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A new type of conserved quantity of Mei symmetry for relativistic nonholonomic mechanical system in phase space

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In this paper, a new type of conserved quantity induced directly from the Mei symmetry for a relativistic nonholonomic mechanical system in phase space is studied. The definition and the criterion of the Mei symmetry for the system are given. The conditions for the existence and form of the new conserved quantity are obtained. Finally, an example is given to illustrate the application of the result.

Keywords: relativity, phase space, Mei symmetry, new type of conserved quantity
PACC: 0320, 0330

1. Introduction

The study of conserved quantities by symmetries is a hot topic in mathematical physics, especially in analytic mechanics. The modern methods to investigate conserved quantities are mainly the Noether symmetry,[1] which means an invariance of the Hamiltonian's action under the infinitesimal transformations of groups, and the Lie symmetry[2] which means an invariance of differential equations under the infinitesimal transformations of groups. They can lead to the Noether conserved quantity and the Hojman conserved quantity directly. In the recent 20 years, many important results about the Noether symmetry and Lie symmetry have been obtained.[3−11] A few years ago, Mei[12,13] studied a new invariance, which means that the form of the equation of motion remains invariant under infinitesimal transformation of groups and is thus called Mei symmetry. The Mei symmetry can lead to the Mei conserved quantity directly.[14−20] Lately, Ref.[21] gave a new type of conserved quantity for a Lagrange system which is also deduced from the Mei symmetry directly but which is more general than the Mei conserved quantity.

In this paper, we study a new type of conserved quantity induced directly from the Mei symmetry for a relativistic nonholonomic mechanical system in phase space. The definition and the criterion of the Mei symmetry for the system are given. The conditions for existence and the form of the new conserved quantity are obtained. The result in this paper has more general significance.

2. The Hamilton canonical equations for a relativistic nonholonomic mechanical system

Consider a relativistic mechanical system composed of N particles, and its configuration is determined by the n generalized coordinates $q_s(s = 1, \cdots, n)$. The system is subject to g ideal nonholonomic constraints of Chetaev type

$$f^*_\beta(t, q, \dot{q}) = 0, \quad (\beta = 1, \cdots, g). \tag{1}$$

Suppose that the constraints are independent of each other, then the limitation of constraints (1) imposed on the virtual displacement is

$$\sum_{s=1}^{n} \frac{\partial f^*_\beta}{\partial \dot{q}_s} \delta q_s = 0. \tag{2}$$

The Euler–Lagrange equations of the relativistic nonholonomic mechanical system can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L^*}{\partial \dot{q}_s} - \frac{\partial L^*}{\partial q_s} = Q^*_s + \Lambda^*_s, \quad (s = 1, \cdots, n), \tag{3}$$

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$$\Lambda_s^* = \Lambda_s^*(t, \boldsymbol{q}, \dot{\boldsymbol{q}}) = \sum_{s=1}^{n} \lambda_\beta \frac{\partial f_\beta^*}{\partial \dot{q}_s}, \tag{4}$$

where $L^* = L^*(t, \boldsymbol{q}, \dot{\boldsymbol{q}}) = T^* - V$ is the generalized Lagrangian of the system. $T^* = \sum_{i=0}^{N} m_{0i}c^2(1 - \sqrt{1 - v_i^2/c^2})$ is the relativistic generalized kinetic energy, m_{0i}=const., Q_s^* are the generalized nonpotential forces.

Introduce relativistic generalized momenta and relativistic generalized Hamiltonian

$$p_s = \frac{\partial L^*}{\partial \dot{q}_s}, \quad H^* = \sum_{s=1}^{n} p_s \dot{q}_s - L^*, \quad (s = 1, \cdots, n), \tag{5}$$

and the Hamilton canonical equations of the relativistic mechanical system are

$$\dot{q}_s = \frac{\partial H^*}{\partial p_s}, \ \dot{p}_s = -\frac{\partial H^*}{\partial q_s} + \tilde{Q}_s^* + \tilde{\Lambda}_s^*, \ (s = 1, \cdots, n), \tag{6}$$

where

$$\begin{aligned} &H^* = H^*(t, \boldsymbol{q}, \boldsymbol{p}), \\ &\tilde{Q}_s^* = \tilde{Q}_s^*(t, \boldsymbol{q}, \boldsymbol{p}) = Q_s^*\left(t, \boldsymbol{q}, \dot{\boldsymbol{q}}\left(t, \boldsymbol{q}, \boldsymbol{p}\right)\right), \\ &\tilde{\Lambda}_s^* = \tilde{\Lambda}_s^*(t, \boldsymbol{q}, \boldsymbol{p}) = \Lambda_s^*(t, \boldsymbol{q}, \dot{q}(t, \boldsymbol{q}, \boldsymbol{p})). \end{aligned} \tag{7}$$

Expanding Eqs.(6), we can obtain

$$\begin{aligned} &\dot{q}_s = g_s(t, \boldsymbol{q}, \boldsymbol{p}), \\ &\dot{p}_s = h_s(t, \boldsymbol{q}, \boldsymbol{p}), \quad (s = 1, \cdots, n). \end{aligned} \tag{8}$$

The nonholonomic constraints (1) in phase space can be expressed as

$$\tilde{f}_\beta^* = \tilde{f}_\beta^*(t, \boldsymbol{q}, \boldsymbol{p}) = f_\beta^*[t, \boldsymbol{q}, \dot{\boldsymbol{q}}(t, \boldsymbol{q}, \boldsymbol{p})] = 0. \tag{9}$$

3. Definition and criterion of the Mei symmetry for the system

Introduce the general infinitesimal transformations for t, q_s, p_s as

$$\begin{aligned} &t^* = t + \varepsilon\tau(t, \boldsymbol{q}, \boldsymbol{p}), \\ &q_s^*(t^*) = q_s(t) + \varepsilon\xi_s(t, \boldsymbol{q}, \boldsymbol{p}), \\ &p_s^*(t^*) = p_s(t) + \varepsilon\eta_s(t, \boldsymbol{q}, \boldsymbol{p}), \end{aligned} \tag{10}$$

where ε is an infinitesimal parameter, and τ, ξ_s, η_s are infinitesimal generators.

Under the transformations (10), the functions H^*, $\tilde{Q}_s^*$, $\tilde{\Lambda}_s^*$ and $\tilde{f}_\beta^*$ become $\bar{H}^* = H^*(t^*, \boldsymbol{q}^*, \boldsymbol{p}^*)$, $\bar{Q}_s^* = \tilde{Q}_s^*(t^*, \boldsymbol{q}^*, \boldsymbol{p}^*)$, $\bar{\Lambda}_s^* = \tilde{\Lambda}_s^*(t^*, \boldsymbol{q}^*, \boldsymbol{p}^*)$ and $\bar{f}_\beta^* = \tilde{f}_\beta^*(t^*, \boldsymbol{q}^*, \boldsymbol{p}^*)$.

Definition Under the infinitesimal transformation (10), if the form of Eqs.(6) and (9) are invariant, that is

$$\dot{q}_s = \frac{\partial \bar{H}^*}{\partial p_s}, \quad \dot{p}_s = -\frac{\partial \bar{H}^*}{\partial q_s} + \bar{Q}_s^* + \bar{\Lambda}_s^*, \tag{11}$$

and

$$\bar{f}_\beta^* = \tilde{f}_\beta^*(t^*, \boldsymbol{q}^*, \boldsymbol{p}^*) = 0, \tag{12}$$

the invariance is called Mei symmetry for relativistic nonholonomic mechanical system in phase space.

Expanding $\bar{H}^*$, $\bar{Q}_s^*$, $\bar{\Lambda}_s^*$, $\bar{f}_\beta^*$ respectively, we have

$$\begin{aligned} \bar{H}^* =& H^*(t^*, \boldsymbol{q}^*, \boldsymbol{p}^*) \\ =& H^*(t, \boldsymbol{q}, \boldsymbol{p}) + \varepsilon X^{(0)}(H^*) + O(\varepsilon^2), \\ \bar{Q}_s^* =& \tilde{Q}_s^*(t^*, \boldsymbol{q}^*, \boldsymbol{p}^*) \\ =& \tilde{Q}_s^*(t, \boldsymbol{q}, \boldsymbol{p}) + \varepsilon X^{(0)}(\tilde{Q}_s^*) + O(\varepsilon^2), \\ \bar{\Lambda}_s^* =& \tilde{\Lambda}_s^*(t^*, \boldsymbol{q}^*, \boldsymbol{p}^*) \\ =& \tilde{\Lambda}_s^*(t, \boldsymbol{q}, \boldsymbol{p}) + \varepsilon X^{(0)}(\tilde{\Lambda}_s^*) + O(\varepsilon^2), \\ \bar{f}_\beta^* =& \tilde{f}_\beta^*(t^*, \boldsymbol{q}^*, \boldsymbol{p}^*) \\ =& \tilde{f}_\beta^*(t, \boldsymbol{q}, \boldsymbol{p}) + \varepsilon X^{(0)}(\tilde{f}_\beta^*) + O(\varepsilon^2), \end{aligned} \tag{13}$$

where

$$X^{(0)} = \tau\frac{\partial}{\partial t} + \xi_s\frac{\partial}{\partial q_s} + \eta_s\frac{\partial}{\partial p_s}. \tag{14}$$

Substituting Eqs.(13) into Eqs.(11) and (12), using Eqs.(6), ignoring ε^2 and the higher-order infinitesimal terms, we obtain

$$\begin{aligned} &\frac{\partial}{\partial p_s}[X^{(0)}(H^*)] = 0, \\ &\frac{\partial}{\partial q_s}[X^{(0)}(H^*)] = X^{(0)}(\tilde{Q}_s^* + \tilde{\Lambda}_s^*) \end{aligned} \tag{15}$$

and

$$X^{(0)}[\tilde{f}_\beta^*(t, \boldsymbol{q}, \boldsymbol{p})] = 0. \tag{16}$$

Then, we have the following criterion.

Criterion For relativistic nonholonomic mechanical system, Eqs.(6) and (9), in phase space, if the infinitesimal generators τ, ξ_s, η_s satisfy Eqs.(15) and (16), the corresponding invariance is Mei symmetry of the system.

4. A new type of conserved quantity of Mei symmetry for the system

The conditions for the existence and form of a new type of conserved quantity, which is induced directly from Mei symmetry of the relativistic nonholonomic mechanical system in phase space, are given in the following theorem.

Theorem For the relativistic nonholonomic mechanical system Eqs.(6) and (9), in phase space, if the infinitesimal generators τ, ξ_s, η_s of Mei symmetry and a gauge function $G_{\rm M} = G_{\rm M}(t, \boldsymbol{q}, \boldsymbol{p})$ satisfy

$$\begin{aligned} & X^{(0)}(p_s)\frac{\bar{\rm d}}{{\rm d}t}\xi_s + \frac{\bar{\rm d}}{{\rm d}t}[X^{(0)}(p_s)]\xi_s \\ & - \frac{\partial X^{(0)}(H^*)}{\partial t}f - \frac{\partial X^{(0)}(H^*)}{\partial q_s}\xi_s \\ & - X^{(0)}(H^*)\frac{\bar{\rm d}f}{{\rm d}t} + X^{(0)}(\tilde{Q}_s^* + \tilde{\Lambda}_s^*)(\xi_s - \dot{q}_s f) \\ & + \frac{\bar{\rm d}}{{\rm d}t}G_{\rm M} = 0, \end{aligned} \tag{17}$$

where

$$\frac{\bar{\rm d}}{{\rm d}t} = \frac{\partial}{\partial t} + \frac{\partial H^*}{\partial p_s}\frac{\partial}{\partial q_s} - \left(\frac{\partial H^*}{\partial q_s} - \tilde{Q}_s^* - \tilde{\Lambda}_s^*\right)\frac{\partial}{\partial p_s}, \tag{18}$$

then Mei symmetry of the system will lead to the following new conserved quantity

$$I_{\rm M} = X^{(0)}(p_s)\xi_s - X^{(0)}(H^*)f + G_{\rm M} = \text{const.}, \tag{19}$$

where $f = f(t, \boldsymbol{q}, \boldsymbol{p})$ which is called a coordination function is a random function to make $G_{\rm M}$ exist.

Proof Using Eqs.(17) and (18), we have

$$\begin{aligned} \frac{\bar{\rm d}}{{\rm d}t}I_{\rm M} =& \frac{\bar{\rm d}}{{\rm d}t}X^{(0)}(p_s)\xi_s + X^{(0)}(p_s)\frac{\bar{\rm d}}{{\rm d}t}\xi_s - \frac{\bar{\rm d}}{{\rm d}t}X^{(0)}(H^*)f - X^{(0)}(H^*)\frac{\bar{\rm d}}{{\rm d}t}f \\ & - X^{(0)}(p_s)\frac{\bar{\rm d}}{{\rm d}t}\xi_s - \frac{\bar{\rm d}}{{\rm d}t}[X^{(0)}(p_s)]\xi_s + \frac{\partial}{\partial t}[X^{(0)}(H^*)]f + \frac{\partial}{\partial q_s}[X^{(0)}(H^*)]\xi_s \\ & + X^{(0)}(H^*)\frac{\bar{\rm d}}{{\rm d}t}f - X^{(0)}(\tilde{Q}_s^* + \tilde{\Lambda}_s^*)\left(\xi_s - \frac{\partial H^*}{\partial p_s}f\right) \\ =& \left\{\frac{\partial}{\partial q_s}[X^{(0)}(H^*)] - X^{(0)}(\tilde{Q}_s^* + \tilde{\Lambda}_s^*)\right\}\left(\xi_s - \frac{\partial H^*}{\partial p_s}f\right) \\ & + \frac{\partial}{\partial p_s}[X^{(0)}(H^*)]\left(\frac{\partial H^*}{\partial p_s} - \tilde{Q}_s^* - \tilde{\Lambda}_s^*\right)f. \end{aligned} \tag{20}$$

Substituting Eqs.(15) into Eq.(20), we obtain

$$\frac{\bar{\rm d}}{{\rm d}t}I_{\rm M} = 0. \tag{21}$$

Special case If $f = \tau$, Eqs.(17) and (19) respectively become

$$\begin{aligned} & X^{(0)}(p_s)\frac{\bar{\rm d}}{{\rm d}t}\xi_s + \frac{\bar{\rm d}}{{\rm d}t}[X^{(0)}(p_s)]\xi_s - X^{(0)}[X^{(0)}(H^*)] \\ & - X^{(0)}(H^*)\frac{\bar{\rm d}\tau}{{\rm d}t} + X^{(0)}(\tilde{Q}_s^* + \tilde{\Lambda}_s^*)\left(\xi_s - \frac{\partial H^*}{\partial p_s}\tau\right) \\ & + \frac{\bar{\rm d}}{{\rm d}t}G_{\rm M} = 0, \end{aligned} \tag{22}$$

$$I_{\rm M} = \sum_{s=1}^{n} X^{(0)}(p_s)\xi_s - X^{(0)}(H^*)\tau + G_{\rm M} = \text{const.} \tag{23}$$

Equation (22) is the structural equation from which Mei symmetry of the relativistic nonholonomic mechanical system in phase space can lead to the Mei conserved quantity (23).

We call the new type of conserved quantity (19) a generalized Mei conserved quantity of the relativistic nonholonomic mechanical system in phase space.

5. An illustrate example

The generalized Lagrangian of relativistic mechanical system is

$$\begin{aligned} L^* =& m_0c^2(1 - \sqrt{1 - (\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2)/c^2}) \\ & - m_0gq_3 - \frac{1}{2}m_0gt^2, \end{aligned} \tag{24}$$

with the nonholonomic constraints of Chetaev type

$$f_\beta^* = \dot{q}_1^2 + \dot{q}_2^2 - \dot{q}_3^2 = 0. \tag{25}$$

We study the Mei symmetry and the generalized Mei conserved quantity of the relativistic nonholonomic mechanical system in phase space.

We have

$$\begin{aligned} & \sqrt{1 - (\dot{q}_1^2 + \dot{q}_1^2 + \dot{q}_1^2)/c^2} \\ =& \frac{1}{\sqrt{1 + (p_1^2 + p_2^2 + p_3^2)/m_0^2c^2}}, \end{aligned} \tag{26}$$

and we can obtain the Hamiltonian of the system as

follows:

$$H^* = m_0c^2(\sqrt{1+(p_1^2+p_2^2+p_3^2)/m_0^2c^2}-1) + m_0gq_3+\frac{1}{2}m_0gt^2. \tag{27}$$

So Eqs.(6) can be written as

$$\begin{aligned}
\dot{q}_1 &= \frac{p_1}{m_0}\cdot\frac{1}{\sqrt{1+(p_1^2+p_2^2+p_3^2)/m_0^2c^2}},\\
\dot{q}_2 &= \frac{p_2}{m_0}\cdot\frac{1}{\sqrt{1+(p_1^2+p_2^2+p_3^2)/m_0^2c^2}},\\
\dot{q}_3 &= \frac{p_3}{m_0}\cdot\frac{1}{\sqrt{1+(p_1^2+p_2^2+p_3^2)/m_0^2c^2}},\\
\dot{p}_1 &= 2\lambda\frac{p_1}{m_0}\cdot\frac{1}{\sqrt{1+(p_1^2+p_2^2+p_3^2)/m_0^2c^2}},\\
\dot{p}_2 &= 2\lambda\frac{p_2}{m_0}\cdot\frac{1}{\sqrt{1+(p_1^2+p_2^2+p_3^2)/m_0^2c^2}},\\
\dot{p}_3 &= -mg-2\lambda\frac{p_3}{m_0}\cdot\frac{1}{\sqrt{1+(p_1^2+p_2^2+p_3^2)/m_0^2c^2}}.
\end{aligned} \tag{28}$$

The constraints equation (25) can be expressed as

$$\tilde{f} = \frac{p_1^2+p_2^2-p_3^2}{m_0^2}\cdot\frac{1}{1+(p_1^2+p_2^2+p_3^2)/m_0^2c^2} = 0. \tag{29}$$

Using Eqs.(28) and (29), we can obtain

$$\lambda = -\frac{m_0^2g}{4p_3}\cdot\sqrt{1+(p_1^2+p_2^2+p_3^2)/m_0^2c^2}. \tag{30}$$

From Eqs.(6) and (8) we have

$$\begin{aligned}
g_1 &= \dot{q}_1 = \frac{p_1}{m_0}\cdot\frac{1}{\sqrt{1+(p_1^2+p_2^2+p_3^2)/m_0^2c^2}},\\
g_2 &= \dot{q}_2 = \frac{p_2}{m_0}\cdot\frac{1}{\sqrt{1+(p_1^2+p_2^2+p_3^2)/m_0^2c^2}},\\
g_3 &= \dot{q}_3 = \frac{p_3}{m_0}\cdot\frac{1}{\sqrt{1+(p_1^2+p_2^2+p_3^2)/m_0^2c^2}},\\
h_1 &= \dot{p}_1 = -\frac{m_0gp_1}{2p_3},\\
h_2 &= \dot{p}_2 = -\frac{m_0gp_2}{2p_3},\quad h_3 = \dot{p}_3 = -\frac{1}{2}m_0g,\\
\tilde{\Lambda}_1 &= -\frac{m_0gp_1}{2p_3},\quad \tilde{\Lambda}_2 = -\frac{m_0gp_2}{2p_3},\quad \tilde{\Lambda}_3 = \frac{m_0g}{2}.
\end{aligned} \tag{31}$$

After calculation, we obtain

$$\begin{aligned}
&X^{(0)}(H) = m_0gt\tau+m_0g\xi_3+\frac{1}{m_0}\cdot\frac{1}{\sqrt{1+(p_1^2+p_2^2+p_3^2)/m_0^2c^2}}\cdot(\eta_1p_1+\eta_2p_2+\eta_3p_3),\\
&X^{(0)}(\tilde{Q}_1^*+\tilde{\Lambda}_1^*) = -\eta_1\frac{m_0g}{2p_3}+\eta_3\frac{m_0gp_1}{2p_3^2},\quad X^{(0)}(\tilde{Q}_2^*+\tilde{\Lambda}_2^*) = -\eta_2\frac{m_0g}{2p_3}+\eta_3\frac{m_0gp_2}{2p_3^2},\\
&X^{(0)}(\tilde{Q}_3^*+\tilde{\Lambda}_3^*) = 0, X^{(0)}(\tilde{f}_\beta) = \frac{2}{m_0^2}\cdot\frac{1}{1+(p_1^2+p_2^2+p_3^2)/m_0^2c^2}\cdot(\eta_1p_1+\eta_2p_2-\eta_3p_3).
\end{aligned} \tag{32}$$

Choose the infinitesimal generators as

$$\begin{aligned}
\tau &= 1,\\
\xi_1 &= \frac{p_1}{\sqrt{1+(p_1^2+p_2^2+p_3^2)/m_0^2c^2}},\\
\xi_2 &= \frac{p_2}{\sqrt{1+(p_1^2+p_2^2+p_3^2)/m_0^2c^2}},\\
\xi_3 &= \frac{p_3}{\sqrt{1+(p_1^2+p_2^2+p_3^2)/m_0^2c^2}},\\
\eta_1 &= -\frac{m_0^2gp_1}{2p_3},\quad \eta_2 = -\frac{m_0^2gp_2}{2p_3},\\
\eta_3 &= -\frac{1}{2}m_0^2g,
\end{aligned} \tag{33}$$

then we have

$$\begin{aligned}
&X^{(0)}(H) = m_0gt,\\
&X^{(0)}(\tilde{Q}_1^*+\tilde{\Lambda}_1^*) = 0,\\
&X^{(0)}(\tilde{Q}_2^*+\tilde{\Lambda}_2^*) = 0,\\
&X^{(0)}(\tilde{Q}_3^*+\tilde{\Lambda}_3^*) = 0, X^{(0)}(\tilde{f}_\beta) = 0.
\end{aligned} \tag{34}$$

It is easy to verify that Eqs.(15) and (16) hold true, so the infinitesimal generators (33) are Mei symmetrical.

Using Eq.(17), we have

$$\frac{\bar{\mathrm{d}}G_{\mathrm{M}}}{\mathrm{d}t} = m_0gf+m_0gt\frac{\bar{\mathrm{d}}f}{\mathrm{d}t}-\frac{\bar{\mathrm{d}}}{\mathrm{d}t}[X^{(0)}(p_s)\xi_s]. \tag{35}$$

Choose

$$f = \frac{p_3}{m_0 g}, \tag{36}$$

so we have

$$\begin{aligned} G_{\rm M} =& m_0 q_3 \sqrt{1 + (p_1^2 + p_2^2 + p_3^2)/m_0^2 c^2} \\ & - \frac{1}{4} m_0 g t^2 + \frac{m_0 g}{2c^2} q_3^2 \\ & + m_0^2 g \cdot \frac{p_3}{\sqrt{1 + (p_1^2 + p_2^2 + p_3^2)/m_0^2 c^2}}. \end{aligned} \tag{37}$$

From Eq.(19), we obtain

$$\begin{aligned} I_{\rm M} =& m_0 q_3 \sqrt{1 + (p_1^2 + p_2^2 + p_3^2)/m_0^2 c^2} \\ & - \frac{1}{4} m_0 g t^2 + \frac{m_0 g}{2c^2} q_3^2 - p_3 t = \text{const.} \end{aligned} \tag{38}$$

Choosing

$$f = \frac{p_3 + \frac{1}{2} m_0 g t}{m_0 g}, \tag{39}$$

we gain

$$\begin{aligned} G_{\rm M} =& m_0 q_3 \sqrt{1 + (p_1^2 + p_2^2 + p_3^2)} \\ & + \frac{1}{4} m_0 g t^2 + \frac{m_0 g}{2c^2} q_3^2 \\ & + m_0^2 g \cdot \frac{p_3}{\sqrt{1 + (p_1^2 + p_2^2 + p_3^2)/m_0^2 c^2}}. \end{aligned} \tag{40}$$

From Eq.(19), we can obtain

$$\begin{aligned} I_{\rm M} =& m_0 q_3 \sqrt{1 + (p_1^2 + p_2^2 + p_3^2)} \\ & + \frac{1}{4} m_0 g t^2 + \frac{m_0 g}{2c^2} q_3^2 - p_3 t - \frac{1}{2} m_0 g t^2 \\ =& \text{const.} \end{aligned} \tag{41}$$

When $f = \tau$, we have

$$\begin{aligned} G_{\rm M} =& m_0^2 g \cdot \frac{p_3}{\sqrt{1 + (p_1^2 + p_2^2 + p_3^2)/m_0^2 c^2}} \\ & + p_3 + \frac{3}{2} m_0 g t. \end{aligned} \tag{42}$$

According to the above theorem, we have

$$I_{\rm M} = p_3 + \frac{1}{2} m_0 g t = \text{const.}, \tag{43}$$

which is the Mei conserved quantity with Mei symmetry for the relativistic nonholonomic mechanical system in phase space.

6. Conclusion

In this paper, we have studied the new type of conserved quantity which is induced directly from Mei symmetry for the relativistic nonholonomic mechanical system in phase space. The conditions for the existence and form of the new conserved quantity have been obtained. The result in this paper can be of general significance. It can be applied not only in relativistic mechanical systems but also in classical mechanical systems. When $v \ll c$, we will obtain the result for the classical nonholonomic mechanical system in phase space from this paper. When $f = \tau$, the new type of conserved quantity we obtain in this paper will turn into the Mei conserved quantity of the system. Furthermore, since the choice of the coordination function is multiform, much more conserved quantities of Mei symmetry for relativistic nonholonomic mechanical system in phase space can be obtained.

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