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Third-order ghost interference with thermal light*

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A scheme for third-order ghost interference with thermal light is proposed. The visibility and resolution of the interference fringe related to the bandwidth of the spatial frequency spectrum of the source are analysed. The results show that the visibility of the third-order ghost interference fringe is much higher than that of the second-order one.

Keywords: ghost interference, ghost imaging, high-order field correlation
PACC: 4225H, 4250

1. Introduction

About one decade ago, ghost interference and imaging (coincidence interference and imaging) were first observed by using entangled photon pairs generated in spontaneous parametric down-conversion (SPDC) process.[1,2] These effects are of great importance in potential applications such as quantum metrology, quantum lithography, and quantum holography.[3−6] Abouraddy *et al*[7] discussed the role of entanglement in ghost imaging, and showed that entanglement is a prerequisite for achieving distributed ghost imaging. However, Bennink *et al*[8] showed in their experiments that coincidence imaging and interference can be also performed with a classical coherent light source under certain conditions. Recent theoretical and experimental studies showed that a thermal (or quasi-thermal) light source can play a role similar to that of an entangled light source in ghost imaging, ghost interference, and sub-wavelength interference.[9−29] Gatti *et al*[9] proposed theoretically that ghost imaging can be realized by using a thermal light source with classical correlation. At the same time, Cheng and Han[10] studied the incoherent coincidence imaging and its application in x-ray diffraction. Cao *et al*[11] discussed geometrical optics in correlated imaging system, and first pointed out that classical thermal light source looks like a phase-conjugate mirror in the correlated imaging, and hence can exhibit ghost imaging without using any lenses. The first experimental demonstrations of ghost imaging with pseudo-thermal light were reported by Valencia *et al*[12] and Ferri *et al.*[13] Subsequently, the experimental observation of the lensless imaging was demonstrated by Scarcelli *et al*[14] and Basano *et al.*[15] Moreover, Wang and Cao[16] demonstrated theoretically that a thermal light source can produce subwavelength double-slit interference in joint intensity measurement. Cai and Zhu[17] studied ghost interference and imaging with incoherent and partially coherent light radiation. The experimental performance of sub-wavelength interference with pseudo-thermal light were soon carried out by Scarcelli *et al*[18] and Xiong *et al.*[19] Furthermore, these effects were observed experimentally with true thermal light.[20,21] Bache *et al*[26] reported that coherent imaging with incoherent classical thermal light is able to produce the interference pattern of a pure phase object in an experiment, and Zhang *et al*[27,28] demonstrated experimentally that lensless Fourier-transform ghost imaging of a pure phase object with classical incoherent light.

Very recently, Wen *et al*[30] presented a theoretical analysis of the third-order ghost imaging and interference by a three-photon entangled state. Agafonov *et al*[31] discussed, both experimentally and theoretically, the classical visibility limit for three- and

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four-photon interference and showed that both third- and fourth-order interference patterns from classical source have a much higher visibility than their second-order analogous. Ou *et al*[32] and Bai *et al*[33] studied third-order ghost imaging with thermal light. In this paper, we study a third-order ghost interference scheme using a thermal light source. Unlike the entangled-photon case where higher-order correlation effect requires preparation of a multi-photon entangled state, which is somewhat difficult, thermal light possesses an obvious advantage that it has been provided with higher-order field correlation. We show that the third-order field correlation is able to improve the visibility of ghost interference. We analyse how the visibility and resolution of the third-order ghost interference fringes are related to the bandwidth of the spatial frequency spectrum of thermal light.

2. Theoretical analysis

The scheme for the third-order ghost interference with thermal light is shown in Fig.1. The thermal light is split into two beams by a beam splitter (BS1). The reflected beam propagates through path 1 to detector D_1. The transmitted light is split by a second beam splitter (BS2), and then the two outgoing beams propagate through paths 2 and 3 to detectors D_2 and D_3, respectively. The beams in paths 1 and 2 propagate freely. In path 3, however, there is a double-slit with slit distance d and slit width b between BS2 and detector D_3. The coincidence counting rate of the three detectors is proportional to the third-order correlation function $G^{(3)}(x_1, x_2, x_3)$. We consider thermal light described by $E(x,z,t) = \int E(q)\exp[\mathrm{i}(qx)]dq\exp[\mathrm{i}(kz-\omega t)]$, where $E(q)$ is a stochastic variable obeying Gaussian statistics, $k = 2\pi/\lambda$ is the wave number, λ is the wavelength of light, and q is the transverse wave vector satisfying $q \ll k$. For simplicity, here we consider one-dimensional transverse wave vector.

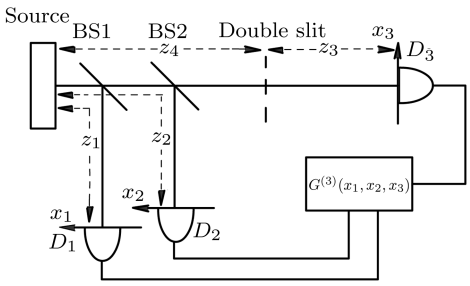


Fig.1. The schematic setup for third-order ghost interference with thermal light.

According to the Wiener–Khintchine theorem, the first-order field correlation of the source above satisfies

$$\langle E^*(q)E(q')\rangle = S(q)\delta(q-q'), \tag{1}$$

where $S(q)$ is the power spectrum of the spatial frequency. For any random variable with Gaussian statistics, all high-order correlations can be expressed in terms of the first-order ones.[34] In particular, the third-order field correlation of thermal light can be written as

$$\begin{aligned}&\langle E^*(q_1)E^*(q_2)E^*(q_3)E(q_3')E(q_2')E(q_1')\rangle\\ =\ & S(q_1)S(q_2)S(q_3)\{\delta(q_1-q_1')[\delta(q_2-q_2')\delta(q_3-q_3')+\delta(q_2-q_3')\delta(q_3-q_2')]+\delta(q_1-q_2')[\delta(q_2-q_1')\\ &\times\delta(q_3-q_3')+\delta(q_2-q_3')\delta(q_3-q_1')]+\delta(q_1-q_3')\times[\delta(q_2-q_1')\delta(q_3-q_2')+\delta(q_2-q_2')\delta(q_3-q_1')]\},\end{aligned} \tag{2}$$

where Eq.(1) has been used. The third-order field correlation can be acquired by setting the two beam splitters (BS1 and BS2) to divide the input beam as shown in Fig.1. We denote $F(q_m)$ $(m = 1, 2, 3)$ as the three output fields of the two beam splitters. For the classical thermal light source, the third-order correlation of the output fields $\langle F^*(q_1)F^*(q_2)F^*(q_3)F(q_3')F(q_2')F(q_1')\rangle$ can be expressed by the correlation of the input field given by Eq.(2). Let $h_m(x,x')$ $(m = 1, 2, 3)$ be the impulse response function for path m, then the third-order correlation function at the detection planes is given by

$$\begin{aligned}G^{(3)}(x_1,x_2,x_3) =\ & \int h_1^*(x_1,-q_1)h_1(x_1,-q_1')h_2^*(x_2,-q_2)h_2(x_2,-q_2')h_3^*(x_3,-q_3)h_3(x_3,-q_3')\\ &\times\langle E^*(q_1)E^*(q_2)E^*(q_3)E(q_3')E(q_2')E(q_1')\rangle\,\mathrm{d}q_1\mathrm{d}q_1'\mathrm{d}q_2\mathrm{d}q_2'\mathrm{d}q_3\mathrm{d}q_3',\end{aligned} \tag{3}$$

where $h_m(x,q) = (1/\sqrt{2\pi})\int h_m(x,x')\exp(-\mathrm{i}qx')\mathrm{d}x'$. Substituting Eq.(2) into Eq.(3), we arrive at

$$G^{(3)}(x_1,x_2,x_3) = G^{(1)}(x_1,x_1)G^{(1)}(x_2,x_2)G^{(1)}(x_3,x_3) + G^{(1)}(x_1,x_1)\left|G^{(1)}(x_2,x_3)\right|^2$$

$$+G^{(1)}(x_2,x_2)\left|G^{(1)}(x_1,x_3)\right|^2+G^{(1)}(x_3,x_3)\left|G^{(1)}(x_1,x_2)\right|^2$$
$$+\left[G^{(1)}(x_1,x_2)G^{(1)}(x_2,x_3)G^{(1)}(x_3,x_1)+\text{c.c.}\right], \quad (4)$$

where

$$G^{(1)}(x_m,x_n)=\int S(q)h_m^*(x_m,-q)h_n(x_n,-q)\mathrm{d}q,\ m=1,2,3;n=1,2,3. \quad (5)$$

The first term in Eq.(4) is an uncorrelated intensity product of three positions, contributing a background. The other terms exhibit the field correlations between different transverse positions which may contain coherent information. For example, in our scheme of Fig.1, the second, third, and last terms involve the interference information of double-slit, but the fourth term does not.

For simplicity, we assume that the two beam splitters are close to the source. The impulse response functions of paths 1 and 2 are written as[11]

$$h_n(x_n,q)=\frac{1}{\sqrt{2\pi}}\exp\left[\mathrm{i}(kz_n-qx_n-\frac{q^2z_n}{2k})\right],\quad n=1,2. \quad (6)$$

where z_1 and z_2 are the distances from the source to detectors D_1 and D_2, respectively, and the impulse response function of path 3 can be expressed as

$$h_3(x_3,q)=\sqrt{\frac{k}{2\pi z_3}}\widetilde{T}\left(q+\frac{kx_3}{z_3}\right)\exp\left[-\mathrm{i}\frac{\pi}{4}+\mathrm{i}k\left(z_3+z_4+\frac{x_3^2}{2z_3}\right)-\mathrm{i}\frac{q^2z_4}{2k}\right)\Bigg], \quad (7)$$

where

$$\widetilde{T}(q)=\frac{1}{\sqrt{2\pi}}\int T(u)\exp(-\mathrm{i}qu)\mathrm{d}u \quad (8)$$

is the Fourier transform of the double-slit transmission function $T(u)$ [$T(u)=1$ within slit width and 0 otherwise]; z_3 and z_4 are the distances from the double-slit to detector D_3 and the source, respectively.

To obtain the analytical form, we first consider the broadband limit, where spectrum $S(q)$ can be regarded as a constant in the integrations. We set detector D_3 at $x_3=0$ and $z_1=z_2=z'$. Under these conditions, we substitute Eqs.(5)–(7) into Eq.(4) and obtain the third-order correlation function

$$G^{(3)}(x_1,x_2,0)\propto\widetilde{T}^2\left(\frac{kx_1}{z'-z_4}\right)+\widetilde{T}^2\left(\frac{kx_2}{z'-z_4}\right), \quad (9)$$

where $\widetilde{T}[kx_1/(z'-z_4)]$ and $\widetilde{T}[kx_2/(z'-z_4)]$ display two double-slit interference fringes by independently scanning transverse positions in paths 1 and 2, respectively; whereas, in the second-order ghost interference scheme, only one fringe is available.[17]

3. Numerical results

In the general case, we assume that the thermal light has a Gaussian type spectrum $S(q)=(\sqrt{2\pi}w)^{-1}\exp\left[-q^2/\left(2w^2\right)\right]$ with the spectral bandwidth w. To evaluate the third-order correlation function given by Eq.(4), we calculate the following integrations using Eqs.(5)–(7)

$$G^{(1)}(x_n,x_n)=\frac{1}{2\pi},\quad n=1,2, \quad (10)$$

$$G^{(1)}(x_3,x_3)=\frac{k}{4\pi^2z_3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}T^*(u)T(u')\exp\left[\mathrm{i}\frac{kx_3}{z_3}(u-u')-\frac{w^2}{2}(u-u')^2\right]\mathrm{d}u\mathrm{d}u', \quad (11)$$

$$G^{(1)}(x_1,x_2)=\frac{1}{2\pi}\sqrt{\frac{k}{k-\mathrm{i}w^2(z_1-z_2)}}\exp\left[\mathrm{i}k(z_2-z_1)-\frac{(x_1-x_2)^2}{2/w^2-2\mathrm{i}(z_1-z_2)/k}\right], \quad (12)$$

$$G^{(1)}(x_n,x_3)=\frac{1}{2\pi}\sqrt{\frac{k}{2\pi z_3}}\sqrt{\frac{k}{k-\mathrm{i}w^2(z_n-z_4)}}\exp\left[-\mathrm{i}\frac{\pi}{4}+\mathrm{i}k(z_3+z_4-z_n)\right]$$
$$\times\int_{-\infty}^{\infty}T(u)\exp\left[-\frac{(x_n-u)^2}{2/w^2-2\mathrm{i}(z_n-z_4)/k}-\mathrm{i}\frac{kx_3u}{z_3}\right]\mathrm{d}u,\ n=1,2. \quad (13)$$

In order to compare the ghost interference schemes between the second-order and third-order field correlation, we assume that detectors 1 and 2 are scanned along $x_1 = x_2 = x$ while detector 3 is fixed at $x_3 = 0$. Again, $z_1 = z_2 = z'$ is set. This is equivalent to the case where the third-order correlation is recorded by only two detectors: detector 1 (or 2) is redundant and can be taken away. Substituting Eqs.(10)–(13) into Eq.(4), we obtain

$$G^{(3)}(x,x,0) = \frac{k}{8\pi^4 z_3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} T^*(u)T(u')\exp\left[-\frac{w^2}{2}(u-u')^2\right]\mathrm{d}u\mathrm{d}u' + \frac{k}{4\pi^4 z_3}\sqrt{\frac{k^2}{k^2+(z'-z_4)^2w^4}}\times\left|\int_{-\infty}^{\infty} T(u)\exp\left[-\frac{(x-u)^2}{2/w^2-2\mathrm{i}(z'-z_4)/k}\right]\mathrm{d}u\right|^2, \quad (14)$$

Using Eq.(14), we plot in Fig.2 the normalized third-order interference fringes for different normalized bandwidths $W = wb$ of the thermal light source. The parameters for Fig.2 and the following Fig.3 are $\lambda = 632.8\,\mathrm{nm}$, $b = 10\,\mu\mathrm{m}$, $d = 30\,\mu\mathrm{m}$, $z_1 = 50\,\mathrm{mm}$, $z_2 = 50\,\mathrm{mm}$, $z_3 = 40\,\mathrm{mm}$, and $z_4 = 10\,\mathrm{mm}$. From Fig.2, we can see that as the bandwidth increases, the resolution of the fringes increases while the visibility decreases. In other words, better resolution is accompanied by lower visibility. This is similar to the second-order ghost interference.[17] Since the slit-width b characterizes the resolution of the double-slit, the interference fringe emerges when the bandwidth of the thermal light source has the order of $w = 1/b$, or $W = 1$.

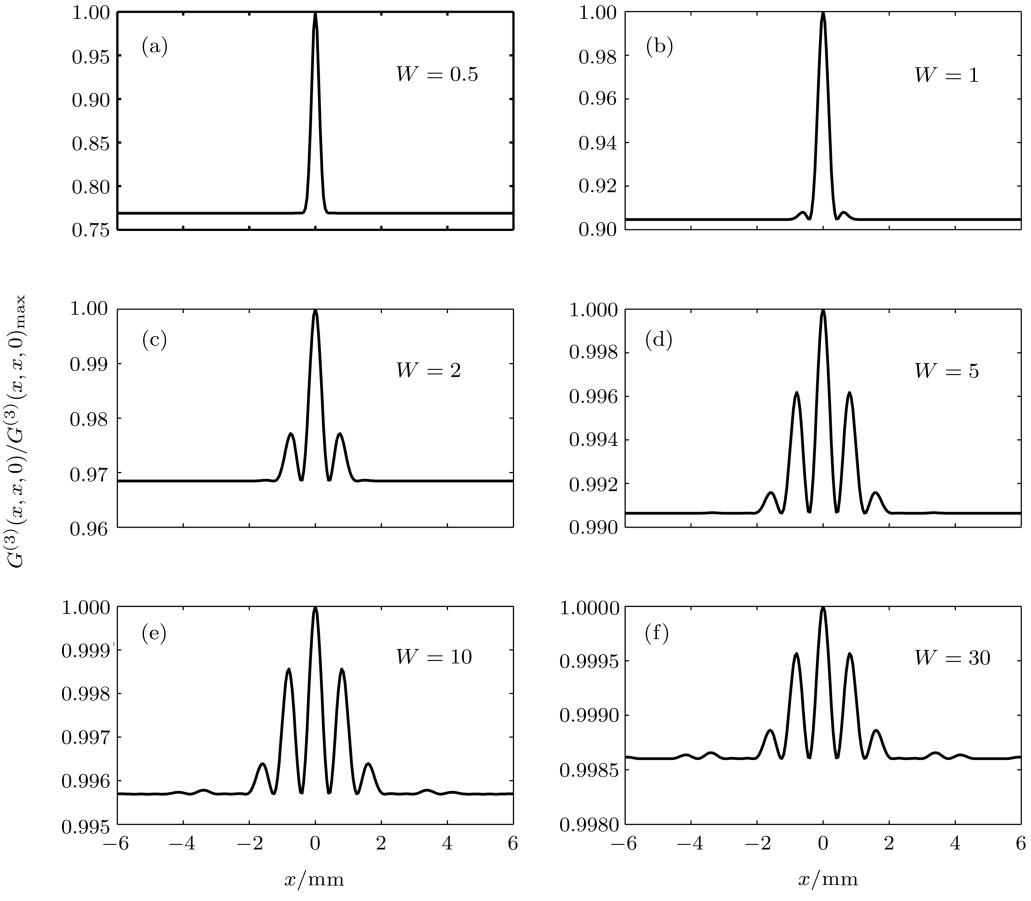


Fig.2. Third-order ghost interference fringes for different normalized bandwidths $W = wb$ of the spatial-frequency spectrum of thermal light.

For the sake of comparison, we calculate the second-order correlation function between paths 1 and 3 under the same conditions

$$G^{(2)}(x,0) = \frac{k}{8\pi^3 z_3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} T^*(u)T(u')\exp\left[-\frac{w^2}{2}(u-u')^2\right]\mathrm{d}u\mathrm{d}u' + \frac{k}{8\pi^3 z_3}\sqrt{\frac{k^2}{k^2+(z_1-z_4)^2w^4}}\times\left|\int_{-\infty}^{\infty} T(u)\exp\left[-\frac{(x-u)^2}{2/w^2-2\mathrm{i}(z_1-z_4)/k}\right]\mathrm{d}u\right|^2. \quad (15)$$

Comparison between Eqs.(14) and (15) shows that the expressions are nearly the same apart from an overall $1/\pi$ factor and another factor of 2 in the interference term (the second term in the expressions). This is because all high-order correlations in Gaussian statistics are expressed in terms of the first-order ones and the conditions $x_1 = x_2$ and $z_1 = z_2$ are applied. Hence the two identical interference terms bring about a factor 2.

We define the visibilities of the second- and third-order interference fringes as

$$V_1 = \frac{G^{(2)}(x,0)_{\max} - G^{(2)}(x,0)_{\min}}{G^{(2)}(x,0)_{\max} + G^{(2)}(x,0)_{\min}}, \quad (16)$$

$$V_2 = \frac{G^{(3)}(x,x,0)_{\max} - G^{(3)}(x,x,0)_{\min}}{G^{(3)}(x,x,0)_{\max} + G^{(3)}(x,x,0)_{\min}} \quad (17)$$

respectively, where $G^{(3)}(x,x,0)_{\max}$ and $G^{(2)}(x,0)_{\max}$ are defined as the highest peaks of the fringe while $G^{(3)}(x,x,0)_{\min}$ and $G^{(2)}(x,0)_{\min}$ are the values of background . We plot the visibilities of the second- and third-order interference fringes as V_1 and V_2, respectively, as functions of the normalized bandwidth $W = wb$ in Fig.3. We can see in Fig.2 that the fringe is visible from $W = 1$, so that we set the abscissa of Fig.3 starting from $W = 1$. As the bandwidth increases, both visibilities V_1 and V_2 decrease. Taking into account the fact that the background is very high as compared to the interference term (see Fig.2, where the background is higher than 90% of the total when the fringe is visible), one immediately obtains from Eqs.(14)–(17) that the visibility of the third-order interference fringes is nearly twice as high as the second-order ones, as also shown by the numerical plots in Fig.3.

The increase of visibility for higher-order correlation can be understood in the general way. According to Schwarz inequality

$$\left|G^{(1)}(x_r,x_s)\right|^2 \le G^{(1)}(x_r,x_r)G^{(1)}(x_s,x_s) \quad (r,s=1,2,3), \quad (18)$$

the maximum value of the interference term is equal to the corresponding intensity product, i.e. the background. In the second-order correlation case, the minimum is, of course, the background while the maximum is twice the background. As a result, the maximum visibility of the second-order interference for thermal light is 33.3%. This happens when the thermal light beam as a whole illuminates the double-slit.[16] Similarly, in the third-order correlation function shown in Eq.(4), there are five interference terms versus a background. Therefore, the visibility in the third-order correlation is limited by 5/7. To reach this limit, all the photons involving in the third-order correlation must pass through the double-slit, such as the scheme indicated in Ref.[31]. However, in the ghost interference scheme when one of the daughter beams travels freely, the intensity background increases greatly. Thus the visibility is far from its maximum value. The enhancement of visibility in higher-order correlation is mainly due to the contribution of multi-interference terms which are set to be equal to one another.

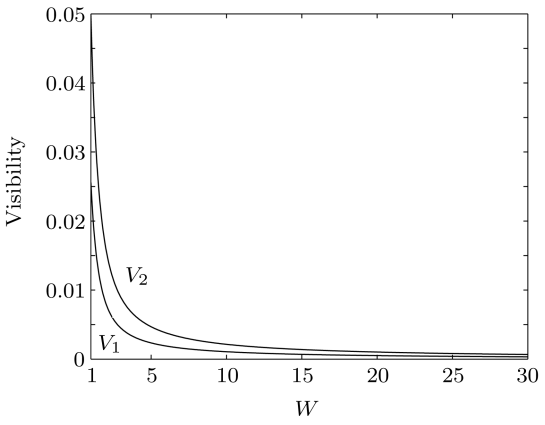


Fig.3. Visibilities V_1 and V_2, corresponding to the second- and third-order interference fringes, respectively, as functions of the normalized bandwidth $W = wb$.

4. Conclusion

In conclusion, we have discussed theoretically the third-order ghost interference scheme with thermal

light. Our analysis has shown that the interference fringes are strongly dependent on the bandwidth of the spatial spectrum of the thermal light source. When the bandwidth is small, we have an incomplete interference pattern with high visibility. As the bandwidth increases, we can obtain a more complete interference pattern with better resolution, but worse visibility. However, we have demonstrated that the visibility of the third-order ghost interference is much higher than that of the second-order one.

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