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The solution of a wedge disclination dipole interacting with an annular inclusion and the force acting on the disclination dipole*

Song Hao-Peng(宋豪鹏), Fang Qi-Hong(方棋洪), and Liu You-Wen(刘又文)†

College of Mechanics and Aerospace, Hunan University, Changsha 410082, *China*

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The interaction between a wedge disclination dipole and an elastic annular inclusion is investigated. Utilizing the Muskhelishvili complex variable method, the explicit series form solutions of the complex potentials in the matrix and the inclusion region are derived. The image force acting on the disclination dipole centre is also calculated. The influence of the location of the disclination dipole and the thickness of the annular inclusion as well as the elastic dissimilarity of materials upon the equilibrium position of the disclination dipole is discussed in detail. The results show that a stable equilibrium point of the disclination dipole near the inclusion is found for certain combinations of material constant. Moreover, the force on the disclination dipole is strongly affected by the position of the disclination dipole and the thickness of annular inclusion. The repulsion force increases (or the attraction force reduces) with the increase of the thickness of the annular inclusion. An appropriate critical value of the thickness of the annular inclusion may be found to change the direction of the force on the disclination dipole. The present solutions include previous results as special cases.

Keywords: wedge disclination dipole, annular inclusion, image force
PACC: 6170G, 6170Q, 6170J

1. Introduction

The concept of disclination is recognized to be very useful in condensed matter physics. It has been developing intensively for liquid crystals, magnetics materials, biological objects, superconductors and superfliud helium, etc. Disclinations were originally introduced into the mechanics of deformed solids together with dislocations as linear imperfections. A unit disclination is a special and very strong source of internal elastic stress and possesses an energy which is very large in comparison with the energy of a dislocation. However in the cases of self-screened disclination configurations, such as disclination dipoles and disclination loops, the energies may be relatively small and disclinations can exist in crystalline and amorphous materials.[1] Disclination elastic field determines many physical properties of the deformation process of materials. As a result, considerable efforts have been made to obtain the elastic stress fields and energies produced by screened disclinations.[1−22]

It is well known that inclusions either unwanted or deliberately introduced can dramatically change the properties of materials. The elastic interaction between inclusions and dislocations disclinations is of paramount importance in studying the mechanical behaviour of many composite materials.[23,24] Exhilaratingly, the hollow inclusions (e.g. carbon nanotubes) not only reduce the weight of the composite materials but also improve their mechanical properties. In the present work, the complex variable method is introduced to deal with the boundary value problem involving the interaction between a wedge disclination dipole and an elastic annular inclusion. The force on the disclination dipole centre is calculated. The effect of the location of the disclination dipole as well as the material mismatch on the equilibrium position of the disclination dipole is discussed in detail. The results reveal that the equilibrium position has a strong dependence on the mismatch of the shear moduli and Poisson's ratios between the matrix and the inclusion.

2. Problem statement and general treatment

The physical problem to be considered is shown in Fig.1. Let an infinite matrix S^- with the elastic

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†Corresponding author. E-mail: liuyouw8294@sina.com

properties υ_2 and μ_2 contain an elastic annular inclusion S^+ of inner and outer radii $1/R$ and 1 with the elastic properties υ_1 and μ_1, where $\mu_j (j = 1, 2)$ is the shear modulus and υ_j is the Poisson's ratio, respectively. The wedge disclination dipole consists of a positive disclination with strength ω at $z_1 (= x_1 + \mathrm{i}y_1 = z_0 - a\,\mathrm{e}^{\mathrm{i}\varphi})$ and a negative disclination with the same strength at $z_2 (= x_2 + \mathrm{i}y_2 = z_0 + a\,\mathrm{e}^{\mathrm{i}\varphi})$ in the infinite matrix so that the dipole arm has the length $2a$. Both disclination lines are perpendicular to the xoy-plane. The circular interface L between the inclusion and the matrix is perfectly bonded. All quantities with subscripts '1' and '2' refer respectively to the region S^+ and S^-.

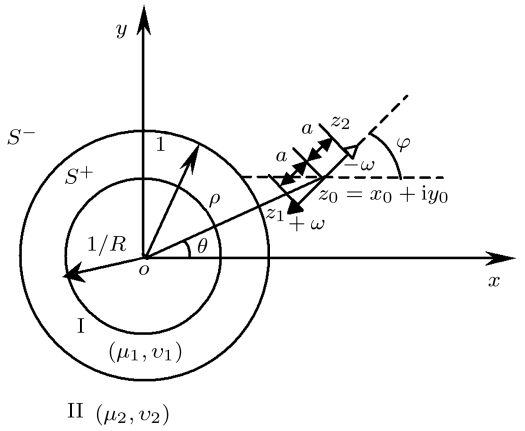


Fig.1. A wedge disclination dipole near an elastic annular inclusion.

For the present problem, displacement components $u_j, v_j\,(j = 1, 2)$, stress components σ_{xxj}, $\sigma_{yyj}, \sigma_{xyj}$ and resultant force $F_{xj} + \mathrm{i}F_{yj}$ can be expressed in terms of the analytic function $\varphi_j(z)$, $\psi_j(z)$ with respect to z as

$$2G_j(u_j + \mathrm{i}v_j) = \kappa\varphi_j(z) - z\overline{\varphi_j'(z)} - \overline{\psi_j(z)}, \quad (1)$$

$$\sigma_{xxj} + \sigma_{yyj} = 2[\varphi_j'(z) + \overline{\varphi_j'(z)}], \quad (2)$$

$$\sigma_{yyj} - \sigma_{xxj} + 2\mathrm{i}\sigma_{xyj} = 2[\bar{z}\varphi_j''(z) + \psi_j'(z)], \quad (3)$$

$$F_{xj} + \mathrm{i}F_{yj} = -\mathrm{i}[\varphi_j(z) + z\overline{\Phi_j(z)} + \overline{\psi_j(z)}]_{\mathrm{A}}^{\mathrm{B}}. \quad (4)$$

A perfect bonding between dissimilar materials implies continuity conditions of stresses and displacements. The stress continuity can be integrated to resultant force continuity conditions. Formally, the boundary conditions at the interface between region S^+ and S^- can be expressed as follows:

$$\begin{aligned} &F_{x1} + \mathrm{i}F_{y1} = F_{x2} + \mathrm{i}F_{y2}, \\ &u_1 + \mathrm{i}v_1 = u_2 + \mathrm{i}v_2, \quad |t| = 1, \end{aligned} \quad (5)$$

Substituting Eq.(1) and Eq.(4) into Eq.(5) yields

$$\begin{aligned} &\varphi_1(t) + t\overline{\Phi_1(t)} + \overline{\psi_1(t)} \\ &= \varphi_2(t) + t\overline{\Phi_2(t)} + \overline{\psi_2(t)}, \quad |t| = 1, \end{aligned} \quad (6)$$

$$\begin{aligned} &G_2[\kappa_1\varphi_1(t) - t\overline{\varphi_1'(t)} - \overline{\psi_1(t)}] \\ &= G_1[\kappa_2\varphi_2(t) - t\overline{\varphi_2'(t)} - \overline{\psi_2(t)}], \quad |t| = 1. \end{aligned} \quad (7)$$

Considering that the traction force on $|t| = 1/R$ is zero, from Eq.(4), we have

$$\varphi_1(t) + t\overline{\Phi_1(t)} + \overline{\psi_1(t)} = 0, \quad |t| = 1/R. \quad (8)$$

For the convenience of analysis, the following new analytic functions are introduced in the corresponding regions according to the Schwarz symmetry principle,

$$\varphi_{1*}(z) = -z\bar{\Phi}_1(1/z) - \bar{\psi}_1(1/z),\ 1 < |z| < R, \quad (9)$$

$$\varphi_{2*}(z) = -z\bar{\Phi}_2(1/z) - \bar{\psi}_2(1/z),\ |z| < 1, \quad (10)$$

using Eqs.(9) and (10), the boundary conditions Eqs.(6) and (7) can be finally rewritten as

$$\begin{aligned} &[\varphi_1(t) + \varphi_{2*}(t)]^+ \\ &= [\varphi_2(t) + \varphi_{1*}(t)]^-, \quad |t| = 1, \end{aligned} \quad (11)$$

$$\begin{aligned} &[G_2\kappa_1\varphi_1(t) - G_1\varphi_{2*}(t)]^+ \\ &= [G_1\kappa_2\varphi_2(t) - G_2\varphi_{1*}(t)]^-, \quad |t| = 1. \end{aligned} \quad (12)$$

3. Solutions of the problem

According to Ref.[21], if a wedge disclination dipole is located at an arbitrary point z_0 inside the matrix, complex potentials $\varphi_2'(z)$ and $\psi_2'(z)$ in the matrix can be written as

$$\varphi_2'(z) = G_2'(z) + \varphi_{20}'(z), \quad (13)$$

$$\psi_2'(z) = H_2'(z) + \psi_{20}'(z), \quad (14)$$

where $G_2'(z) = \dfrac{D_2\omega}{2}[\ln(z - z_1) - \ln(z - z_2)]$, $H_2'(z) = \dfrac{D_2\omega}{2}\left(\dfrac{\bar{z}_2}{z - z_2} - \dfrac{\bar{z}_1}{z - z_1}\right)$, and $D_2 = G_2/2\pi(1 - \upsilon_2)$ and $\varphi_{20}'(z)$ and $\psi_{20}'(z)$ are holomorphic in the region S^-.

Combined with Eq.(10), Eq.(13) arrives at

$$\varphi_{2*}(z) = G_{2*}(z) + \varphi_{20*}(z), \quad (15)$$

where $G_{2*}(z) = \dfrac{D_2\omega}{2}\left[(z - z_2)\ln\left(\dfrac{1}{z} - \bar{z}_2\right) - (z - z_1)\ln\left(\dfrac{1}{z} - \bar{z}_1\right)\right]$.

Neglecting the constant term representing the rigid displacement, $\varphi_1(z)$, $\varphi_{1*}(z)$ can be expanded into a Laurent series in the annular region,

$$\begin{aligned}\varphi_1(z) &= G_P(z) + G_N(z),\\ \varphi_{1*}(z) &= G_{\mathrm{P}*}(z) + G_{N*}(z),\end{aligned} \tag{16}$$

where $G_P(z) = \sum_{k=1}^{\infty} a_k z^k$, $G_N(z) = \sum_{k=1}^{\infty} b_k z^{-k}$, $G_{\mathrm{P}*}(z) = \sum_{k=1}^{\infty} c_k z^k$ and $G_{N*}(z) = \sum_{k=1}^{\infty} d_k z^{-k}$, a_k, b_k, c_k, d_k are complex constants.

Considering the resultant force continuity conditions and the jump value of sectionally holomorphic functions, the following relations can be obtained

$$[\varphi_{20*}(t) + \varphi_1(t)]^+ - [\varphi_{20*}(t)]^- = G_P(t) + G_N(t), \quad |t| = 1/R, \tag{17}$$
$$[\varphi_{20*}(t) + \varphi_1(t)]^+ - [\varphi_{1*}(t) + \varphi_{20}(t)]^- = -G_{2*}(t) + G_2(t), \quad |t| = 1, \tag{18}$$
$$[\varphi_{20}(t))]^+ - [\varphi_{1*}(t) + \varphi_{20}(t)]^- = -G_{P*}(t) - G_{N*}(t), \quad |t| = R. \tag{19}$$

With reference to Eqs.(17), (18), (19), the following expressions can be obtained by using Cauchy integral,

$$\varphi_{20*}(z) = -G_P(z) + \frac{D_2\omega}{2}[(z-z_1)\ln(z-z_1) - (z-z_2)\ln(z-z_2)] + G_{\mathrm{P}*}(z), \quad |z| < 1, \tag{20}$$
$$\varphi_{20}(z) = G_N(z) + \frac{D_2\omega}{2}\left[(z-z_2)\ln\left(\frac{1}{z} - \bar{z}_2\right) - (z-z_1)\ln\left(\frac{1}{z} - \bar{z}_1\right)\right] - G_{N*}(z), \quad |z| > 1. \tag{21}$$

Similarly, considering the displacement continuity conditions and the jump value of sectionally holomorphic functions, we can find that

$$[G_2\kappa_1\varphi_1(t) - G_1\varphi_{20*}(t)]^+ - [-G_1\varphi_{20*}(t)]^- = G_2\kappa_1[G_P(t) + G_N(t)], \quad |t| = \frac{1}{R}, \tag{22}$$
$$[G_2\kappa_1\varphi_1(t) - G_1\varphi_{20*}(t)]^+ - [G_1\kappa_2\varphi_{20}(t) - G_2\varphi_{1*}(t)]^- = G_1G_{2*}(t) + G_1\kappa_2G_2(t), \quad |t| = 1, \tag{23}$$
$$[G_1\kappa_2\varphi_{20}(t)]^+ - [G_1\kappa_2\varphi_{20}(t) - G_2\varphi_{1*}(t)]^- = G_2[G_{P*}(t) + G_{N*}(t)], \quad |t| = R, \tag{24}$$

which result in

$$G_1\varphi_{20*}(z) = G_2\kappa_1G_P(z) - G_1\kappa_2G_2(z) + G_2G_{P*}(z), \quad |z| < 1, \tag{25}$$
$$G_1\kappa_2\varphi_{20}(z) = G_2\kappa_1G_N(z) - G_1G_{2*}(z) + G_2G_{N*}(z), \quad |z| > 1, \tag{26}$$

where

$$G_2(z) = \frac{D_2\omega}{2}[(z-z_1)\ln(z-z_1) - (z-z_2)\ln(z-z_2)], G_{2*}(z) = \frac{D_2\omega}{2}\left[(z-z_2)\ln\left(\frac{1}{z} - \bar{z}_2\right) - (z-z_1)\ln\left(\frac{1}{z} - \bar{z}_1\right)\right].$$

Combining Eq.(20) with Eq.(25) gives

$$(G_1 + G_2\kappa_1)G_P(\zeta) + (G_2 - G_1)G_{\mathrm{P}*}(z) = G_1(1+\kappa_2)\frac{D_2\omega}{2}[(z-z_1)\ln(z-z_1) - (z-z_2)\ln(z-z_2)], \tag{27}$$

$$\varphi_{20*}(z) = -\frac{G_2\kappa_1 + G_2}{G_2 - G_1}G_P(z) + \frac{G_2 + G_1\kappa_2}{G_2 - G_1}G_2(z), \tag{28}$$

Substituting Eq.(21) into Eq.(26) yields

$$\varphi_{20}(z) = \frac{(G_2\kappa_1 + G_1)}{G_1(\kappa_2+1)}G_N(z) + \frac{G_2 - G_1}{G_1(\kappa_2+1)}G_{N*}(z), \tag{29}$$

$$\begin{aligned}&(G_1\kappa_2 - G_2\kappa_1)G_N(z) - (G_1\kappa_2 + G_2)G_{N*}(z)\\ &= -G_1(\kappa_2+1)\frac{D_2\omega}{2}\left[(z-z_2)\ln\left(\frac{1}{z} - \bar{z}_2\right) - (z-z_1)\ln\left(\frac{1}{z} - \bar{z}_1\right)\right].\end{aligned} \tag{30}$$

From Eq.(8) and noting that on $|t| = 1/R$, $t\bar{t} = 1/R^2$, $\varphi_1(t) = -t\bar{\varphi'}_1(1/R^2 t) - \bar{\psi}_1(1/R^2 t)$, namely, $\varphi_1(t)$ and $-t\bar{\varphi'}_1(1/R^2 t) - \bar{\psi}_1(1/R^2 t)$ get across circle $|t| = 1/R$ and extend into each other, we can deduce the following expressions,

$$\varphi_1(z) = -z\bar{\varphi'}_1\left(\frac{1}{R^2 z}\right) - \bar{\psi}_1\left(\frac{1}{R^2 z}\right)$$

and

$$\bar{\psi}_1\left(\frac{1}{z}\right) = -\frac{z}{R^2}\bar{\varphi'}_1\left(\frac{1}{z}\right) - \varphi_1\left(\frac{z}{R^2}\right). \tag{31}$$

Substituting Eq.(31) into Eq.(9) yields

$$\varphi_{1*}(z) = \frac{z}{R^2}\bar{\varphi'}_1\left(\frac{1}{z}\right) + \varphi_1\left(\frac{z}{R^2}\right) - z\bar{\varphi'}_1\left(\frac{1}{z}\right). \tag{32}$$

The substitution of Eqs.(16), (27) and (30) into Eq.(32) respectively yields

$$\begin{aligned}
&\left(\frac{1}{R^2} - 1\right) z\bar{\varphi'}_1\left(\frac{1}{z}\right) + G_P\left(\frac{z}{R^2}\right) + G_N\left(\frac{z}{R^2}\right) \\
&= \frac{G_1(1+\kappa_2)}{G_2 - G_1}\frac{D_2\omega}{2}[(z - z_1)\ln(z - z_1) - (z - z_2)\ln(z - z_2)] - \frac{G_1 + G_2\kappa_1}{G_2 - G_1}G_P(z) + \frac{(G_1\kappa_2 - G_2\kappa_1)}{G_1\kappa_2 + G_2}G_N(z) \\
&\quad + \frac{G_1(\kappa_2 + 1)}{G_1\kappa_2 + G_2}\frac{D_2\omega}{2}\left[(z - z_2)\ln\left(\frac{1}{z} - \bar{z}_2\right) - (z - z_1)\ln\left(\frac{1}{z} - \bar{z}_1\right)\right].
\end{aligned} \tag{33}$$

Comparison of the coefficients of the same power terms in the Eq.(33) yields

$$\begin{aligned}
a_1 &= \frac{\left(\frac{1}{R^2} - 1\right) M_4\frac{D_2\omega}{2}\ln\frac{\bar{z}_1}{\bar{z}_2} - \left(\frac{1}{R^2} + M_2\right) M_4\frac{D_2\omega}{2}\ln\frac{z_1}{z_2}}{\left(\frac{1}{R^2} - 1\right)^2 - \left(\frac{1}{R^2} + M_2\right)^2}, \quad a_2 = \frac{M_4\frac{D_2\omega}{2}\left(-\frac{1}{2z_1} + \frac{1}{2z_2}\right)}{\frac{1}{R^4} + M_2}, \\
a_{k+2} &= \frac{k\left(\frac{1}{R^2} - 1\right)\bar{L}_{k2} + (R^{2k} - M_1)L_{k1}}{\left(1 - \frac{1}{R^2}\right)^2 k(k+2) + (R^{2k} - M_1)\left(\frac{1}{R^{2(k+2)}} + M_2\right)}, \; b_k = \frac{L_{k2} - \left(\frac{1}{R^2} - 1\right)(k+2)\bar{a}_{k+2}}{R^{2k} - M_1},
\end{aligned} \tag{34}$$

where

$$\begin{aligned}
L_{k1} &= M_4\frac{D_2\omega}{2}\left[-\frac{1}{(k+1)(k+2)z_1^{k+1}} + \frac{1}{(k+1)(k+2)z_2^{k+1}}\right], \\
L_{k2} &= M_3\frac{D_2\omega}{2}\left[\frac{z_2}{k\bar{z}_2^k} - \frac{z_1}{k\bar{z}_1^k} + \frac{1}{(k+1)\bar{z}_1^{k+1}} - \frac{1}{(k+1)\bar{z}_2^{k+1}}\right], \\
M_1 &= \frac{G_1\kappa_2 - G_2\kappa_1}{G_1\kappa_2 + G_2}, \quad M_2 = \frac{G_1 + G_2\kappa_1}{G_2 - G_1}, \quad M_3 = \frac{G_1(\kappa_2 + 1)}{G_1\kappa_2 + G_2}, \quad M_4 = \frac{G_1(1+\kappa_2)}{G_2 - G_1}.
\end{aligned}$$

Considering Eqs.(13), (15), (16), (19) and (27)–(30), the complex variables can be written as

$$\begin{aligned}
\varphi_2(z) &= G_2(z) + \sum_{k=1}^{\infty}\left[\frac{G_2\kappa_1 + G_1}{G_1(\kappa_2 + 1)} + \frac{G_2 - G_1}{G_1(\kappa_2 + 1)}\frac{G_1\kappa_2 - G_2\kappa_1}{G_1\kappa_2 + G_2}\right] b_k z^{-k} \\
&\quad + \frac{G_2 - G_1}{G_1\kappa_2 + G_2}\frac{D_2\omega}{2}\left[(z - z_2)\ln\left(\frac{1}{z} - \bar{z}_2\right) - (z - z_1)\ln\left(\frac{1}{z} - \bar{z}_1\right)\right],
\end{aligned} \tag{35}$$

$$\varphi_{2*}(z) = G_{2*}(z) + \frac{G_2\kappa_1 + G_2}{G_1 - G_2}\sum_{k=1}^{\infty} a_k z^k + \frac{G_1\kappa_2 + G_2}{G_2 - G_1}\frac{D_2\omega}{2}[(z - z_1)\ln(z - z_1) - (z - z_2)\ln(z - z_2)], \tag{36}$$

$$\varphi_1(z) = \sum_{k=1}^{\infty} a_k z^k + \sum_{k=1}^{\infty} b_k z^{-k}, \tag{37}$$

$$\begin{aligned}
\varphi_{1*}(z) &= \frac{G_1(1+\kappa_2)}{G_2 - G_1}G_2(z) - \frac{G_1 + G_2\kappa_1}{G_2 - G_1}\sum_{k=1}^{\infty} a_k z^k + \frac{G_1\kappa_2 - G_2\kappa_1}{G_1\kappa_2 + G_2}\sum_{k=1}^{\infty} b_k z^{-k} \\
&\quad + \frac{G_1(1+\kappa_2)}{G_1\kappa_2 + G_2}\frac{D_2\omega}{2}\left[(z - z_2)\ln\left(\frac{1}{z} - \bar{z}_2\right) - (z - z_1)\ln\left(\frac{1}{z} - \bar{z}_1\right)\right].
\end{aligned} \tag{38}$$

With $R \to \infty$, the interaction between a wedge disclination dipole and a circular inhomogeneity can be calculated. With reference to Eq.(34), we have

$$a_1 = \frac{M_4}{M_2\left(1-\dfrac{1}{M_2^2}\right)}\frac{D_2\omega}{2}\left(\ln\frac{z_1}{z_2}+\frac{1}{M_2}\ln\frac{\overline{z_1}}{\overline{z_2}}\right),$$
$$a_2 = M_4\frac{D_2\omega}{2}\left(-\frac{1}{2z_1}+\frac{1}{2z_2}\right)/M_2,$$
$$a_{k+2} = \frac{L_{k1}}{M_2}, \qquad b_k = 0,$$

which are in agreement with the previous results of Ref.[21].

4. Force on the disclination dipole

In the present work, the force components per unit length on the disclination dipole centre can be calculated approximatively by using the Peach–Koehler formula[6,19]

$$\begin{aligned} & f_x - \mathrm{i}f_y \\ = \; & 2\omega a\cos\varphi\widehat{\sigma}_{xy} + 2\omega a\sin\varphi\widehat{\sigma}_{yy} \\ & +\mathrm{i}[2\omega a\cos\varphi\widehat{\sigma}_{xx} + 2\omega a\sin\varphi\widehat{\sigma}_{xy}], \end{aligned} \tag{39}$$

where $\widehat{\sigma}_{xx}$, $\widehat{\sigma}_{yy}$ and $\widehat{\sigma}_{xy}$ are the components of the perturbation stress at the disclination dipole centre in Cartesian coordinates. The perturbation stresses can be calculated by the perturbation stress functions $\Phi_1^*(z_0)$, $\Phi_1^{*'}(z_0)$ and $\Psi_1^*(z_0)$ in the region S^+. Referring to Qaissaunee and Santare,[24] equation (39) can be rewritten as

$$\begin{aligned} f_x - \mathrm{i}f_y \; = \; & 2\omega a\,\mathrm{e}^{\mathrm{i}\varphi}[\Phi_2^*(z_0) + \overline{\Phi_2^*(z_0)}] \\ & +2\omega a\,\mathrm{e}^{-\mathrm{i}\varphi}[\overline{z}_0\Phi_2^{*'}(z_0) + \Psi_2^*(z_0)], \end{aligned} \tag{40}$$

where

$$\begin{aligned} \Phi_2^*(z_0) &= \lim_{z\to z_0}[\Phi_2(z) - G_2'(z)] \\ &= \sum_{k=1}^{\infty}\left[-\frac{G_2\kappa_1+G_1}{G_1(\kappa_2+1)} + \frac{G_2-G_1}{G_1(\kappa_2+1)}\frac{G_1\kappa_2-G_2\kappa_1}{G_1\kappa_2+G_2}\right]kb_kz_0^{-(k+1)} \\ &\quad + \frac{G_2-G_1}{G_1\kappa_2+G_2}\frac{D_2\omega}{2}\left[\ln\frac{(z_0-z_2^*)}{(z_0-z_1^*)} + \frac{z_1-z_1^*}{z_0-z_1^*} - \frac{z_1}{z_0} - \frac{z_2-z_2^*}{z_0-z_2^*} + \frac{z_2}{z_0}\right], \end{aligned} \tag{41}$$

$$\begin{aligned} \Phi_2^{*'}(z_0) &= \lim_{z\to z_0}\frac{\mathrm{d}[\Phi_2(z) - G_2'(z)]}{\mathrm{d}z} \\ &= \sum_{k=1}^{\infty}\left[\frac{G_2\kappa_1+G_1}{G_1(\kappa_2+1)} - \frac{G_2-G_1}{G_1(\kappa_2+1)}\frac{G_1\kappa_2-G_2\kappa_1}{G_1\kappa_2+G_2}\right]k(k+1)b_kz_0^{-(k+2)} \\ &\quad + \frac{G_2-G_1}{G_1\kappa_2+G_2}\frac{D_2\omega}{2}\left[\frac{z_1-z_2}{z_0^2} - \frac{1}{z_0-z_1^*} + \frac{1}{z_0-z_2^*} - \frac{z_1-z_1^*}{(z_0-z_1^*)^2} + \frac{z_2-z_2^*}{(z_0-z_2^*)^2}\right], \end{aligned} \tag{42}$$

$$\begin{aligned} \Psi_2^*(z_0) &= \lim_{z\to z_0}[\Psi_2(z) - H_2'(z)] \\ &= \frac{1}{z_0^2}\left\{\sum_{k=1}^{\infty}\left[-\frac{G_2\kappa_1+G_1}{G_1(\kappa_2+1)} + \frac{G_2-G_1}{G_1(\kappa_2+1)}\frac{G_1\kappa_2-G_2\kappa_1}{G_1\kappa_2+G_2}\right]kb_kz_0^{-(k+1)}\right. \\ &\quad \left. + \frac{G_2-G_1}{G_1\kappa_2+G_2}\frac{D_2\omega}{2}\left[\ln\frac{(z_0-z_2^*)}{(z_0-z_1^*)} + \frac{z_1-z_1^*}{z_0-z_1^*} - \frac{z_1}{z_0} - \frac{z_2-z_2^*}{z_0-z_2^*} + \frac{z_2}{z_0}\right]\right\} \\ &\quad - \frac{1}{z_0}\left\{\sum_{k=1}^{\infty}\left[\frac{G_2\kappa_1+G_1}{G_1(\kappa_2+1)} - \frac{G_2-G_1}{G_1(\kappa_2+1)}\frac{G_1\kappa_2-G_2\kappa_1}{G_1\kappa_2+G_2}\right]k(k+1)b_kz_0^{-(k+2)}\right. \\ &\quad \left. + \frac{G_2-G_1}{G_1\kappa_2+G_2}\frac{D_2\omega}{2}\left[\frac{z_1-z_2}{z_0^2} - \frac{1}{z_0-z_1^*} + \frac{1}{z_0-z_2^*} - \frac{z_1-z_1^*}{(z_0-z_1^*)^2} + \frac{z_2-z_2^*}{(z_0-z_2^*)^2}\right]\right\} \\ &\quad + \frac{1}{z_0^2}\left\{\sum_{k=1}^{\infty}k\bar{a}_kz_0^{-(k-1)} + \frac{D_2\omega}{2}\left[\ln\left(\frac{1}{z_0}-z_1\right) - \ln\left(\frac{1}{z_0}-z_2\right)\right]\right\}, \end{aligned} \tag{43}$$

and $z_1^* = 1/\bar{z}_1$, $z_2^* = 1/\bar{z}_2$.

From Eqs.(40)–(44), it is observed that the force acting on the wedge disclination dipole is a rather complicated function of the location of the disclination dipole and the material properties of the two phases. Equilibrium states of disclination dipoles are of primary importance. Here, we will mainly focus on the effect of the material mismatch upon the equilibrium position of the wedge disclination dipole. Without loss of generality, we consider the case that the disclination dipole centre lies on the x-axis ($z_0 = x_0 = \rho$, $\theta = 0°$). The normalized force on the disclination dipole centre is defined as $f_{xo} = f_x/G_2\omega^2 a$.

Figure 2 shows the variation of the normalized force f_{x0} versus z_0 with different ratios of the shear modulus G_1/G_2 at $R = 2$, $v_1 = 0.3$, $v_2 = 0.5$, $a = 0.01$ and $\theta = \varphi = 0°$. It is found that the disclination dipole tends to pass into a soft inclusion (having a low shear modulus). The force f_{x0} is positive first, and then becomes negative when the wedge disclination dipole moves to infinity from the interface along the x-axis under certain material property combinations (e.g. $R = 2$, $G_1/G_2 = 1.95$, $v_1 = 0.3$ and $v_2 = 0.5$). There exists a stable equilibrium point near the interface. The force f_{x0} also achieves a negative maximum value (attraction force) on the x-axis for this case. It is seen that the circular inclusion always repels the dipole if the ratio of the shear modulus G_1/G_2 takes a relatively large value (e.g. $G_1/G_2 = 2.2$). In addition, the attraction force acting on the dipole centre increases with the softening of the inclusion.

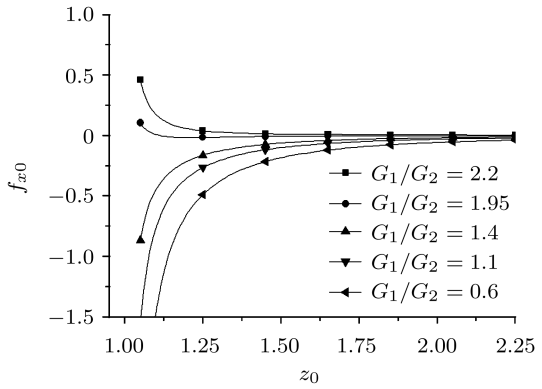


Fig.2. f_{x0} (normalized force) versus z_0 (position) at $R = 2$, $v_1 = 0.3$, $v_2 = 0.5$, $\varphi = 0$, $\theta = 0$, $a = 0.01$.

The variation of f_{x0} with respect to the ratio of the shear modulus G_1/G_2 with different relative position z_0 at $R = 2$, $v_1 = 0.3$, $v_2 = 0.5$, $a = 0.01$ and $\theta = \varphi = 0°$ is depicted in Fig.3. It is found that, if $G_1 \neq G_2$, an appropriate critical value of the shear modulus ratio G_1/G_2 may be found to change the direction of the force f_{x0} when the disclination centre is located near the circular interface, and the location of the disclination dipole is an equilibrium point.

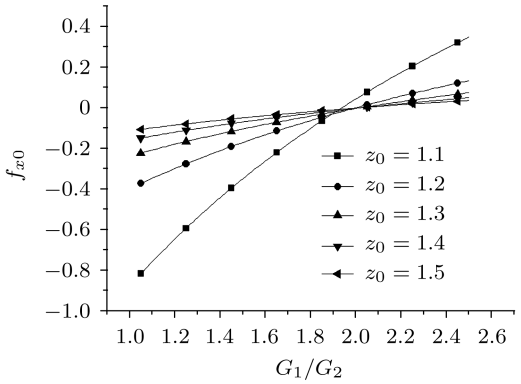


Fig.3. f_{x0} (normalized force) versus G_1/G_2 (ratio of the shear modulus) at $R = 2$, $v_1 = 0.3$, $v_2 = 0.5$, $\varphi = 0$, $\theta = 0$, $a = 0.01$.

The variation of f_{x0} versus R with different ratios of the shear modulus G_1/G_2 is shown in Fig.4 for the case of $z_0 = 1.1$, $v_1 = 0.1$, $v_2 = 0.1$, $a/R = 0.01$ and $\theta = \varphi = 0°$. It is found that, if $G_1/G_2 > 1$, an appropriate critical value of R may be found to change the direction of the force f_{x0}. The repulsion force increases (or the attraction force reduces) with the increase of R (the thickness of the annular inclusion).

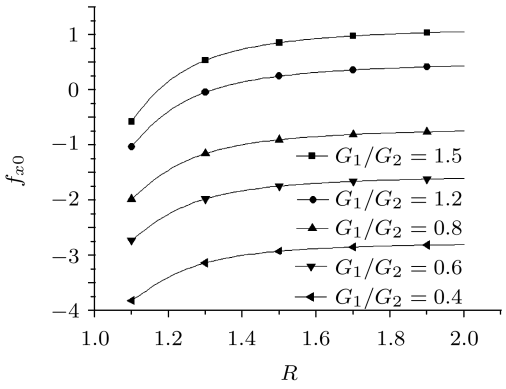


Fig.4. f_{x0} (normalized force) versus R (the thickness of the annular inclusion) at $z_1 = 1.1$, $v_1 = 0.1$, $v_2 = 0.1$, $\varphi = 0$, $\theta = 0$, $a = 0.01$.

5. Conclusion

By using the complex variable method, analytical solutions are derived for complicated potentials due to a wedge dipole located near an annular inclusion. The

force acting on the disclination dipole is also given. The influence of the location of the disclination dipole and the thickness of the annular inclusion as well as the material elastic dissimilarity on the equilibrium position of the disclination dipole is discussed in detail. The results indicate that the equilibrium position has a strong dependence on the mismatch of the shear moduli and Poisson's ratios between the matrix and inclusion. It is found that a stable equilibrium point may be available near the inclusion under certain material property combinations. Furthermore, the repulsion force increases (or the attraction force reduces) with the increase of the thickness of the annular inclusion. An appropriate critical value of the thickness of the annular inclusion may be found to change the direction of the force on the disclination dipole.

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