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Electrohydromechanical analysis based on conductivity gradient in microchannel*

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Fluid manipulation is very important in any lab-on-a-chip system. This paper analyses phenomena which use the alternating current (AC) electric field to deflect and manipulate coflowing streams of two different electrolytes (with conductivity gradient) within a microfluidic channel. The basic theory of the electrohydrodynamics and simulation of the analytical model are used to explain the phenomena. The velocity induced for different voltages and conductivity gradient are computed. The results show that when the AC electrical signal is applied on the electrodes, the fluid with higher conductivity occupies a larger region of the channel and the interface of the two fluids is deflected. It will provide some basic reference for people who want to do more study in the control of different fluids with conductivity gradient in a microfluidic channel.

Keywords: electrohydrodynamics, conductivity gradient, theoretical analysis, numerical simulation
PACC: 4700, 4762

1. Introduction

The lab-on-a-chip (LOC) is a new technology in the micro-electromechanical system (MEMS).[1] In the system of LOC, we have much work to study.[2] Dynamic control of fluids is important in LOC technology, and MEMS devices are often used for this.[3] However, these devices are often complicated to construct and cannot be integrated into LOC easily. Now, the AC electric field-induced flow can be used to move fluids and analyse or produce mixing.[4−7] These techniques include micromechanical methods, direct current (DC) electro-osmosis, electro-wetting, thermo-capillary pumping, and AC electro-osmosis.[8]

Recently, a new phenomenon to control two fluids with conductivity gradient was found.[9] The simply experimental model is shown in Fig.1. The device is made of interdigitated electrodes which occupies the width of the channel, and the two fluids are potassium chloride with conductivity gradient ($\sigma_2 > \sigma_1$). The fluids are laminar flow, which is shown in Fig.2(a). Figure 2(b) shows that when the AC electrical signal(voltage) is applied on the electrodes, the fluid with higher conductivity occupies a larger region of the channel and the interface of the two fluids is deflected. The phenomena are interesting and we try to do some studies about the basic theory and simulation

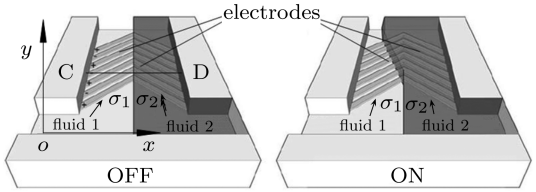


Fig.1. The model of microchannel.

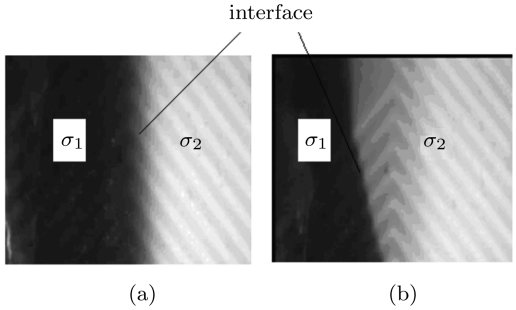


Fig.2. The phenomenon of the experiment.

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2. Basic theory

The experiment shows that there must be a force that drives the fluid. The body force of the fluid, $\boldsymbol{f}_{\rm e}$,[10] is

$$\boldsymbol{f}_{\rm e} = \rho_{\rm q}\boldsymbol{E} - \frac{1}{2}\boldsymbol{E}^2\nabla\varepsilon + \frac{1}{2}\nabla\left[\rho_{\rm m}\left(\frac{\partial\varepsilon}{\partial\rho_{\rm m}}\right)_T \boldsymbol{E}^2\right], \quad (1)$$

where $\rho_{\rm q}$ is the charge density, $\rho_{\rm m}$ is the mass density, ε is the permittivity, T is the temperature, and E is the magnitude of the electric field. The first and second terms are the Coulomb and dielectric forces, respectively, and the last term is electrostriction force, which can be ignored for incompressible fluids.[10] For the system described here, the dielectric term is zero because both liquids have the same permittivity. The main force acting on the free charge at the interface between the two liquids is Coulomb force.

In order to find an easier model, we solve a two dimensional (2D) problem. Figure 1(a) shows that a xoy coordinate is selected and a line named 'CD' that parallels the xoy plane is chosen freely. Figure 3 is the section along the line 'CD'. In fact, the real experimental device is more complex and the electrodes are more than those shown in the Fig.1. Because the size of the electrodes are small enough, it can be assumed that the sections of the electrodes are the same, which is shown in Fig.3.

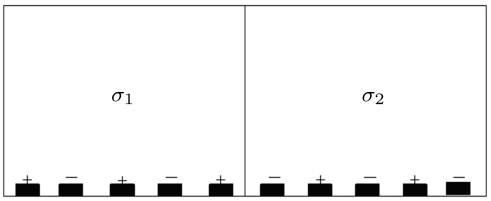


Fig.3. The analytical model.

In order to predigest the model, a pair of electrodes are analysed, which are shown in Fig.4. For a non-uniform electric field, a small region 'abdc' is chosen, where 'oo'' is the centre and also the interface of the two fluids. The small region is nearly uniform.

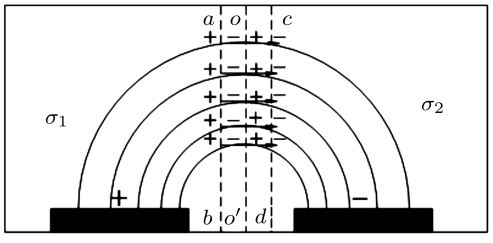


Fig.4. The model with two electrodes.

We assume that the left current density is $\boldsymbol{J}_1$ and right is $\boldsymbol{J}_2$, so

$$\boldsymbol{n}\cdot\boldsymbol{J}_1 = \boldsymbol{n}\cdot\boldsymbol{J}_2, \quad (2)$$

where $\boldsymbol{n}$ is the normal vector to the interface. The left electric field intensity is $\boldsymbol{E}_1$ and right is $\boldsymbol{E}_2$, the equations based on the Ohm's law can be expressed as

$$\boldsymbol{J}_1 = \sigma_1\boldsymbol{E}_1 \quad (3)$$

and

$$\boldsymbol{J}_2 = \sigma_2\boldsymbol{E}_2 \quad (4)$$

respectively.

Substituting Eqs.(3) and (4) into Eq.(2), we can obtain

$$\sigma_1\boldsymbol{E}_1\cdot\boldsymbol{n} = \sigma_2\boldsymbol{E}_2\cdot\boldsymbol{n}. \quad (5)$$

The net charge per unit area at the interface based on the Gauss's law[11] can be described as

$$Q_{\rm surface} = (\varepsilon_2\boldsymbol{E}_2 - \varepsilon_1\boldsymbol{E}_1)\cdot\boldsymbol{n}. \quad (6)$$

Because of the same solute, the two liquids have the same permittivity ($\varepsilon_1 = \varepsilon_2 = \varepsilon$). Thus, Eq.(6) can be changed to

$$Q_{\rm surface} = \varepsilon(\boldsymbol{E}_2 - \boldsymbol{E}_1)\cdot\boldsymbol{n} = \varepsilon\left(\frac{\sigma_1}{\sigma_2} - 1\right)\boldsymbol{E}_1\cdot\boldsymbol{n}. \quad (7)$$

From the Eq.(7), if $\sigma_1 > \sigma_2$, the net charge at the interface is positive and the direction of Coulomb force is right; if $\sigma_1 < \sigma_2$, on the contrary, the net charge is negative and the direction of Coulomb force is left. The reason is that in Fig.3, the direction of electric field is from left to right.

Otherwise, if we change the direction of the electric field, the Eq.(7) will be changed to

$$Q_{\rm surface} = \varepsilon(\boldsymbol{E}_1 - \boldsymbol{E}_2)\cdot\boldsymbol{n} = \varepsilon\left(1 - \frac{\sigma_1}{\sigma_2}\right)\boldsymbol{E}_1\cdot\boldsymbol{n}. \quad (8)$$

In our experiment, $\sigma_1 < \sigma_2$, the net charge at the interface is positive and the electric field is from right to left, so the fluids will get the left Coulomb force, which is the same as before.

Using more electrodes, we find that the prediction is the same. Because the line 'CD' can be chosen freely, so in all the microchannel, the interface of the two fluids will get the left Coulomb force.

3. Simulation

After the theoretical analysis, we know that fluid is driven by the Coulomb force, but the phenomena in the micro-system are complex. We make some simulation in order to compare them with the experimental results.

3.1. Basic equations

For the electrodes, the voltage information can be considered, and the diffuse equation is suitable for the conductivity gradient. The Navier–Stokes equation can be considered about the fluid flows. Then the three equations are coupled and the approximative results of the system simulation can be seen.

3.1.1. Voltage equation

The charge conservation equation is

$$\nabla \cdot \boldsymbol{J} = 0. \tag{9}$$

The relationship between the voltage and electric field intensity is

$$\boldsymbol{E} = -\nabla\phi, \tag{10}$$

where ϕ is the voltage.

The equation based on Ohm's law is

$$\boldsymbol{J} = \sigma\boldsymbol{E}. \tag{11}$$

Substituting Eqs.(10) and (11) into Eq.(9), we can express a new equation as

$$\nabla \cdot (\sigma\nabla\phi) = \nabla\sigma \cdot \nabla\phi + \sigma\nabla^2\phi = 0. \tag{12}$$

3.1.2. Diffusion equation for equivalent conductivity

The fluid has a conductivity that satisfies the diffusion equation[12]

$$D\nabla^2\sigma = \frac{\partial\sigma}{\partial t} + (\boldsymbol{u} \cdot \nabla)\sigma, \tag{13}$$

where D is diffusion coefficient.

3.1.3. Navier–Stokes equation

Assume that the fluid is incompressible, so the Navier–Stokes equation is[11]

$$\rho\frac{\partial\boldsymbol{u}}{\partial t} = \eta\nabla^2\boldsymbol{u} - \nabla\boldsymbol{p} + \boldsymbol{f}_{\mathrm{e}}, \tag{14}$$

where η is the viscosity, $\boldsymbol{p}$ is the pressure and $\boldsymbol{f}_{\mathrm{e}}$ is the body force. Changing the Eq.(12), we can describe a new equation as

$$\nabla^2\phi = -\frac{\nabla\sigma \cdot \nabla\phi}{\sigma}. \tag{15}$$

The body force in Navier–Stokes equation is[11]

$$\boldsymbol{f}_{\mathrm{e}} = \rho\boldsymbol{E} = \varepsilon\nabla^2\phi \cdot \nabla\phi = -\varepsilon\frac{\nabla\sigma \cdot \nabla\phi}{\sigma}\nabla\phi. \tag{16}$$

The relationships of the elements of the three main equations are as follows:

(1) Voltage equation provides electric field intensity $\boldsymbol{E}$ for the Navier–Stokes equation.

(2) Diffusion equation provides the conductivity gradient $\nabla\sigma$ and conductivity σ for the Navier–Stokes equation.

(3) Navier–Stokes equation provides the fluid velocity $\boldsymbol{u}$ for the diffuse equation.

3.2. Simulation results

After making sure the relationship of the equations, we can use commercial software to simulate the model founded in Fig.3.

3.2.1. Voltage simulation

The width of the electrodes is 15 μm and the gap of two adjacent electrodes is 10 μm. In order to get the obvious simulation, we let the conductivities of the left and the right boundary be σ_1 and σ_2, respectively. All the other boundary conditions are electric insulation. The simulation result of the voltage is shown in Fig.5 when the applied voltage is 1V.

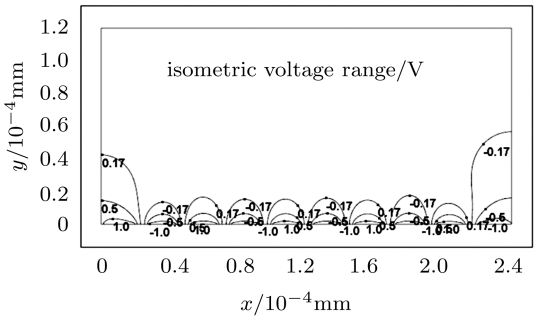


Fig.5. Voltage simulation.

When the voltage is improved gradually, the same isometric voltage range can be found, but the value is not the same. The voltage affects the fluid velocity, which will be studied in the next section.

3.2.2. Time-dependent conductivity simulation

The conductivity ratio is $\sigma_2/\sigma_1 = 2$. The simulation of the conductivity includes two steps:

First, the conductivity is the initial state, which is shown in Fig.6.

Second, Figs.7(a) to 7(d) show that the conductivity is instantaneous and coupling with other equations when the time is 0.01s, 0.1s, 1s and 3s, respectively.

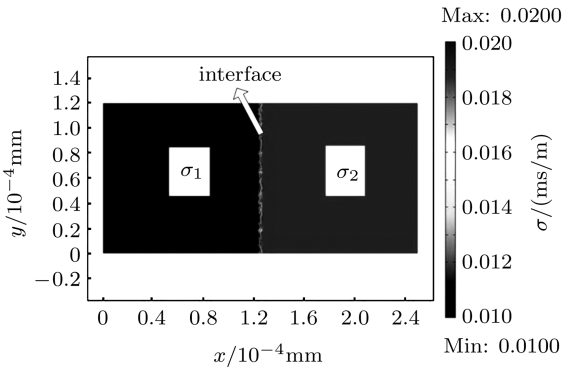


Fig.6. Initial conductivity.

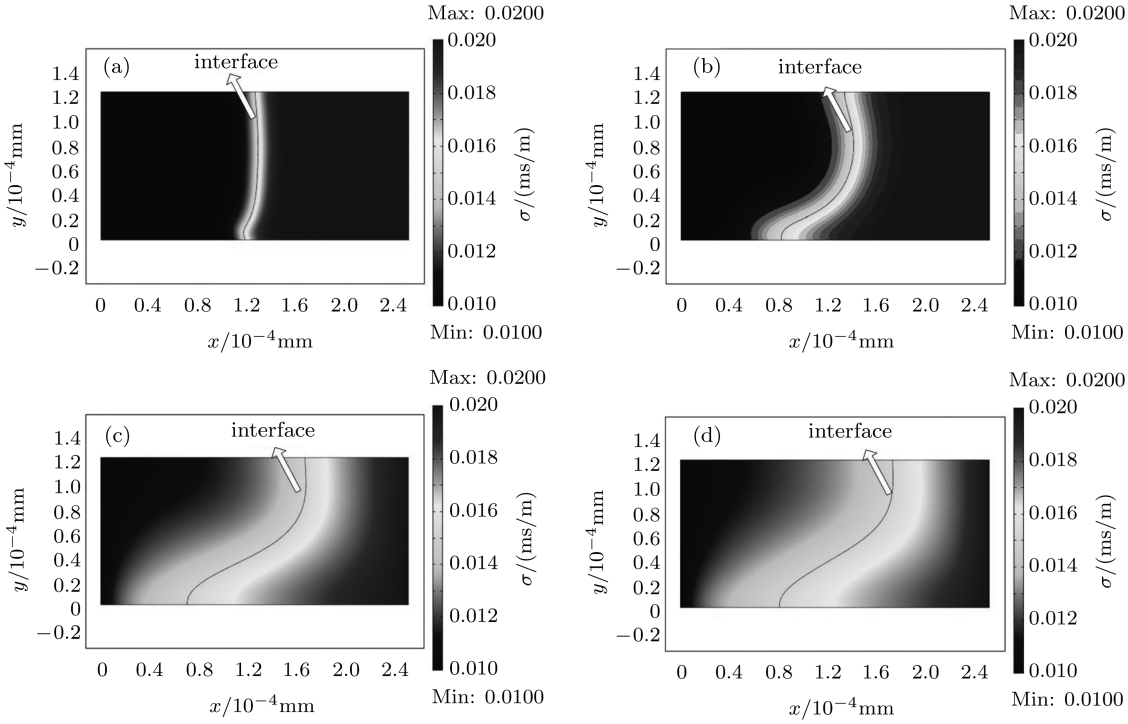


Fig.7. Instantaneous conductivity: (a) $t = 0.01$s; (b) $t = 0.1$s; (c) $t = 1$s; (d) $t = 3$s.

Comparing the Fig.7(c) with Fig.7(d) we can see that after 1s the state of conductivity is nearly steady. Because the left and right boundary conductivities are all constant, finally, conductivity of the whole system is not the same.

From the Figs.7(a) to 7(d) we can see that the fluid with higher conductivity tends to occupy the region close to the electrodes and the interface of the two fluids is deflected. The model simulated has one section paralleled xoy plane and is along the line 'CD'. Because the line can be chosen freely, the phenomena of the whole channel are the same.

From the conductivity simulation, it can be seen that the fluids flow, otherwise, the phenomena from Figs.7(a) to 7(d) cannot be seen. In Fig.8, it is the

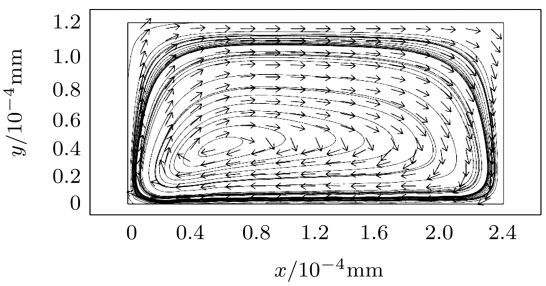


Fig.8. Velocity field.

the steady state of rotative fluid flow, which is different from the experimental results (Fig.2). The reason

is that the boundary condition in the model is closed and the fluids can't flow out.

3.3. Velocity of fluid flow

Some specific factors that affect the fluid velocity are considered in the following analysis.

First, assume that the conductivity ratio is $\sigma_2/\sigma_1 = 2$, and then improve the voltage from 0.01V to 2V gradually. Figure 9 shows the relationship between the voltage and the fluid velocity. The velocity, $\overline{v}$, is an average quantity, calculated by

$$\overline{v} = \frac{\iint v\mathrm{d}s}{\iint \mathrm{d}s} = \frac{\iint v\mathrm{d}s}{S} \tag{17}$$

where $\overline{v}$ is the average velocity, v is the velocity of a random point, and S is the integral of area.

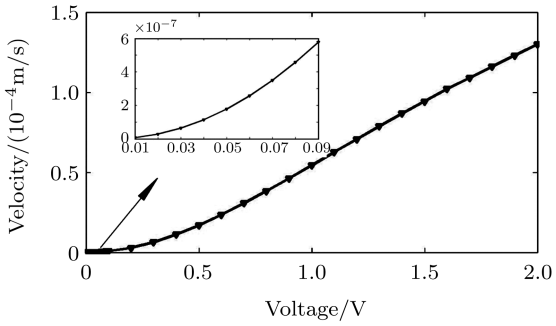


Fig.9. The relationship between voltage and velocity.

From Fig.9 we can see that at range of the low voltage, the velocity nearly scales with $(\nabla\phi)^2$. However, at range of the high voltage, the relationship between voltage and fluid velocity is not the same.

Considering Eq.(15), we can assume that when the voltage is low, the $\nabla\sigma$ and σ change little [12],so the force nearly scales with $(\nabla\phi)^2$, but as the voltage increases, $\nabla\sigma$ and σ are important for the force, so the relationship is changed. In order to avoid the electrolysis, the voltage should not be very high. We let the voltage less than 20V.[11]

Second, we assume that the applied voltage is 1V, simulate the time-dependent velocity. Figure 10 shows that the fluid flow velocity increases quickly at the beginning, then it becomes slow and at last it is steady. Comparing with the Fig.7 we can see the same rule: at the beginning, the interface changes obviously and then after 1s, the position of the interface changes little.

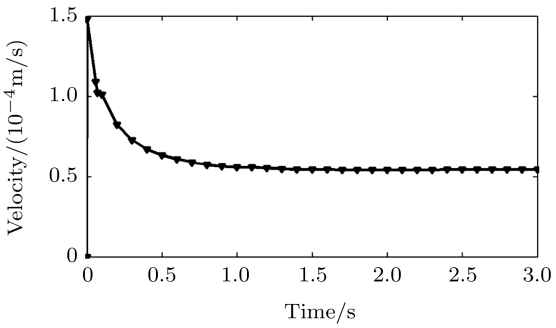


Fig.10. The relationship between time and fluid velocity.

Third, the relationship between the conductivity gradient and the fluid velocity is shown in Fig.11. The voltage is 1V and the σ_2/σ_1 changes from 1 to 10 gradually. When $\sigma_2/\sigma_1 = 1$, the fluid velocity is zero, which also can be calculated by the Eq.(7). The force is proportional to $(\sigma_2-\sigma_1)/(\sigma_2+\sigma_1)$,[9] so when $\sigma_2 >> \sigma_1$, $(\sigma_2-\sigma_1)/(\sigma_2+\sigma_1)$ tends to one, and the force and velocity are nearly constant, too.

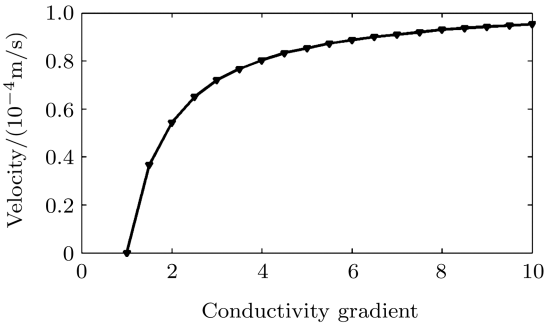


Fig.11. The relationship between conductivity gradient and velocity.

4. Conclusion

Fluid manipulation is an important technology in LOC. In this paper we analyse the phenomena in the microchannel, and find that if the voltage signal is applied on the electrodes, the fluid with higher conductivity occupies a larger region of the channel and the interface of the two fluids is deflected. We make some study in the basic theory, and explain the phenomena. Also, we simulate the system and compare the simulation results with the experimental results. It can be seen that the simulation results are near to the experimental phenomena. The present research can be a useful guide for people who want to do more study in the control of fluids with conductivity gradient.

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