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Dynamics of a pair of electron and hole in semiconductor superlattice under an intense electric field

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This paper investigates the behaviour of a pair of electron and hole in semiconductor superlattice under an external electric field with the consideration of Coulomb interaction. By numerically calculating the corresponding probability in the nearest neighbour tight binding approximation, we find that the single electron (or the hole) can not be dynamically localized due to the Coulomb interaction, while the dynamic localization of exciton (the pair of the electron and hole) still exists. Moreover we find that with the increase of the intensity of electric field, the exciton can be dynamically localized more completely.

Keywords: semiconductor superlattice, dynamic localization, quasi energy
PACC: 7360L, 7320D, 7135

1. Introduction

Over the past decades, quantum dynamics of an electron in low dimensional mesoscopic system under external electric field has been investigated thoroughly both theoretically and experimentally, particularly after the well known Wannier–Stark ladder[1] and its dynamical counterpart Bloch oscillation[2] were first observed experimentally in semiconductor superlattice (SSL) by Mendez *et al*[3] and Feldmann *et al*[4] respectively. Many interesting phenomena such as the dynamic Franz–Keldysh effect,[5] the terahertz (THz) radiation[6] etc. have been found in SSL. Among them is the famous dynamic localization (DL).

DL was first discussed by Dunlap[7] and verified experimentally in optical lattice by Madison.[8] It was predicted by Dunlap that for a purely sinusoidal AC field $E\cos(\omega t)$ in the nearest neighbour tight binding approximation (NNTBA), the DL of an electron can be achieved when the ratio $eEd/\hbar\omega$ reaches the root of zero order Bessel function, i.e. the electron can return to its initial state periodically if $J_0(eEd/\hbar\omega) = 0$, where e and d are the fundamental charge and SSL period width respectively. Later DL was discussed under DC–AC case and investigated by using the Floquet quasi-energy theorem.[9−13] It is found that under combined DC–AC electric field the DL can still be obtained and when DL occurs the quasi-energy miniband will collapse. Recently the case of DL under DC-bichromatic electric field has been discussed and the result is similar with the case of DC–AC electric field and can be generalized to DC-multi-chromatic case.[14−16] The model in NNTBA gives us a direct physical picture of electron's dynamics by observation of probability propagator and the mean-square displacement.

To observe DL in the SSL, people usually use an ultrafast optical pulse to excite the electron-hole pair. Due to the Coulomb interaction, the electron behaviour cannot be considered as a single one and it becomes a many-body problem. Actually the model used by Dunlap is just a gedanken case, because where there is an electron there is a hole. Consequently, the effect of Coulomb interaction on the DL should be considered. There are a number of approaches to treat Coulomb interaction in photo-excited SSL. Among them the semiconductor Bloch equations (SBE)[17] are widely used, which is derived in the Hartree–Fock approximation. Meier *et al*[18] have calculated the time-integrated square of the THz signal generated by a photoexcited SSL as Coulomb interaction is considered, and predicted that the DL can be observed in an applied AC electric field. Later Yan *et al*[19] have given the THz signal and Liu *et al*[20] have investigated the degenerate four-wave-mixing signals in DC–AC electric fields respectively.

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In this paper, we will investigate the behaviour of a pair of electron and hole in SSL under external electric field using the NNTBA when the Coulomb interaction of the electron and the hole is considered. We referred the DL discussed by Dunlap as single–electron DL. If the relative distance between the considered pair of electron and hole returns to its initial state periodically under external electric field, we can say that the DL of exciton is achieved. By numerically solving the Schrödinger equation we find that Coulomb interaction will destroy the DL of the single electron in presence of AC electric field, while the DL of exciton still exists to some extent. When the intensity of the AC electric field is large enough, even though the DL of single particle (the electron or the hole) cannot be completely achieved as the case of neglecting the Coulomb interaction, the electron and hole could be confined to its initial location for a longer time. At the same time the DL of exciton can be obtained more completely.

2. Model and quasi energy miniband

We begin with the following Hamiltonian in NNTBA:

$$\begin{aligned} H &= V_1\left(a^\dagger_{e,m}a_{e,m+1}+a^\dagger_{e,m+1}a_{e,m}\right)\\ &\quad -eE(t)md a^\dagger_{e,m}a_{e,m}\\ &\quad +V_2\left(a^\dagger_{h,n}a_{h,n+1}+a^\dagger_{h,n+1}a_{h,n}\right)\\ &\quad +eE(t)nd a^\dagger_{h,n}a_{h,n}\\ &\quad -V_{\rm c}n_{e,m}n_{h,n}\delta_{m,n}, \end{aligned} \tag{1}$$

where $a^\dagger_{i,m}, a_{i,m}, (i=e,h.)$ are the creation and annihilation operator of a Wannier state of the electron and the hole localized on lattice site m, respectively. $n_{e,m}(n_{h,n})$ is $a^\dagger_{e,m}a_{e,m}(a^\dagger_{h,n}a_{h,n})$. Repeated index automatically represents summation. V_1 and V_2 are the nearest neighbour interaction of the electron and hole respectively. $V_{\rm c}$ is the Coulomb interaction, treated as on one site,[18,19] which means that only when the electron and hole are located on the same lattice site the Coulomb interaction exists. The electric field $E(t)$ is

$$E(t) = E\cos(\omega t). \tag{2}$$

In order to investigate the dynamics of the system, we express the system state $\psi(t)$ as a linear combination of Wannier states $|m,n\rangle \equiv a^\dagger_{e,m}a^\dagger_{h,n}|0,0\rangle$, i.e. $\psi(t)=\sum_{m,n}C_{m,n}|m,n\rangle$. Then the evolution equation for $C_{m,n}$ can be obtained from Eq.(1),

$$\begin{aligned} {\rm i}\frac{{\rm d}C_{m,n}}{{\rm d}t} &= V_1(C_{m+1,n}+C_{m-1,n})-E(t)mC_{m,n}\\ &\quad +V_2(C_{m,n+1}+C_{m,n-1})+E(t)nC_{m,n}\\ &\quad -V_{\rm c}C_{m,n}\delta_{m,n}. \end{aligned} \tag{3}$$

For convenience, we set $e,\hbar,d=1$. Suppose a pair of electron and hole are excited and located at site 0 initially, i.e.

$$C_{m,n}(t=0)=\delta_{m,0}\delta_{n,0}. \tag{4}$$

The quantities of interest are the probability of electron at different lattice sites at different times

$$P_e(m,t)=\sum_n |C_{m,n}(t)|^2, \tag{5}$$

and the mean-square displacement of electron

$$\langle m^2\rangle = \sum m^2 P_{m,t} - \left(\sum mP_{m,t}\right)^2. \tag{6}$$

If the DL occurs, the probability of finding the electron in its initial position would return to 1 after one period of AC electric field. In the case, the mean-square displacement, representing the extension to which the electron disperse, would be bounded. So we observe the quantity $P_{0,e}\equiv P_e(0,t)$ and $\langle m^2\rangle$ to tell the existence of DL. As for DL of single particle, we only consider the electron for the reason that the behaviour of the hole is similar to the electron in nature. For exciton, its initial state is that electron and hole stay at the same site, so we will investigate the behaviour of exciton under external electric field by observing the probability

$$P_{\rm ex}\equiv\sum_m |C_{m,m}(t)|^2, \tag{7}$$

the meaning of which will be discussed in detail in Section IV.

Before investigating the behaviour of a pair of electron and hole we give the quasi energy diagram. In quasi moment representation the Hamiltonian becomes

$$\begin{aligned} H &= 2(V_1+V_2)\cos(k)a^\dagger_k a_k\\ &\quad -{\rm i}a^\dagger_k E(t)\partial_k a_k - \frac{V_{\rm c}}{N}a^\dagger_k a_{k'}, \end{aligned} \tag{8}$$

where N is the number of SSL period and k represents the quasi momentum of relative motion of electron and hole along growth direction. The quasi energy is obtained with the help of Floquet theorem. Figure 1 gives the quasi energy diagram. V_1 and V_2 is set to 1.5 meV and 1 meV respectively, which means that the combined miniband width $\Delta\equiv 4(V_1+V_2)$ is 10 meV.

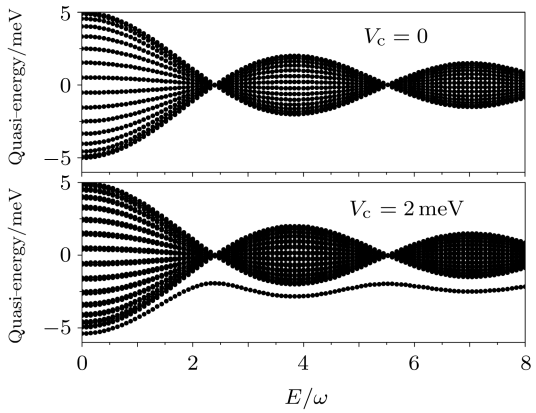


Fig.1. Quasi energy diagram, V_1, V_2 are 1.5 meV and 1 meV respectively. ω is 15 meV. V_c is zero for upper one and 2 meV for lower one.

Coulomb interaction is 2 meV. The superlattice period is 100 nm. First value of E/ω to make quasi-energy collapse is 2.405. Due to the Coulomb interaction, the discrete excitonic levels is separated from the miniband. The quasi-energy diagram indicates that although the Coulomb interaction will result in some discrete energy levels separated from the collapsed miniband, the DL still exists when $J_0(E/\omega) = 0$.

3. Behaviour of electron

When the Coulomb interaction is not considered, i.e. $V_c = 0$, the behaviours of the electron and the hole are not influenced by each other, so that electron (or the hole) would dynamically localized by the external electric field both in AC and in DC–AC case, which is predicted by Dunlap and Zhao. In Fig.2, the case is shown with the solid (red) lines. We set $E = 10$ meV, and choose the appropriate value of ω to satisfy $E/\omega = 2.405$. The probability of $P_{e,0}$ (in Fig.2(a)) is periodic and the displacement $\langle m^2 \rangle$ (in Fig.2(b)) is bounded. When the Coulomb interaction is considered, the probability of electron on site 0 can not return to 1, which means that the electron cannot be dynamically localized. Therefore the mean-square displacement increases infinitely as the time increase. The stronger the Coulomb interaction is, the faster the destruction of dynamic localization is.

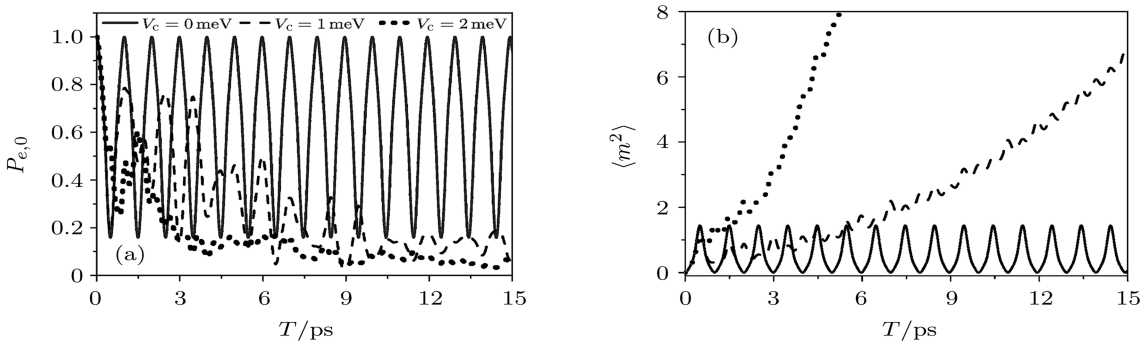


Fig.2. The probability of the electron at the initial site 0 (a) and mean square displacement (b) as function of time with the increase of strength of the Coulomb interaction. E is 10 meV and E/ω is fixed to 2.405. V_c changes as 0, 1, 3 meV.

As Coulomb interaction is considered, we want to find the effect of AC electric field strength on the behaviour of electron. We choose V_c to be 1 meV, and change the value of E while the ratio of E/ω is fixed to 2.405. The probability $P_{e,0}$ and mean square displacement $\langle m^2 \rangle$ were plotted in Figs.3(a) and 3(b) respectively. We can see that with the increase of strength of electric field, the delocalization process is slowed

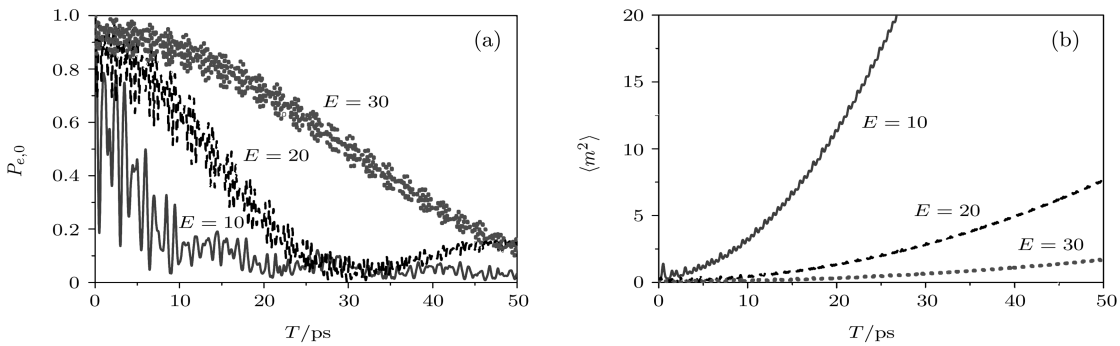


Fig.3. The probability of electron on initial site 0 (a) and mean square displacement (b) as function of time with the increase of intensity of the AC electric field. V_c is 1meV and E is 10, 20, 30 meV respectively while the ratio of E/ω is fixed to 2.405.

down. Although DL cannot be achieved, the AC electric field can prevent the electron from being dynamically delocalized.

We also consider the case under a DC–AC electric field, the results is similar to the AC electric field. The DL of electron cannot be achieved completely even when the $J_1(E/\omega)$ is zero (considering the case $\omega = F$, when F is DC electric field strength).

4. DL of exciton

When the Coulomb interaction is considered, the behaviour of exciton is more interesting than a single electron or hole. The exciton can be thought of as a pair of electron and hole with certain distance connected by Coulomb interaction. After excitation by a pulse, pairs of electron and hole were created at the same site, for example, site 0. On one hand the electron and the hole will disperse in the external field, on the other hand the exciton comes into being due to Coulomb interaction. With the evolution of time, the relative radius of the exciton changes. If the radius of the exciton can return to certain value periodically with the external field, we say that the exciton achieves DL. We investigate the behaviour of the exciton under the external electric field by observing the evolution of $P_{\rm ex}$, which is the probability of finding the pair of electron and hole staying at the same lattice site. We change the value of $V_{\rm c}$ from 1 meV to 10 meV while E and E/ω are fixed to 10 meV and 2.405 respectively. Figure 4 plots the results at three particular values of $V_{\rm c}$, 1, 3, and 10 meV. As long as

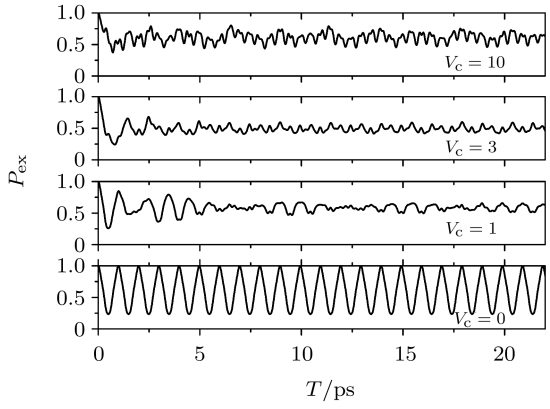


Fig.4. The probability of the pair of electron and hole within the same site as function of time with the increase of strength of the Coulomb interaction. E is 10 meV while the ratio of E/ω is 2.405.

the Coulomb interaction exists, the DL of exciton can be obtained when the ratio of E/ω reaches the root of Bessel function J_0, although the probability cannot reach 1.

Now we show the influence of AC electric field strength on the DL of exciton. The Coulomb interaction $V_{\rm c}$ is fixed to 1 meV. By changing the value of AC electric field E while keeping the ratio of E/ω as 2.405, we get the evolution of probability of $P_{\rm ex}$ with the time. Figure 5 shows that the intensity of AC electric field can affect the degree of DL of the exciton. The influence of intensity of AC electric field on the DL of exciton is consistent with its effect on DL of a single electron (or hole). The reason is that when the intensity of AC electric field is large enough, the electron and hole is almost dynamically localized, which is helpful to the DL of exciton. We also investigated the DL of exciton in the presence of DC–AC electric field, the result is similar to the current case.

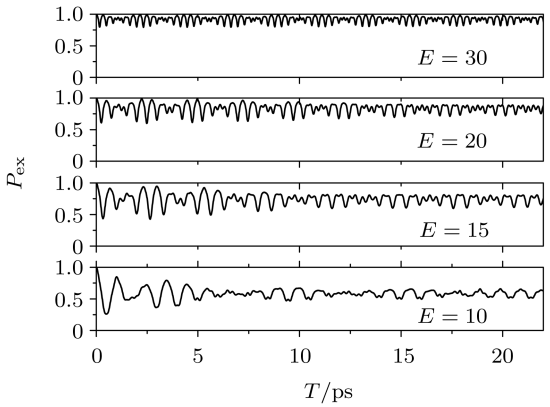


Fig.5. The probability of the pair of electron and hole within the same site as function of time with the increase of intensity of AC electric field. $V_{\rm c}$ is 1 meV while the ratio of E/ω is fixed to 2.405.

5. Conclusion

We investigated the behaviour of a pair of electron and hole under the external electric field with Coulomb interaction between them considered in the NNTBA. For the single electron (or the hole), our conclusion is that the DL of the single particle does not exist due to the Coulomb interaction. The increase of electric field would slow down the process of delocalization. As for the exciton, at the point of $J_0(E/\omega) = 2.405$, the probability of finding the electron and the hole at the same site will reach a certain value, which means that the DL of exciton can be obtained. Stronger the external electric field is, more completely the DL of exciton is realized.

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