathrm{e}^{\mathrm{i}\Theta} = \mathrm{e}^{\mathrm{i}\Theta}, \tag{6}$$

$$\Theta \simeq \mathrm{Im}\left(\oint \xi^*\mathrm{d}\xi\right), \tag{7}$$

where the phase operator Θ is determined by the area of a loop in the squeezing parameter phase space and is independent of the quantized state of the harmonic oscillator. Therefore, when the cavity mode and the vibrational mode are initially in the vacuum state, the geometric phase generated from the squeezed operator reduces to that in Ref.[19]. The squeezed operator S_t will be used to generate an unconventional GQG in the system without cavity decay.

In order to obtain an unconventional GQG in a system under cavity decay, we define a squeezed-like operator $S'(\xi')$ according to the definition of the squeezed operator $S(\xi)$,

$$S'(\xi') = \mathrm{e}^{\xi'(t)a^\dagger b^\dagger - \xi'(-t)ab}, \tag{8}$$

where $\xi'(t)$ is a time-dependent complex parameter. When $\Delta\xi_1'$ and $\Delta\xi_2'$ are approaching zero, the following relation is satisfied approximately(assume that the cavity mode and the vibrational mode are initially in the vacuum state)

$$S'(\Delta\xi_2')S'(\Delta\xi_1') \simeq S'(\Delta\xi_2'+\Delta\xi_1')\exp\left[\frac{1}{2}\left(\Delta\xi_1'(-t)\Delta\xi_2'(t)-\Delta\xi_1'(t)\Delta\xi_2'(-t)\right)\right]. \tag{9}$$

When a path in parametric space consisting of N short straight displacements $\Delta\xi_j'$, we obtain the total operator

$$\begin{aligned} S_t' =& S'(\Delta\xi_N')\cdots S'(\Delta\xi_1') \\ \simeq& S'\left(\sum_{j=1}^{N}\Delta\xi_j'\right) \\ &\times\exp\left[\frac{1}{2}\left(\sum_{j=2}^{N}\Delta\xi_j'(t)\sum_{k=1}^{j-1}\Delta\xi_k'(-t)-\sum_{j=2}^{N}\Delta\xi_j'(-t)\sum_{k=1}^{j-1}\Delta\xi_k'(t)\right)\right]. \end{aligned} \tag{10}$$

In the case of $N\to\infty$, we have

$$S_t' \simeq S'\left(\int_\gamma \mathrm{d}\xi'(t)\right)\mathrm{e}^{-A}\mathrm{e}^{\mathrm{i}\Theta'}, \tag{11}$$

where

$$\begin{aligned} \Theta' &\simeq \frac{1}{2}\mathrm{Im}\left[\int_\gamma \xi'(-t)\mathrm{d}\xi'(t)-\int_\gamma \xi'(t)\mathrm{d}\xi'(-t)\right], \\ A &\simeq -\frac{1}{2}\mathrm{Re}\left[\int_\gamma \xi'(-t)\mathrm{d}\xi'(t)-\int_\gamma \xi'(t)\mathrm{d}\xi'(-t)\right]. \end{aligned} \tag{12}$$

And for a closed path, we have

$$S_t' \simeq S'(0)\mathrm{e}^{-A}\mathrm{e}^{\mathrm{i}\Theta'} = \mathrm{e}^{-A}\mathrm{e}^{\mathrm{i}\Theta'}, \tag{13}$$

where

$$\begin{aligned} \Theta' &\simeq \frac{1}{2}\mathrm{Im}\left[\oint \xi'(-t)\mathrm{d}\xi'(t)-\oint \xi'(t)\mathrm{d}\xi'(-t)\right], \\ A &\simeq -\frac{1}{2}\mathrm{Re}\left[\oint \xi'(-t)\mathrm{d}\xi'(t)-\oint \xi'(t)\mathrm{d}\xi'(-t)\right]. \end{aligned} \tag{14}$$

Therefore we have obtained a squeezed-like operator S_t' which will be used to generate an unconventional GQG in the system under the cavity decay.

3. The model for ion trap-optical cavity system

We consider two identical three-level trapped ions which are placed inside a high finesse cavity, and each ion has one excited state $|r\rangle$ and two ground states $|e\rangle$ and $|g\rangle$. The two ions are both driven with a laser of frequency ω_L and a single-mode cavity field of frequency ω_c. Assume that the state $|g\rangle$ is not affected during the interaction. In the rotating-wave approximation, the Hamiltonian for this system is given by[11] (assume $\hbar=1$, $\phi=0$)

$$\begin{aligned} H =& \omega_0\sum_{j=1,2}S_{z,j}+\omega_\mathrm{c}a^\dagger a+\nu b^\dagger b \\ &+\sum_{j=1,2}[\Omega(S_j^+\mathrm{e}^{-\mathrm{i}[\omega_\mathrm{L}t-\eta_\mathrm{L}(b^\dagger+b)]}+\mathrm{H.c.}) \\ &+g(S_j^++S_j^-)(a^\dagger+a)\sin[\eta_\mathrm{c}(b^\dagger+b)]], \end{aligned} \tag{15}$$

where $S_{z,j}=\frac{1}{2}(|r_j\rangle\langle r_j|-|e_j\rangle\langle e_j|)$, $S_j^+=|r_j\rangle\langle e_j|$, and $S_j^-=|e_j\rangle\langle r_j|$, and $b^\dagger(b)$ and $a^\dagger(a)$ are the creation(annihilation) operators for the vibrational mode and cavity mode, respectively, and ω_0 is the transition frequency for the $|e_j\rangle\leftrightarrow|r_j\rangle$, ν is the vibrational mode frequency, and Ω is the Rabi frequency. $\eta_\mathrm{c}(\eta_\mathrm{L})$ is the Lamb–Dicke parameter which depends on the cavity field (external laser field). Consider that $\omega_0=\omega_\mathrm{L}$ and the resolved sideband regime, the vibrational mode frequency ν and cavity mode frequency ω_c are much larger than $\Delta=\omega_0-\omega_\mathrm{c}-\nu$, Ω, and g(i.e., ν, $\omega_\mathrm{c}\gg\Delta$, Ω, and g), and the Lamb–Dicke limit $\eta\ll1$. We can discard the rapidly oscillating terms and expand the interaction picture Hamiltonian to the first order in η,

$$\begin{aligned} H_I' =& \sum_{j=1,2}\Omega(S_j^++S_j^-) \\ &+\eta g(S_j^+ab\mathrm{e}^{\mathrm{i}\Delta t}+S_j^-a^\dagger b^\dagger\mathrm{e}^{-\mathrm{i}\Delta t}). \end{aligned} \tag{16}$$

Performing the transformation $\mathrm{e}^{\mathrm{i}H_0t}H_i\mathrm{e}^{-\mathrm{i}H_0t}$, we obtain

$$\begin{aligned} H_i' =& \mathrm{e}^{\mathrm{i}H_0t}H_i\mathrm{e}^{-\mathrm{i}H_0t} \\ =& \frac{\eta g}{2}\sum_{j=1,2}[|+\rangle_{jj}\langle+|-|-\rangle_{jj}\langle-| \\ &+\mathrm{e}^{\mathrm{i}2\Omega t}|+\rangle_{jj}\langle-| \\ &-\mathrm{e}^{-\mathrm{i}2\Omega t}|-\rangle_{jj}\langle+|]ab\mathrm{e}^{\mathrm{i}\Delta t}+\mathrm{H.c.}, \end{aligned} \tag{17}$$

where

$$H_0=\sum_{j=1,2}\Omega(S_j^++S_j^-), \tag{18}$$

and

$$H_i=\sum_{j=1,2}\eta g(S_j^+ab\mathrm{e}^{i\Delta t}+S_j^-a^\dagger b^\dagger\mathrm{e}^{-\mathrm{i}\Delta t}), \tag{19}$$

with $|\pm\rangle_j = (|e\rangle \pm |r\rangle)_j/\sqrt{2}$. In the strong driving regime $\Omega \gg \Delta, \eta g$, we can neglect the terms oscillating fast in Eq.(17). So the effective Hamiltonian can be expressed as

$$H_{\rm eff} = \frac{\eta g}{2}\sum_{j=1,2}(|+\rangle_{jj}\langle +| - |-\rangle_{jj}\langle -|) \times (ab{\rm e}^{{\rm i}\Delta t} + a^\dagger b^\dagger {\rm e}^{-{\rm i}\Delta t}). \quad (20)$$

From the above Hamiltonian, the terms that related to the evolution of the encoded qubit states $|g\rangle_j$ and $|-\rangle_j$ are

$$H'_{\rm eff} = -\frac{\eta g}{2}\sum_{j=1,2}(ab{\rm e}^{{\rm i}\Delta t} + a^\dagger b^\dagger {\rm e}^{-{\rm i}\Delta t})|-\rangle_{jj}\langle -|. \quad (21)$$

From the expression of the squeezed operator in Eq.(1), the corresponding evolution operator of the states for an infinitesimal interval dt can be expressed as

$$U({\rm d}t) = S({\rm d}\xi) = {\rm e}^{a^\dagger b^\dagger {\rm d}\xi - ab{\rm d}\xi^*} = {\rm e}^{-{\rm i}H'_{\rm eff}{\rm d}t}, \quad (22)$$

where

$${\rm d}\xi = i\frac{\eta g}{2}{\rm e}^{-{\rm i}\Delta t}{\rm d}t. \quad (23)$$

We note that only the qubit states including a single state $|-\rangle_j$ will evolve according to $S({\rm d}\xi)$, and that including l states $|-\rangle_j (l = 1, 2, \cdots, N)$ will evolve according to $S(l{\rm d}\xi)$, while the other states will not evolve. After an interaction time t, the squeezing parameter $\xi(t)$ and the phase shifts Θ can be written as

$$\xi(t) = \int_0^t {\rm i}\frac{\eta g}{2}{\rm e}^{-{\rm i}\Delta t}{\rm d}t = \frac{\eta g}{2\Delta}(1 - {\rm e}^{-{\rm i}\Delta t}),$$
$$\xi_l(t) = l\xi(t), \quad (24)$$

and

$$\Theta = {\rm Im}\left(\int_\gamma \xi^* {\rm d}\xi\right) = -\frac{(\eta g)^2}{4\Delta^2}[\Delta t - \sin(\Delta t)],$$
$$\Theta_\lambda = {\rm Im}\left(\int_\gamma \lambda\xi^* {\rm d}\lambda\xi\right) = \lambda^2\Theta, \quad (25)$$

respectively.

4. The unconventional GQG in ideal cavity

From the Eq.(25), the condition for a periodical evolution is $\Delta t = 2\pi m$, where m is a positive integer. Assume that the centre-of-mass vibrational mode and the cavity mode are initially prepared in the vacuum state $|0, 0\rangle$. After an interaction time t, the states of the system evolve to

$$|g\rangle_1|g\rangle_2|0\rangle|0\rangle \longrightarrow |g\rangle_1|g\rangle_2|0\rangle|0\rangle,$$
$$|g\rangle_1|-\rangle_2|0\rangle|0\rangle \longrightarrow {\rm e}^{{\rm i}\Theta}S(\xi)|g\rangle_1|-\rangle_2|0\rangle|0\rangle,$$
$$|-\rangle_1|g\rangle_2|0\rangle|0\rangle \longrightarrow {\rm e}^{{\rm i}\Theta}S(\xi)|-\rangle_1|g\rangle_2|0\rangle|0\rangle,$$
$$|-\rangle_1|-\rangle_2|0\rangle|0\rangle \longrightarrow {\rm e}^{{\rm i}4\Theta}S(2\xi)|-\rangle_1|-\rangle_2|0\rangle|0\rangle. \quad (26)$$

If the interaction time satisfies $\Delta t = 2\pi m$, the above states become

$$|g\rangle_1|g\rangle_2|0\rangle|0\rangle \longrightarrow |g\rangle_1|g\rangle_2|0\rangle|0\rangle,$$
$$|g\rangle_1|-\rangle_2|0\rangle|0\rangle \longrightarrow {\rm e}^{{\rm i}\Theta}|g\rangle_1|-\rangle_2|0\rangle|0\rangle,$$
$$|-\rangle_1|g\rangle_2|0\rangle|0\rangle \longrightarrow {\rm e}^{{\rm i}\Theta}|-\rangle_1|g\rangle_2|0\rangle|0\rangle,$$
$$|-\rangle_1|-\rangle_2|0\rangle|0\rangle \longrightarrow {\rm e}^{{\rm i}4\Theta}|-\rangle_1|-\rangle_2|0\rangle|0\rangle, \quad (27)$$

where $\Theta = -m\pi(\eta g)^2/2\Delta^2$. Therefore, the states evolution for the system decided by H'_I is

$$|g\rangle_1|g\rangle_2|0\rangle|0\rangle \longrightarrow |g\rangle_1|g\rangle_2|0\rangle|0\rangle,$$
$$|g\rangle_1|-\rangle_2|0\rangle|0\rangle \longrightarrow {\rm e}^{{\rm i}(\Theta+\Omega t)}|g\rangle_1|-\rangle_2|0\rangle|0\rangle,$$
$$|-\rangle_1|g\rangle_2|0\rangle|0\rangle \longrightarrow {\rm e}^{{\rm i}(\Theta+\Omega t)}|-\rangle_1|g\rangle_2|0\rangle|0\rangle,$$
$$|-\rangle_1|-\rangle_2|0\rangle|0\rangle \longrightarrow {\rm e}^{{\rm i}(4\Theta+2\Omega t)}|-\rangle_1|-\rangle_2|0\rangle|0\rangle. \quad (28)$$

After performing the single-qubit operation $|-\rangle_j \rightarrow {\rm e}^{-{\rm i}(\Theta+\Omega t)}|-\rangle_j (j = 1, 2)$, we obtain the states of the system

$$|g\rangle_1|g\rangle_2|0\rangle|0\rangle \longrightarrow |g\rangle_1|g\rangle_2|0\rangle|0\rangle,$$
$$|g\rangle_1|-\rangle_2|0\rangle|0\rangle \longrightarrow |g\rangle_1|-\rangle_2|0\rangle|0\rangle,$$
$$|-\rangle_1|g\rangle_2|0\rangle|0\rangle \longrightarrow |-\rangle_1|g\rangle_2|0\rangle|0\rangle,$$
$$|-\rangle_1|-\rangle_2|0\rangle|0\rangle \longrightarrow {\rm e}^{{\rm i}2\Theta}|-\rangle_1|-\rangle_2|0\rangle|0\rangle. \quad (29)$$

Choosing $2\Theta = -\pi$, we obtain a π-phase gate.

5. The unconventional GQG under the cavity decay

When we consider the cavity decay, the Hamiltonian that describes the system in Section 3 can be expressed as

$$H = \omega_0\sum_{j=1,2}S_{z,j} + \omega_{\rm c}a^\dagger a + \nu b^\dagger b - \frac{\rm i}{2}\kappa a^\dagger a + \sum_{j=1,2}[\Omega(S_j^+{\rm e}^{-{\rm i}[\omega_L t - \eta_L(b^\dagger + b)]} + {\rm H.c.}) + g(S_j^+ + S_j^-)(a^\dagger + a) \times \sin[\eta_{\rm c}(b^\dagger + b)])], \quad (30)$$

where κ is the decay rate of the cavity. Assume that the cavity mode and the vibrational mode are initially in the vacuum state. By the similar derivation as that

in Section 3, the effective Hamiltonian under the conditions of $\Omega \gg \Delta, \eta g$ and $\kappa t \ll \Delta t$ is

$$H_{\rm eff}^{\kappa} = \frac{\eta g}{2} \sum_{j=1,2} (|+\rangle_{jj}\langle +| - |-\rangle_{jj}\langle -|) \times (ab{\rm e}^{{\rm i}\Delta t}{\rm e}^{-\frac{\kappa}{2}t} + a^{\dagger}b^{\dagger}{\rm e}^{-{\rm i}\Delta t}{\rm e}^{\frac{\kappa}{2}t}). \tag{31}$$

The terms that related to the evolution of the encoded qubit states $|g\rangle_j$ and $|-\rangle_j$ are

$$H_{\rm eff}^{\kappa'} = -\frac{\eta g}{2} \sum_{j=1,2} |-\rangle_{jj}\langle -| \times (ab{\rm e}^{{\rm i}\Delta t}{\rm e}^{-\frac{\kappa}{2}t} + a^{\dagger}b^{\dagger}{\rm e}^{-{\rm i}\Delta t}{\rm e}^{\frac{\kappa}{2}t}). \tag{32}$$

So the evolution operator of the states during the infinitesimal interval ${\rm d}t$ is

$$\begin{aligned} U'({\rm d}t) =& S'({\rm d}\xi') = {\rm e}^{{\rm d}\xi'(t)a^{\dagger}b^{\dagger} - {\rm d}\xi'(-t)ab} \\ =& {\rm e}^{-{\rm i}H_{\rm eff}^{\kappa'}{\rm d}t}, \end{aligned} \tag{33}$$

where

$${\rm d}\xi'(t) = {\rm i}\frac{\eta g}{2}{\rm e}^{(-{\rm i}\Delta + \frac{\kappa}{2})t}{\rm d}t. \tag{34}$$

After an interaction time t, the squeezing parameter $\xi'(t)$, phase shift Θ', and the additional term A can be expressed as

$$\begin{aligned} \xi'(t) &= \frac{\eta g({\rm i}\kappa - 2\Delta)}{\kappa^2 + 4\Delta^2}[{\rm e}^{(-{\rm i}\Delta + \frac{\kappa}{2})t} - 1], \\ \xi_l'(t) &= l\xi'(t), \end{aligned} \tag{35}$$

$$\begin{aligned} \Theta' &= -\frac{\eta^2 g^2}{(4\Delta^2 + \kappa^2)^2}\left[(4\Delta^2 - \kappa^2)\left(\Delta t - \sin\Delta t\cosh\frac{\kappa t}{2}\right) + 4\Delta\kappa\left(\frac{\kappa t}{2} - \cos\Delta t\sinh\frac{\kappa t}{2}\right)\right], \\ \Theta_l' &= l^2\Theta', \end{aligned} \tag{36}$$

and

$$\begin{aligned} A &= \frac{\eta^2 g^2}{(4\Delta^2 + \kappa^2)^2}\left[(4\Delta^2 - \kappa^2)\left(\cos\Delta t\sinh\frac{\kappa t}{2} - \frac{\kappa t}{2}\right) + 4\Delta\kappa\left(\Delta t - \sin\Delta t\cosh\frac{\kappa t}{2}\right)\right], \\ A_l &= l^2 A, \end{aligned} \tag{37}$$

respectively. The required condition for periodical evolution is $\Delta t = 2\pi m$, where m is a positive integer. After an interaction time $t = 2\pi m/\Delta$, the evolutions of the states are

$$\begin{aligned} |g\rangle_1|g\rangle_2|0\rangle|0\rangle &\longrightarrow |g\rangle_1|g\rangle_2|0\rangle|0\rangle, \\ |g\rangle_1|-\rangle_2|0\rangle|0\rangle &\longrightarrow {\rm e}^{-A}{\rm e}^{{\rm i}(\Theta' + \Omega t)}S_t'(\xi')|g\rangle_1|-\rangle_2|0\rangle|0\rangle \\ &\longrightarrow {\rm e}^{-A}{\rm e}^{{\rm i}(\Theta' + \Omega t)}|g\rangle_1|-\rangle_2|0\rangle|0\rangle, \\ |-\rangle_1|g\rangle_2|0\rangle|0\rangle &\longrightarrow {\rm e}^{-A}{\rm e}^{{\rm i}(\Theta' + \Omega t)}S_t'(\xi')|-\rangle_1|g\rangle_2|0\rangle|0\rangle \\ &\longrightarrow {\rm e}^{-A}{\rm e}^{{\rm i}(\Theta' + \Omega t)}|-\rangle_1|g\rangle_2|0\rangle|0\rangle, \\ |-\rangle_1|-\rangle_2|0\rangle|0\rangle &\longrightarrow {\rm e}^{-4A}{\rm e}^{{\rm i}(4\Theta' + 2\Omega t)}S_t'(2\xi')|-\rangle_1|-\rangle_2|0\rangle|0\rangle \\ &\longrightarrow {\rm e}^{-4A}{\rm e}^{{\rm i}(4\Theta' + 2\Omega t)}|-\rangle_1|-\rangle_2|0\rangle|0\rangle. \end{aligned} \tag{38}$$

Performing a single-qubit operation $|-\rangle_j \to {\rm e}^{-{\rm i}(\Theta' + \Omega t)}|-\rangle_j (j = 1, 2)$, we obtain the states of the system

$$\begin{aligned} &|g\rangle_1|g\rangle_2|0\rangle|0\rangle \longrightarrow |g\rangle_1|g\rangle_2|0\rangle|0\rangle, && |g\rangle_1|-\rangle_2|0\rangle|0\rangle \longrightarrow {\rm e}^{-A}|g\rangle_1|-\rangle_2|0\rangle|0\rangle, \\ &|-\rangle_1|g\rangle_2|0\rangle|0\rangle \longrightarrow {\rm e}^{-A}|-\rangle_1|g\rangle_2|0\rangle|0\rangle, && |-\rangle_1|-\rangle_2|0\rangle|0\rangle \longrightarrow {\rm e}^{-4A}{\rm e}^{{\rm i}2\Theta'}|-\rangle_1|-\rangle_2|0\rangle|0\rangle. \end{aligned} \tag{39}$$

Choosing $2\Theta' = -\pi$, we obtain a π-phase gate under the influence of the cavity decay. And the condition

$$\frac{\eta^2 g^2\left\{(4\Delta^2 - \kappa^2)\left[2\pi m - \sin(2\pi m)\cosh\frac{\kappa\pi m}{\Delta}\right] + 4\Delta\kappa\left[\frac{\kappa\pi m}{\Delta} - \cos(2\pi m)\sinh\frac{\kappa\pi m}{\Delta}\right]\right\}}{(4\Delta^2 + \kappa^2)^2} = \frac{\pi}{2} \tag{40}$$

should be satisfied.

6. Discussion and summary

We have numerically calculated the fidelity F and the success probability P in the case of the cavity decay by using the expressions in Ref.[20]. The curves of F, P versus $\frac{\kappa}{\eta g}$ with m and $\frac{\Delta}{\eta g}$ being fixed are shown in Fig.1. From Fig.1 the smaller the cavity decay is, the higher F and P are. For the first(second) group curves $m = 2$ and $\frac{\Delta}{\eta g} = 2\left(m = 10 \text{ and } \frac{\Delta}{\eta g} = 5\right)$, when $\frac{\kappa}{\eta g} \leq 0.008\left(\frac{\kappa}{\eta g} \leq 0.026\right)$, both the fidelity and the success probability exceed 0.99, while for $\frac{\kappa}{\eta g} \geq 1.96\left(\frac{\kappa}{\eta g} \geq 1.17\right)$, both the fidelity and the success probability reach 0.25, in this case, our scheme fails. This is to say, when the condition $\frac{\kappa}{\eta g} \ll \frac{\Delta}{\eta g}$ is satisfied, our scheme works well. When $\frac{\kappa}{\eta g} \leq 0.66$, the higher F and P appear for $m = 10$ compared to $m = 2$, while the higher F and P appear for $m = 2$ when $\frac{\kappa}{\eta g} > 0.66$. We have also investigated the influences of the period parameter m on F and P when the values for $\frac{\kappa}{\eta g}$ and $\frac{\Delta}{\eta g}$ are fixed. We find that the larger the value for m is, the smaller F and P are.

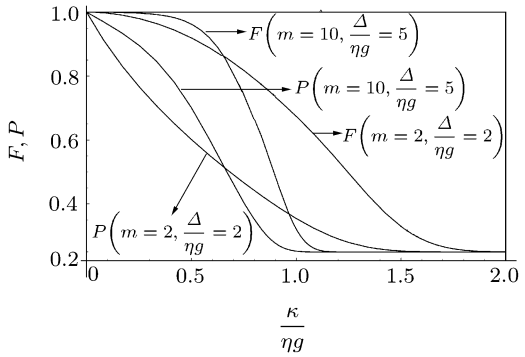


Fig.1. The curves of the fidelity F and the success probability P versus $\frac{\kappa}{\eta g}$ when $\frac{\Delta}{\eta g} = 2$, $m = 2$ and $\frac{\Delta}{\eta g} = 5$, $m = 10$.

Then we discuss the time for implementing the unconventional GQG. The gate time t is expressed as $t = 2\pi/\Delta = 2\pi/(\omega_0 - \omega_c - \nu)$ (assume $m = 1$). Consider that the recently reported coherence time is about 1ms for qubits based on Ca^+ ion[29] and the optical cavity decay time is about 2μs.[30,31] We can tune the frequencies ω_0, ω_c, and ν, appropriately, to ensure that not only the restrictive condition ν, $\omega_c \gg \Delta$, Ω, and g is satisfied but also the value for Δ is large enough so that the gate time is shorter than the coherence time and the optical cavity decay time. It is worth mentioning that another restrictive condition $\kappa t \ll \Delta t$ must be satisfied to avoid the quantum jump.

Consider that a coherent state can be obtained by acting the displacement operator $D(\alpha)$ on a vacuum state and a squeezed vacuum state can be obtained by acting the squeezed operator $S(\xi)$ on a vacuum state. A comparison of the schemes based on squeezed operators with that based on displacement operators shows that the former have some advantages in some applications. For example, in the nonclassical squeezed states of lights that have potential applications in optical communications and in gravitational detection,[32] the quantum fluctuations of one quadrature can be suppressed below the vacuum level. On the other hand, we should note that the present scheme must be implemented under the limited situation of $\Delta\xi_i' \to 0$.

In summary, we have implemented an ideal unconventional geometric phase gate via the interaction of the trapped ions with the cavity field and the external laser field based on squeezed operators. The property of the squeezed operator makes it finish the same job (i.e., realize the unconventional GQG) as the displacement operator in certain systems. Our scheme uses the creation and annihilation operators for the vibrational mode of the laser and the cavity mode to construct the squeezed operator and breaks out the bondage of the previous two-photon[24,26] or the bichromatic field-mode[25] interaction in the atom–cavity system to realize the unconventional GQG in the ion trap-optical cavity system. We have also considered the nonideal case, i.e., realize the unconventional GQG under the cavity decay. By calculating the corresponding fidelity and the success probability, we find that the present scheme is suitable for the smaller cavity decay.

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