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Towards the understanding of 1^1P_1 meson mass spectrum

Feng Xue-Chao(冯学超)[a)†], Jiang Feng-Chun(蒋逢春)[a)],
Chang Tong-Qin(常同钦)[a)], and Feng Jun-Lan(冯军兰)[b)]

[a)] *Department of Technology and Physics, Zhengzhou University of Light Industry, Zhengzhou* 450002, *China*

[b)] *College of Science, Zhongyuan University of Technology, Zhengzhou* 450007, *China*

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Based on the meson–meson mixing and Regge trajectory, this paper establishes the mass relations which can describe the mass spectrum of 1^1P_1 meson state. Using these mass relations, it obtains the mass of K_{1B}, $h_1(1380)$ and $h_c(1P)$ to be 1358.5 MeV, 1468 MeV and 3543.9 MeV, respectively. The results are compared with other theoretical results and should be tested by experiments in the future.

Keywords: Regge trajectory, meson–meson mixing, mass spectrum
PACC: 1440, 3520X

1. Introduction

Quantum chromodynamics (QCD) is accepted as a successful theory of the strong interactions in particle physics. However, the understanding of the strong interactions is far from completeness [1] and it is difficult to interpret the particle properties for the experimental data from the first principles. Therefore, people build different models e.g. quenched lattice gauge theory,[2] the Dyson–Schwinger formalism,[3] constituent quark model,[4] to explain the properties of experimental data. In these models, the constituent quark model has been used to study the low-lying hadron spectrum and is probably the most successful phenomenological model of hadron structure. In the last decade, the study of the meson mass spectrum has attracted much attention, because it provides a clue to understand the theory of QCD. In this work, we first review the assignment of 1^1P_1 meson state in the $q\bar{q}$ quark model (Table 1).

Table 1. Assignment of the 1^1P_1 meson state in PDG[5] ($n(= u, d)$, s, c denote the constituent quark).

$^{2s+1}l_j$	$n\bar{n}$	$n\bar{n}(s\bar{s})$	$n\bar{s}$	$c\bar{c}$	$c\bar{n}$	$c\bar{s}$
1^1P_1	$b_1(1235)$	$h_1(1170)$ $h_1(1380)$	K_{1B}	$h_c(1P)$	$D_1(2420)$	$D_{s1}(2536)$

According to the new edition of Particle Data Group (PDG),[5] the $b_1(1235)$, $h_1(1170)$, $D_1(2420)$, $D_{s1}(2536)$ have been established in the experiment in the 1^1P_1 meson state. However, the $h_1(1380)$, K_{1B} and $h_c(1P)$ are problematic and remain interesting. Due to the kaons having no definite C-Parity, the spin-singlet and spin-triplet can mix via the spin-orbit interaction to form the physical state. Therefore, PDG suggests that the isodoublet of the 1^1P_1 and 1^3P_1 are nearly equal (45°) mix of the $K_1(1270)$ and $K_1(1400)$. In order to compare the theoretical prediction with experimental data, the mixing angle of K_{1A} and K_{1B} has been estimated by many approaches.[6−10]

The $h_1(1170)$ is identified as the isoscalar $(u\bar{u} + d\bar{d})$ member in the 1^1P_1 meson state. However, the partner $h_1(1380)$ is problematic which has been reported by only two experiments, LASS[11] (with mass 1380 ± 20 MeV and width 80 ± 30 MeV) and Crystal Barrel[12] (with mass 1440 ± 60 MeV and width 170 ± 80 MeV). According to the mass and dominant decay to KK^*, Barnes *et al* suggest that $h_1(1380)$ is the $s\bar{s}$ partner of $h_1(1170)$.[13] On the other hand, the mass of h_1 (1380) from other phenomenological models[14,15] is larger than 1386 ± 19 MeV, which im-

†Corresponding author. E-mail: fxchao@zzuli.edu.cn

plies that the assignment of $h_1(1380)$ needs to be further examined experimentally.

The spectrum of charmonium states is one of the most exciting areas of high energy physics, since it provides fruitful information to understand the interaction between heavy quark–antiquark. In the 1^1P_1 meson state, the $h_c(1P)$ is still based on very weak experimental signals. The $h_c(1P)$ has been observed in the decay $\psi(2s) \to \pi^0 h_c$ with $h_c \to \gamma\eta_c$ by CLEO.[16] It has also been seen by Fermilab Experiment E835 in the reaction $p\bar{p} \to h_c \to \gamma\eta_c \to \gamma\gamma\gamma$.[17]

In this work, basing on the fact that the isoscalar states $h_1(1170)$ and $h_1(1380)$ can mix to form the physical states in the $q\bar{q}$ quark model and the linear Regge trajectory, we establish the new mass relations which relate the mass spectrum of meson state with constituent quark masses. Inserting the corresponding constituent quark masses, we investigate the mass spectrum of the 1^1P_1 meson nonet.

2. Meson-meson mass mixing matrix of isoscalar states and Regge phenomenology

In the quark model, the two isoscalar states with the same J^{PC} will mix to form the physical isoscalar states. We can establish the mass-squared matrix in the $s\bar{s}$ and $N = (u\bar{u} + d\bar{d})/\sqrt{2}$ basis:[18]

$$M^2 = \begin{pmatrix} M^2_{b_1(1235)} + 2A_{nn} & \sqrt{2}A_{ns} \\ \sqrt{2}A_{ns} & 2M^2_{K_{1B}} - M^2_{b_1(1235)} + A_{ss} \end{pmatrix}, \tag{1}$$

where $M_{b_1(1235)}$ and $M_{K_{1B}}$ are the masses of isovector and isodoublet states of the 1^1P_1 meson nonet, respectively; A_{nn}, A_{ns} and A_{ss} are the mixing parameters which describe the $q\bar{q} \leftrightarrow q'\bar{q}'$ transition amplitudes. In order to reduce the number of parameters, we adopt the similar expression of the transition amplitudes in the $q\bar{q} \leftrightarrow q'\bar{q}'$ process which is widely used in literatures.[18−20]

$$\begin{aligned} A_{nn} &= \frac{\Lambda}{m_n m_n}, \\ A_{ss} &= \frac{\Lambda}{m_s m_s}, \\ A_{ns} &= \frac{\Lambda}{m_n m_s}, \end{aligned} \tag{2}$$

where Λ is a phenomenological parameter. Based on the isospin symmetry, we have $m_u = m_{\bar{u}} = m_d = m_{\bar{u}}$, $m_s = m_{\bar{s}}$ (m_n, m_s denote the masses of light quark u,d and s respectively).

In the 1^1P_1 meson nonet, we assume that the physical states $h_1(1170)$ and $h_1(1380)$ are the eigenstates of mass-squared matrix and the masses square of $M^2_{h_1(1170)}$ and $M^2_{h_1(1380)}$ are the eigenvalues, respectively. The physical states $h_1(1170)$ and $h_1(1380)$ can be related to the $s\bar{s}$ and $N = (u\bar{u} + d\bar{d})/\sqrt{2}$ by

$$\begin{pmatrix} |h_1(1170)\rangle \\ |h_1(1380)\rangle \end{pmatrix} = U \begin{pmatrix} |N\rangle \\ |S\rangle \end{pmatrix}, \tag{3}$$

and the unitary matrix U can be described by

$$UM^2U^\dagger = \begin{pmatrix} M^2_{h_1(1170)} & 0 \\ 0 & M^2_{h_1(1380)} \end{pmatrix}. \tag{4}$$

According to the relations (1), (2), (3) and (4), we will obtain:

$$\begin{aligned} &2\frac{\Lambda}{m_n^2} + 2M^2_{K_{1B}} + \frac{\Lambda}{m_s^2} \\ &= M^2_{h_1(1170)} + M^2_{h_1(1380)}, \end{aligned} \tag{5}$$

$$\begin{aligned} &\left(M^2_{b_1(1235)} + 2\frac{\Lambda}{m_n^2}\right)\left(2M^2_{K_{1B}} - M^2_{b_1(1235)} + \frac{\Lambda}{m_s^2}\right) \\ &-2\frac{\Lambda}{m_s^2 m_n^2} = M^2_{h_1(1170)} * M^2_{h_1(1380)}. \end{aligned} \tag{6}$$

In the relations (5) and (6), the masses of K_{1B} and $h_1(1380)$ are related with constituent quarks mass and phenomenological parameter Λ.

Regge theory is cornered with the particle spectrum, the forces between particles, and the high energy behaviour of scattering amplitudes. Hence Regge theory has become an important theory for the particle phenomenology.[21] Due to the fact that Regge trajectories can offer an effective way for the assignment and classification of meson states, it also becomes an active field with many new particles and resonances, which were observed in the experiment in the last decade. In the Ref.[22], the authors investigated the mass of different meson multiplets, and suggest that the quasi-linear Regge trajectories could describe the meson mass spectrum. Using the phenomenological formulae, Khruschov[23] deprive from the Regge trajectories, predict the masses of excited meson states. Anisovich *et al*[24] show that meson states can fit to the quasi-linear Regge trajectories with good accuracy.

According to the hadron with a set of given quantum number belonging to a quasi-linear trajectory, we will have the following relation:[22]

$$J = \alpha_{i\bar{i'}}(0) + \alpha'_{i\bar{i'}} M^2_{i\bar{i'}}, \tag{7}$$

where $i\bar{i'}$ refers to the quark (antiquark) flavour, J and $M_{i\bar{i'}}$ are respectively the spin and mass of the $i\bar{i'}$ meson. The parameters $\alpha'_{i\bar{i'}}$ and $\alpha_{i\bar{i'}}(0)$ are respectively the slope and intercept of the trajectory. The intercepts and slopes can be parametrized by[22,25]

$$\alpha_{i\bar{i}}(0) + \alpha_{j\bar{j}}(0) = 2\alpha_{i\bar{j}}(0), \tag{8}$$

$$\frac{1}{\alpha'_{i\bar{i'}}} + \frac{1}{\alpha'_{j\bar{j'}}} = \frac{2}{\alpha'_{i\bar{j'}}}. \tag{9}$$

The relation (8) is satisfied in two-dimensional QCD,[26] the dual-analytic model,[27] and the quark bremsstrahlung model.[28] The relation (9) is deprived from the topological and the $q\bar{q}$-string picture of hadrons.[29] According to the available data of meson states, Burakovsky construct a slope formula (10) for all quarks flavours:[30]

$$\frac{\pi}{4}\alpha'_{ji} + \frac{\pi}{4}\sqrt{\alpha'}\frac{m_i + m_j}{2}\alpha'_{ji} = \alpha', \tag{10}$$

where m_i, m_j are the corresponding constituent quark masses, and the $\alpha' = 0.88\,\mathrm{GeV}^{-2}$ is the standard Regge slope in the light quark sector.

From the relations (7)–(10), we obtained the following relations:

$$\frac{m^2_{b_1(1235)}\alpha'}{2 + 2\sqrt{\alpha'}m_n} + \frac{m^2_{h_c(1P)}\alpha'}{2 + 2\sqrt{\alpha'}m_c} = \frac{2m^2_{D_1(2420)}\alpha'}{2 + \sqrt{\alpha'}(m_n + m_c)}, \tag{11}$$

$$\frac{m^2_{h_1(1380)}\alpha'}{2 + 2\sqrt{\alpha'}m_s} + \frac{m^2_{h_c(1P)}\alpha'}{2 + 2\sqrt{\alpha'}m_c} = \frac{2m^2_{D_{s1}(2536)}\alpha'}{2 + \sqrt{\alpha'}(m_s + m_c)}. \tag{12}$$

3. Results and conclusions

Using the relations (5), (6), (11) and (12), we can obtain the masses of K_{1B}, $h_1(1380)$ and $h_c(1P)$ in the 1^1P_1 meson state. In this paper, we use the average values of the constituent quark mass as input parameters (See Table 2). The mass of $b_1(1235)$, $h_1(1170)$, $D_1(2420)$, $D_{s1}(2536)$ used are taken from the new edition of PDG.[5] The results are presented in Table 3.

Table 2. Constituent quark mass(MeV) in different phenomenological models.

Mass	Ref.[31]	Ref.[32]	Ref.[33]	Ref.[25]	Ref.[34]	average
$m_n(n = u, d)$	360	337.5	311	290	310	321
m_s	540	486	487	460	483	491.2
m_c	1710	1550	1592	1650		1625.5

Table 3. Mass (MeV) of K_{1B}, $h_1(1380)$ and $h_c(1P)$ in the 1^1P_1 meson state.

Mass	Our results	Ref.[5]	Ref.[14]	Ref.[15]	Ref.[35]	Ref.[36]	Ref.[37]	Ref.[38]
$M_{h_1(1380)}$	1468	1386	1495.18	1470				
$M_{K_{(1B)}}$	1358.5		1370.03	1340	1368	1356		
$M_{h_c(1P)}$	3543.9	3525.93		3520			3526	3529

In summary, on the basis of the meson–meson mass mixing matrix and liner Regge trajectory, we predict the mass of K_{1B}, $h_1(1380)$ and $h_c(1P)$ in the 1^1P_1 meson state. We determine the mass of $h_1(1380)$ to be (1468 MeV), the prediction is larger than 1386 ± 19 MeV in the PDG. The result is consistent with the value (1470 MeV) predicted by Godfrey and Isgur.[15] Moreover, since the assignment of $h_c(1P)$ is still based on the very weak experimental signals, we suggest that this state should be reexamined in the experiment in the future. In this work, we also predict the mass of the K_{1B}, the prediction should be useful to determine the mixing angle between the $K_1(1270)$ and $K_1(1400)$. The results should be tested by new experiment in the future.

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