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Scheme for realizing assisted cloning of an unknown two-atom entangled state via cavity QED

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This paper proposes a scheme where one can realize quantum cloning of an unknown two-atom entangled state with assistance of a state preparer in cavity QED. The first stage of the scheme requires usual teleportation. In the second stage of the scheme, with the assistance of the preparer, the perfect copies of an unknown atomic entangled state can be produced.

Keywords: quantum cloning, two-atom entangled state, cavity QED
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Quantum entanglement has generated much interest in quantum information processing such as quantum teleportation,[1] quantum dense coding,[2] quantum computation,[3] quantum cryptography,[4] and so on. In recent years, the possibility of cloning quantum states approximately has attracted much attention. A quantum state can not be cloned exactly because of the no-cloning theorem.[5,6] However, quantum cloning approximately is necessary in quantum information.[7] Though exact cloning is not possible, in the literature various cloning machines have been proposed. Universal quantum cloning machines was originally addressed by Bužek and Hillery.[8] The deterministic state-dependent cloning machine, proposed first by Hillery and Bužek,[9] is designed to generate approximate clones of states belonging to a finite set. Gisin and Massar[10] and Bruβ *et al*[11] constructed the universal qubit cloner that maximizes the local fidelity. The probabilistic cloning machine was first considered by Duan and Guo[12] using a general unitary-reduction operation with a postelection of the measurement results. Murao *et al*[13] proposed the quantum telecloning process combining quantum teleportation and optimal quantum cloning from one input to M outputs. Moreover, Pati[14] proposed a scheme of assisted cloning and orthogonal complementing of an unknown state. In Ref.[14], Pati studied the possibility of copying and complementing an unknown state perfectly with 'minimal assistance' from the state preparer (Victor). Pati shows that Victor can use the entangled state left after the teleportation process and send one classical bit to Alice to create either a copy or an orthogonal-complement copy of the unknown state. Inspired by the scheme of Ref.[14], we presented[15,16] two proposals where one can realize quantum cloning of an unknown two-particle entangled state and its orthogonal complement state with assistance.

Recently, cavity quantum electrodynamics (QED) technology has been widely applied to the quantum information field. This is due to the fact that cold and localized atoms are not only the important resource of entanglement, but also well suited for storing quantum information in long lived internal states. Zheng and Guo[17] proposed a scheme for the two-atom entanglement preparation in cavity QED. Osnaghi *et al*[18] have experimentally demonstrated this scheme by the use of two Rydberg atoms crossing a nonresonant cavity. Many authors[19−25] proposed schemes for the teleportation of unknown atomic states. In particular, Riebe *et al*[26] and Barrett *et al*[27] have implemented quantum teleportation of atomic qubits with an average fidelity of 75% by using Ca^+ ions in a linear Paul trap and with an average fidelity of 78% using Be^+ ions in a segmented ion trap, respectively. Recently, Zheng[28] proposed a scheme for the approximate conditional teleportation of an unknown atomic state in cavity QED. Some authors[29−39] have presented schemes for realizing teleportation of atomic states via cavity QED. In this paper, we propose a scheme which can produce copies of an unknown atomic entangled state with assistance in cavity QED.

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We first describe how to prepare a two-atom entangled state. Consider two identical two-level atoms simultaneously interacting with a single-mode cavity and driven by a classical field. The interaction Hamiltonian of the atom–cavity can be expressed as[40]

$$H = \omega_0 \sum_{j=1}^{2} S_{z,j} + \omega_1 a^+ a + \sum_{j=1}^{2} [g(a^+ S_j^- + a S_j^+) + \Omega(S_j^+ \mathrm{e}^{-\mathrm{i}\omega_2 t} + S_j^- \mathrm{e}^{\mathrm{i}\omega_2 t})], \quad (1)$$

where a^+ and a are the creation operator and annihilation operator of the cavity mode respectively, g is atom–cavity coupling strength, $S_j^- = |g\rangle_{jj}\langle e|$, $S_j^+ = |e\rangle_{jj}\langle g|$ and $S_{z,j} = \frac{1}{2}(|e\rangle_{jj}\langle e| - |g\rangle_{jj}\langle g|)$ are atomic operators. $|e\rangle_j$ and $|g\rangle_j$ are the excited and ground states of jth atom, respectively. ω_0, ω_1,ω_2 and Ω are atomic transition frequency ($e \leftrightarrow g$), cavity frequency, driving field frequency and the Rabi frequency of the classical field, respectively. In the case of large detuning $\delta \gg g/2$ (g limit) and strong driving field $2\Omega \gg \delta$, the effective Hamiltonian of this interaction system is[40]

$$H_{\mathrm{eff}} = \lambda \left[\frac{1}{2} \sum_{j=1}^{2} (|e\rangle_{jj}\langle e| + |g\rangle_{jj}\langle g|) + \sum_{j,k=1, j\neq k}^{2} (S_j^+ S_k^+ + S_j^+ S_k^- + \mathrm{H.c.}) \right], \quad (2)$$

where $\lambda = g^2/2\delta$, $\delta = \omega_0 - \omega_1$ and $\omega_0 = \omega_2$. If two atoms are simultaneously sent into the cavity then they interact with the cavity. Using the evolution operator of the system $U(t) = \mathrm{e}^{-\mathrm{i}H_0 t}\mathrm{e}^{-\mathrm{i}H_{\mathrm{e}} t}$ with $H_0 = \sum_{j=1}^{2} \Omega(S_j^+ + S_j^-)$ and H_{e} is the effective Hamiltonian, we easily verify the evolutions by adjusting the interaction time $\lambda t = \pi/4$ and modulating the driving field $\Omega t = \pi$,

$$\begin{aligned} |gg\rangle_{12} &\rightarrow \frac{1}{\sqrt{2}}(|gg\rangle_{12} - \mathrm{i}|ee\rangle_{12}), \\ |ge\rangle_{12} &\rightarrow \frac{1}{\sqrt{2}}(|ge\rangle_{12} - \mathrm{i}|eg\rangle_{12}), \\ |eg\rangle_{12} &\rightarrow \frac{1}{\sqrt{2}}(|eg\rangle_{12} - \mathrm{i}|ge\rangle_{12}), \\ |ee\rangle_{12} &\rightarrow \frac{1}{\sqrt{2}}(|ee\rangle_{12} - \mathrm{i}|gg\rangle_{12}). \end{aligned} \quad (3)$$

Thus we have prepared the four maximally bipartite entangled states from the product states of the two atoms.

Suppose that Alice has an unknown input two-atom entangled state $|\phi\rangle_{12} = \alpha|ee\rangle_{12} + \beta|gg\rangle_{12}$ from a state preparer Victor, with α as a real number and β as a complex number, and $|\alpha|^2 + |\beta|^2 = 1$. Alice wants to transmit $|\phi\rangle_{12}$ to receiver Bob. Assume Alice and Bob share two two-atom entangled states, as the quantum channel, give by

$$\begin{aligned} |\psi\rangle_{34} &= \frac{1}{\sqrt{2}}(|ee\rangle - \mathrm{i}\,|gg\rangle)_{34}, \\ |\psi\rangle_{56} &= \frac{1}{\sqrt{2}}(|ee\rangle - \mathrm{i}\,|gg\rangle)_{56}, \end{aligned} \quad (4)$$

where the atoms 3 and 5 belong to the sender Alice and the other two atoms 4 and 6 belong to Bob.

The initial state of the whole system composed of six atoms is given by

$$\begin{aligned} |\Psi\rangle &= |\phi\rangle_{12} \otimes |\psi\rangle_{34} \otimes |\psi\rangle_{56} \\ &= \frac{1}{4}\big[|\Phi^+\rangle_{13}|\Phi^\pm\rangle_{25}(\alpha|ee\rangle \pm_2 \beta|gg\rangle)_{46} \\ &\quad + |\Phi^+\rangle_{13}|\Psi^\pm\rangle_{25}(-\beta|ge\rangle \mp_2 \alpha|eg\rangle)_{46} \\ &\quad + |\Phi^-\rangle_{13}|\Phi^\pm\rangle_{25}(\alpha|ee\rangle \mp_2 \beta|gg\rangle)_{46} \\ &\quad + |\Phi^-\rangle_{13}|\Psi^\pm\rangle_{25}(\beta|ge\rangle \mp_2 \alpha|eg\rangle)_{46} \\ &\quad + |\Psi^+\rangle_{13}|\Phi^\pm\rangle_{25}(-\alpha|ge\rangle \pm_2 \beta|eg\rangle)_{46} \\ &\quad + |\Psi^+\rangle_{13}|\Psi^\pm\rangle_{25}(\beta|ee\rangle \pm_2 \alpha|gg\rangle)_{46} \\ &\quad + |\Psi^-\rangle_{13}|\Phi^\pm\rangle_{25}(\alpha|ge\rangle \mp_2 \beta|eg\rangle)_{46} \\ &\quad + |\Psi^-\rangle_{13}|\Psi^\pm\rangle_{25}(\beta|ee\rangle \mp_2 \alpha|gg\rangle)_{46}\big], \end{aligned} \quad (5)$$

where $|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|ee\rangle \pm \mathrm{i}\,|gg\rangle)$ and $|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|ge\rangle \pm \mathrm{i}\,|eg\rangle)$ are four Bell states of atom pairs (1, 3) and (2, 5).

In teleportation of entangled states, the joint Bell-state measurement (BSM) is very complex for experimental realization.[26] In order to avoid the difficulty, here we investigate separate atomic measurements to achieve the goal of joint BSM by using optical cavity.

Alice will send the two atoms 1, 3 into an optical cavity which is introduced beforehand. Without loss of generality, we assume that the quantum state of atom pair (1, 3) is

$$|\Phi^+\rangle_{13} = \frac{1}{\sqrt{2}}(|ee\rangle_{13} + \mathrm{i}\,|gg\rangle_{13}). \quad (6)$$

Two atoms will interact simultaneously with the cavity mode. With the choice of $\lambda t = \frac{\pi}{4}$ and $\Omega t = \pi$, according to Eq.(3) the state of the atom pair will become

$$\begin{aligned} |\Phi^+\rangle_{13} &= \frac{1}{\sqrt{2}}(|ee\rangle_{13} + \mathrm{i}\,|gg\rangle_{13}) \rightarrow \frac{1}{2}[(|ee\rangle_{13} \\ &\quad -\mathrm{i}\,|gg\rangle_{13}) + \mathrm{i}(|gg\rangle_{13} - \mathrm{i}\,|ee\rangle_{13}) \\ &= |ee\rangle_{13}. \end{aligned} \quad (7)$$

In this way, Alice can achieve the goal of BSM by separate atomic measurement. Similarly the other three Bell-states of the atom pairs (1, 3) would evolve into

$$\begin{aligned}
\left|\Phi^{-}\right\rangle_{13} &= \frac{1}{\sqrt{2}}\left(|ee\rangle_{13} - \mathrm{i}\,|gg\rangle_{13}\right) \rightarrow -\mathrm{i}\,|gg\rangle_{13}\,, \\
\left|\Psi^{+}\right\rangle_{13} &= \frac{1}{\sqrt{2}}\left(|ge\rangle_{13} + \mathrm{i}\,|eg\rangle_{13}\right) \rightarrow |ge\rangle_{13}\,, \\
\left|\Psi^{-}\right\rangle_{13} &= \frac{1}{\sqrt{2}}\left(|ge\rangle_{13} - \mathrm{i}\,|eg\rangle_{13}\right) \rightarrow -8\,|eg\rangle_{13}\,. \quad (8)
\end{aligned}$$

The four Bell-states of atom pair (2, 5) can also be distinguished successfully in the same way. After these separate atomic measurements, Alice will inform Bob her measurement results through a classical channel. Then Bob can reconstruct the original atomic entangled state by corresponding unitary transformation.

To create a copy of the unknown two-atom entangled state $|\phi\rangle$, Alice needs assistance of Victor. We assume, without loss of generality, the quantum states of atom pairs (1, 3) and (2, 5) are $\left|\Phi^{+}\right\rangle_{13}\left|\Phi^{-}\right\rangle_{25}$. Alice sends atoms 1 and 2 to Victor and keeps atoms 3, and 5 in her possession. Then Victor will send the atoms 1 and 2 into an optical cavity. The evolution of the total system composed of atom pairs (1, 3) and (2, 5) can be expressed as

$$\begin{aligned}
&\left|\Phi^{+}\right\rangle_{13}\left|\Phi^{-}\right\rangle_{25} \\
&= \frac{1}{2}(|ee\rangle_{13} + \mathrm{i}\,|gg\rangle_{13})(|ee\rangle_{25} - \mathrm{i}\,|gg\rangle_{25}) \\
&\rightarrow \mathrm{e}^{-\mathrm{i}\lambda t}\,|ee\rangle_{35}\left[\cos\lambda t\,|ee\rangle_{12} - \mathrm{i}\sin\lambda t\,|gg\rangle_{12}\right] \\
&\quad -\mathrm{i}\mathrm{e}^{-\mathrm{i}\lambda t}\,|eg\rangle_{35}\left[\cos\lambda t\,|eg\rangle_{12} - \mathrm{i}\sin\lambda t\,|ge\rangle_{12}\right] \\
&\quad +\mathrm{i}\mathrm{e}^{-\mathrm{i}\lambda t}\,|ge\rangle_{35}\left[\cos\lambda t\,|ge\rangle_{12} - \mathrm{i}\sin\lambda t\,|eg\rangle_{12}\right] \\
&\quad +\mathrm{e}^{-\mathrm{i}\lambda t}\,|gg\rangle_{35}\left[\cos\lambda t\,|gg\rangle_{12} - \mathrm{i}\sin\lambda t\,|ee\rangle_{12}\right], \quad (9)
\end{aligned}$$

here Victor makes the Rabi frequency Ω satisfy the condition $\Omega t = \pi$ through modulating the classical field. Then Victor will measure the state of atoms 1 and 2. If his measurement outcome is $|ee\rangle_{12}$, the state of atoms 3 and 5 will become

$$|\psi\rangle_{35} = \mathrm{e}^{-\mathrm{i}\lambda t}(\cos\lambda t\,|ee\rangle_{35} - \mathrm{i}\sin\lambda t\,|gg\rangle_{35}). \quad (10)$$

Since Victor has completely known the parameters α and β, he can select the atomic velocity to meet the condition $\cos\lambda t = \alpha$ (or $\sin\lambda t = \beta$) and obtain a conclusive interaction time. After the measurement, Victor sends the result of measurement to Alice through a classical channel with two classical bits, then Alice knows that the state of her atoms 3 and 5 has been found in the entangled state

$$|\psi\rangle_{35} = \mathrm{e}^{-\mathrm{i}\lambda t}(\alpha\,|ee\rangle_{35} - \mathrm{i}\beta\,|gg\rangle_{35}). \quad (11)$$

After a rotation operation, the state in Eq.(11) will become

$$\alpha\,|ee\rangle_{35} + \beta\,|gg\rangle_{35}\,. \quad (12)$$

It is clear from Eq.(12) that Alice gets a complement copy of the original unknown state $|\phi\rangle$.

From Eq.(9), if the measurement results of Victor are $|eg\rangle_{12}$, $|ge\rangle_{12}$ and $|gg\rangle_{12}$, respectively, then Alice can successfully achieve a complement copy of the unknown atomic state $|\phi\rangle_{35}$ via similar operations on atoms 3 and 5.

In conclusion, we have proposed a scheme which can produce perfect copies of an unknown two-atom entangled state, with assistance of Victor (a state preparer) in cavity QED. This scheme includes two stages. The first stage of the scheme requires usual teleportation of an unknown atomic entangled state without the Bell-state measurement. In the second stage, with the assistance of the preparer, the quantum cloning of the unknown atomic entangled state can be realized.

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