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Asymptotic solution for a class of sea–air oscillator model for El Niño–southern oscillation*

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The El Niño–Southern Oscillation (ENSO) is an interannual phenomenon involved in the tropical Pacific Ocean–atmosphere interactions. In this paper, an asymptotic method of solving the nonlinear equation for the ENSO model is used. And based on a class of oscillator of ENSO model, the approximate solution of a corresponding problem is studied by employing the perturbation method. Firstly, an ENSO model of nonlinear time delay equation of equatorial Pacific is introduced, Secondly, by using the perturbed method, the zeroth and first order asymptotic perturbed solutions are constructed. Finally, from the comparison of the values for a figure, it is seen that the first asymptotic perturbed solution using the perturbation method has a good accuracy. And it is proved from the results that the perturbation method can be used as an analytic operation for the sea surface temperature anomaly in the equatorial Pacific of the atmosphere-ocean oscillation for the ENSO model.

Keywords: nonlinear, perturbation method, El Niño-Southern Oscillation Model
PACC: 0200

1. Introduction

El Niño–Southern Oscillation (ENSO) is an interannual phenomenon involved in the tropical Pacific ocean–atmosphere interactions. And it is a very attractive object of study in the international academic circles.[1−6] We have considered also some problems of this class in atmospheric physics, ocean weather, dynamic system and so on,[7−21] the oscillatory nature of ENSO requires both positive and negative ocean-atmosphere feedbacks. An initial positive sea surface temperature (SST) anomaly in the equatorial eastern Pacific can reduce the zonal SST gradient and hence the strength of the southern oscillation circulation, resulting in weaker trade winds around the equator. The weaker trade winds in turn bring on the ocean circulation changes which reinforce the SST anomaly. This positive ocean–atmosphere feedback or coupled ocean–atmosphere instability leads the equatorial Pacific to a never-ending warm state occurring.

The conceptual oscillator model should consider the variations of both the eastern and western Pacific anomaly patterns. As will be shown below, the oscillator model can be reduced to a simple oscillator model by further making simplification and assumptions.

Recently many scholars have investigated the relevant nonlinear problems. Approximate methods have been developed and improved, including the method of averaging, boundary layer method, matched asymptotic expansion method, and multiple scales method.[22−24] Using the perturbation method, Mo *et al* considered a class of nonlinear perturbation problems.[25−27] In this paper we consider a class of ENSO model by using a simple and special perturbation method.

2. An ENSO Model

The western Pacific oscillator emphasizes the role of the western Pacific anomaly patterns in ENSO. In particular, off-equatorial SST variations initiate equatorial wind anomalies in the western Pacific. These

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wind anomalies force ocean to proceed eastward to affect anomalies in the equatorial eastern Pacific. We can understand why the slow ocean dynamical adjustment does not necessarily need the role of wave propagation.

The delayed oscillator model does not consider the effect of the western Pacific on ENSO. It is assumed that winds in the western Pacific do not affect the SST anomalies in the eastern Pacific and the atmosphere is in a steady state or the Niño-4 (over 5°S–5°N, 160°E–150^{0}W) zonal wind stress anomalies are linearly proportional to the Niño-3 (over 5°S–5°N, 150°–90°W) SST anomalies. We can obtain the following nonlinear time delay model of equatorial Pacific:[5]

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{ad}{R}T - \frac{bd}{R}T(t-\eta) - cT^3, \tag{1}$$

where T is the Niño-3 SST anomaly, η is a time delay for waves to travel to the western boundary and return to the eastern Pacific, a is a coefficient representing the positive feedback between T, b is a coefficient representing the negative feedback due to waves reflection at the western boundary, d is a positive coefficient that relates the Niño-3 SST anomalies to the Niño-4 zonal wind stress anomalies, R is a damping coefficient and c is a cubic damping coefficient. And we assume that η is a small parameter.

3. Perturbed solution

Let

$$T = \sum_{i=0}^{\infty} T_i(t)\eta^i. \tag{2}$$

Substituting Eq.(2) into Eq.(1), expanding the nonlinear term in η, and equating the coefficients of the same powers in η, for η^0 we obtain

$$\frac{\mathrm{d}T_0}{\mathrm{d}t} = \frac{(a-b)d}{R}T_0 - cT_0^3. \tag{3}$$

Let $z = T_0^{-2}$. From Eq.(3), we have

$$\frac{\mathrm{d}z}{\mathrm{d}t} + \frac{2(a-b)d}{R}z - 2c = 0.$$

Thus $z(t) = C_0 \exp\left(-\dfrac{2(a-b)d}{R}t\right) + \dfrac{cR}{(a-b)d}$, where C_0 is an arbitrary constant. That is

$$T_0(t) = \left(\left(T(0)^{-2} - \frac{cR}{(a-b)d}\right)\exp\left(-\frac{2(a-b)d}{R}t\right) + \frac{cR}{(a-b)d}\right)^{-1/2}, \tag{4}$$

where $T(0)$ is initial value of $T(t)$.

Substituting Eq.(2) into Eq.(1), for η^1 we have

$$\begin{aligned}\frac{\mathrm{d}T_1}{\mathrm{d}t} = {} & \left[\frac{(a-b)d}{R} - 3c\left(\left(T(0)^{-2} - \frac{cR}{(a-b)d}\right)\exp\left(-\frac{2(a-b)d}{R}t\right) + \frac{cR}{(a-b)d}\right)^{-1}\right]T_1 \\ & + \frac{(a-b)bd^2}{R^2}\left(\exp\left(-\frac{2(a-b)d}{R}t\right)\right) \\ & \times \left[\left(T(0)^{-2} - \frac{cR}{(a-b)d}\right)\exp\left(-\frac{2(a-b)d}{R}t\right) + \frac{cR}{(a-b)d}\right]^{-3/2}.\end{aligned} \tag{5}$$

From Eq.(5), and with zero initial value, then we obtain

$$T_1(t) = \frac{(a-b)bd^2}{R^2}t\exp\left(-\frac{2(a-b)d}{R}t\right)\left[\left(T(0)^{-2} - \frac{cR}{(a-b)d}\right)\exp\left(-\frac{2(a-b)d}{R}t\right) + \frac{cR}{(a-b)d}\right]^{-3/2}.$$

Thus from Eq.(2), the first order asymptotic perturbed solution of Eq.(1) is

$$\begin{aligned}T(t) = {} & \left(\left(T(0)^{-2} - \frac{cR}{(a-b)d}\right)\exp\left(-\frac{2(a-b)d}{R}t\right) + \frac{cR}{(a-b)d}\right)^{-1/2} \\ & \times \left[1 + \frac{(a-b)bd^2\eta}{R^2}t\left(\left(T(0)^{-2} - \frac{cR}{(a-b)d}\right) + \frac{cR}{(a-b)d}\exp\left(\frac{2(a-b)d}{R}t\right)\right)^{-1}\right] + O(\eta^2).\end{aligned} \tag{6}$$

We also can obtain higher-order asymptotic perturbed solution for Eq.(1) by using the same method.

4. Accuracy of approximate solution

In order to evaluate the precision of expansion (6) of the asymptotic perturbed solution to Eq.(1), we give a set of parameters as follows:[5] $a = 1.5 \times 10^2 ^\circ\mathrm{C\,m^2N^{-1}yr^{-1}}$, $b = 2.5 \times 10^2 ^\circ\mathrm{C\,m^2N^{-1}yr^{-1}}$, $d = 3.6 \times 10^{-2} ^\circ\mathrm{C\,m^{-2}Nyr^{-1}}$, $R = 2.0\,\mathrm{yr^{-1}}$, $c = 0$, $C_0 = 1$, $C_1 = 0$.

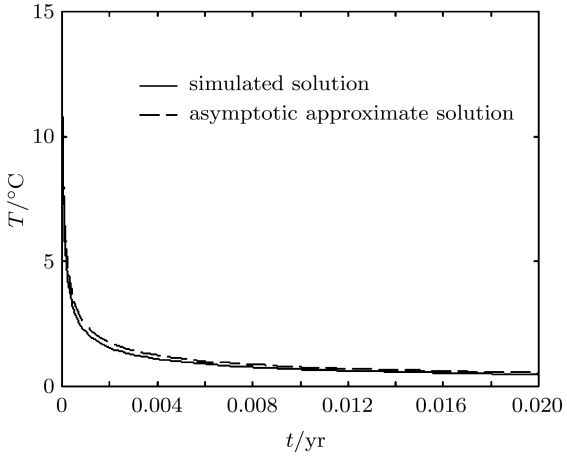


Fig.1. A comparison of the values between T_{num} and T_{asy}.

Now we establish a comparison of the values between T_{num} for the numerical calculation and T_{asy} of the asymptotic perturbed expansion (6) in the case of different values $T(^\circ\mathrm{C})$ and $t(\mathrm{yr})$. We obtain the following curves:

From the Fig.1, we can see that the first approximate perturbed solution T_{asy} using the perturbation method has a good accuracy.

5. Conclusions

1. The ENSO is a complicated natural phenomenon. Hence we need to reduce basic models for the sea–air oscillator and solve them by using the approximate method. The perturbation method is a simple and valid method.

2. The perturbation method is an asymptotic analytic method, which differs from general numerical method. The expansions of solution through the perturbation method can be kept in the analytic operation. Thus, from Eq.(6), we can further study the fixed quality and quantitative behaviours of the temperature anomaly in the equatorial Pacific. That remains for future work.

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