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Statistical-mechanical analysis of multiuser channel capacity with imperfect channel state information*

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In this paper, the effect of imperfect channel state information at the receiver, which is caused by noise and other interference, on the multi-access channel capacity is analysed through a statistical-mechanical approach. Replica analyses focus on analytically studying how the minimum mean square error (MMSE) channel estimation error appears in a multiuser channel capacity formula. And the relevant mathematical expressions are derived. At the same time, numerical simulation results are demonstrated to validate the Replica analyses. The simulation results show how the system parameters, such as channel estimation error, system load and signal-to-noise ratio, affect the channel capacity.

Keywords: statistical mechanics, channel capacity, minimum mean square error channel estimation, code division multiple access (CDMA)

PACC: 0550, 7510H

1. Introduction

Statistical mechanics, a main branch of physics, has been extensively studied. However, these studies mainly focus on the area of physics. Nowadays, it is found that statistical mechanics can be extended to the fields of communications and information theory, such as the analysis of error correcting codes,[1−7] and to the areas of code division multiple access (CDMA) communication,[8,9] and multi-input multi-output (MIMO) techniques.[10] Thus, extensively applying statistical mechanical approach to communication theory is a great piece of news to theoretical physicists as well as the scientists in the area of information science.

Large system analyses of various wireless communications, including multiple access systems, have attracted significant attention in recent years.[8−12] The analysis of multiuser detections with the large system limit (here, 'large system limit' means that the number of users and the spreading factor are infinite) can be found in Refs.[8–10].

Tanaka[11] presented an approximate analysis of direct sequence code division multiple access (DS-CDMA) with optimum detection which is asymptotic when the CDMA system is within the large system limit. And in the analysis in Ref.[11] it was assumed that the power of each user in the system was perfectly controlled. Later on, Guo and verdü [12] generalized the results in Ref.[11] to the case of each user with different powers. Both of the calculations in Refs.[11,12] were based on the replica method. The replica method has been developed and now become a standard tool to analyse disordered systems in statistical physics.[13−20]

However, Tanaka [11] restricted the analysis to the additive white Gaussian noise channel on the assumption that the channel state information was perfectly known at the receiver, which is impractical in actual wireless communication systems. Therefore, it is of interest to analyse large-system CDMA systems with imperfect channel estimates due to the existence of noise and other interference.

In this paper, we focus on analysing the capacity of a multiuser frequency-selective fading channel, where the effect of imperfect channel state information on the capacity is also taken into account.

The paper is organized as follows. In Section 2 the general system model is described and a channel estimation error is defined. In Section 3 the interpretation of the relation between multiuser communication and statistical physics is presented. In Section 4 analytical approaches to the capacity of multiuser frequency-selective fading channel with imperfect channel esti-

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mation are reviewed and a general solution based on the replica method is given. And numerical results are shown in Section 5. Finally, conclusions and outlooks are shown in Section 6.

2. General transmission model

The communication model for a DS-CDMA system is written as[11,21]

$$\boldsymbol{r} = \frac{1}{\sqrt{N}}\overline{S}\boldsymbol{x} + n, \tag{1}$$

where $\overline{S} = (\overline{s}_1, \overline{s}_2, \ldots, \overline{s}_K)$ is an $(N + L - 1) \times K$ matrix; the k-th column is the authentic spreading sequence or practical spreading sequence of user k (we will define authentic spreading sequence and practical spreading sequence in the following paragraph 2.1); $\boldsymbol{x} = (x_1, x_2, \ldots, x_K)^{\mathrm{T}}$ and $\boldsymbol{r} = (r_1, r_2, \ldots, r_K)^{\mathrm{T}}$ are the transmitted bit sequence and the received bit sequences respectively; $n = (n_1, n_2, \ldots, n_K)^{\mathrm{T}}$ is an additive white Gaussian noise sequence with a mean value of zero and a variance of σ_0^2. Moreover, $\overline{S}, \boldsymbol{x}, n$ are mutually independent.

2.1. Authentic spreading sequence and practical spreading sequence

A frequency-selective fading channel can be modelled as a discrete time domain transected filter,[21] whose taps are described as a vector of length $\boldsymbol{L}$, i.e.

$$\boldsymbol{h} \equiv (h_0, h_1, \ldots, h_{L-1})^{\mathrm{T}}.$$

In a CDMA system, the spreading sequence of user k is defined as

$$\boldsymbol{s}_k \equiv (s_k^1, \ldots, s_k^{\mathrm{N}})^{\mathrm{T}},$$

and the matrix of signature sequence is defined as

$$\boldsymbol{S} \equiv (s_1, \ldots, s_K).$$

To demonstrate the multi-path propagation of fading channel, we define the authentic spreading sequence as the convolution of spreading sequence and the channel coefficient

$$\breve{s}_k[n] \triangleq s_k[n] * h_k[n] = \sum_{l=0}^{L-1} s_k[n-l]h_k[l],$$
$$n \in \{0, 1, \ldots, N + L - 2\}. \tag{2}$$

Authentic spreading sequence underlines the idealization of perfect channel state information that is known at the receiver. However, this circumstance is always impractical because the channel is usually estimated and a channel estimation error is present. When the channel estimation is applied at the receiver, the practical spreading sequence is defined as

$$\tilde{s}_k[n] \triangleq s_k[n] * \tilde{h}_k[n] = \sum_{l=0}^{L-1} s_k[n-l]\tilde{h}_k[l],$$
$$n \in \{0, 1, \ldots, N + L - 2\}, \tag{3}$$

where $\tilde{h}_k[n]$ denotes the estimation of channel coefficient.

2.2. MMSE channel estimation

We assume that the channel fading coefficients $\tilde{h}_k[l]$ are attained by applying a train of sequences in the transmitted signals and estimating at the receiver via MMSE channel estimation algorithm. The channel estimation error is given by

$$\varepsilon_k[l] = h_k[l] - \tilde{h}_k[l], \quad k = 1, 2, \ldots, K, \tag{4}$$

where $\varepsilon_k[l]$ is a zero-mean complex Gaussian random variable whose variance is denoted as $V_{\mathrm{ar}}\{\varepsilon_k[l]\}$. According to expressions (2)–(4), the error of practical spreading sequence is given as

$$\begin{aligned} \Gamma_k[l] &= \breve{S}_k[l] - \tilde{S}_k[l] \\ &= \sum_{l=0}^{L-1} s_k[n-l]\varepsilon_k[l]. \end{aligned} \tag{5}$$

From expression (5) it follows that $\Gamma_k[l]$ is also a zero-mean complex Gaussian random variable and its variance is denoted as σ_S^2.

3. The relation between multiuser communication and spin glass

In order to take advantage of the concepts and methods of statistical physics, an artificial thermodynamic system must be constructed. Here, we must emphasize that the artificial thermodynamic system is not an existing physical system and it has no temperature. For convenience, we assume the temperature of the system T to be unity, i.e. $T = 1$.

For the sake of creating the artificial thermodynamic system, we begin our discussion with the posterior distribution $P(x \,|r, \bar{S})$, i.e. the distribution of x conditional on received data r given $\bar{S}$, which is the basic quantity for analysing the multiuser detection.

The posterior distribution $P(x\,|r,\bar{S})$ is given by expression (6) in Ref.[11] and we quote it as follows:

$$P(x\,|r,\bar{S}) = \left[Z(r,\bar{S})\right]^{-1} p(x) \times \exp\left(-\frac{1}{2\sigma^2}\left\|r - N^{-1/2}\bar{S}x\right\|^2\right), \quad (6)$$

where

$$Z(r,\bar{S}) = \sum_x p(x)\exp\left(-\frac{1}{2\sigma^2}\left\|r - N^{-1/2}\bar{S}x\right\|^2\right). \quad (7)$$

Let

$$H(x) = \frac{1}{2\sigma^2}\left\|r - N^{-1/2}\bar{S}x\right\|^2 - \log p(x) \quad (8)$$

then expression (6) will be rewritten as

$$P(x\,|r,\bar{S}) = \left[Z(r,\bar{S})\right]^{-1}\exp\left[-H(x)\right]. \quad (9)$$

An interesting observation is that the term on right-hand side of expression (9) is the same as Boltzmann's law. Thus, the artificial thermodynamic system is obtained, in which Hamiltonian $H(x)$ and partition function $Z(r,\bar{S})$ can be expressed by expressions (8) and (7).

4. Statistical-mechanical analysis

In the above section, the relation between multiuser communication and statistical physics is analysed and an artificial thermodynamic system is created. In this section, we intend to introduce the replica method to compute the free energy of the artificial thermodynamic system, which has a close relation to the capacity of frequency-selective fading channel. The replica method analysis is under the framework of the large system limit. Here, the large system limit means that the number of users and spreading gain are finite but the ratio between them keeps fixed, i.e. the asymptotic limit $(K,N)\to\infty$ with $K/N\to\beta>0$, i.e. a positive constant.

4.1. Free energy and channel capacity

Consider a basic fully synchronous K-user CDMA channel with a perfect power control as depicted in expression (1). When the channel state information is known to the receiver and let $\bar{S}=\breve{S}$, the input–output relation is written as

$$\boldsymbol{r} = \frac{1}{\sqrt{N}}\breve{S}\boldsymbol{x} + \sigma_0 n. \quad (10)$$

The vital point of computing the multi-access capacity is to calculate the system free energy based on the self-averaging assumption,[21,22] which is given by

$$F_K(\boldsymbol{r},\breve{S}) \triangleq K^{-1}\log Z(\boldsymbol{r},\breve{S}) = \lim_{K\to\infty}\frac{1}{K}\int_{R^N} E\left\{P(\boldsymbol{r}|\breve{S})\log Z(\boldsymbol{r},\breve{S})\right\}\mathrm{d}\boldsymbol{r}, \quad (11)$$

where

$$Z(\boldsymbol{r},\breve{S}) \triangleq \sum_x p(x)\exp\left(-\frac{1}{2\sigma^2}\left\|r - N^{-1/2}\breve{S}x\right\|^2\right), \quad (12)$$

is the partition function.

The quantity $P(\boldsymbol{x})$ in expression (12) is the prior distribution for the information bits. The notation $E(\bullet)$ in expression (11) denotes the average over the randomness of the spreading sequences. And $P(\boldsymbol{r}|\breve{S})$ is the probability of observing $\boldsymbol{r}$ as the output of the true channel, for a given authentic spreading sequence $\breve{S}$, it is given as

$$P(\boldsymbol{r}\,\big|\boldsymbol{x},\breve{S}) = (2\pi\sigma_0^2)^{-\frac{N}{2}}\exp\left(-\frac{1}{2\sigma_0^2}\left\|r - N^{-1/2}\breve{S}x\right\|^2\right). \quad (13)$$

Using the replica method , the free energy can be calculated by

$$F_K(\boldsymbol{r},\breve{S}) = \lim_{K\to\infty}\left(\lim_{u\to 0}\frac{\partial}{\partial u}K^{-1}\log\Xi_u\right), \quad (14)$$

where

$$\Xi_u = \sum_{x_0,x_1,\ldots,x_u}\prod_{a=0}^{u}p(x_a)\times\left\{\frac{1}{\sqrt{2\pi\sigma_0^2}}\int_{\mathrm{R}}\overline{w_1}\mathrm{d}r\right\}^N, \quad (15)$$

with

$$w_1 = \exp\left[-\frac{1}{2\sigma_0^2}\left(r - \frac{1}{\sqrt{N}}\sum_{k=1}^{K}\breve{s}_k x_{0k}\right)^2\right] \times\prod_{a=1}^{u}\exp\left[-\frac{1}{2\sigma^2}\left(r - \frac{1}{\sqrt{N}}\sum_{k=1}^{K}\breve{s}_k x_{ak}\right)^2\right]. \quad (16)$$

It is seen from expression (16) that it is convenient to introduce the following two parameters in order to compute the free energy:

$$v_0 = \frac{1}{\sqrt{K}}\sum_{k=1}^{K}\breve{s}_k x_{0k},$$
$$v_a = \frac{1}{\sqrt{K}}\sum_{k=1}^{K}\breve{s}_k x_{ak}, \quad (17)$$

and it is easy to prove that the above two parameters are both Gaussian random variables with a mean value of 0 and a convariance $Q_{ab} \triangleq \overline{v_a v_b}$. Then we have

$$\Xi_u = \int_{[-1,1]^{u(u+1)/2}} \exp(K\beta^{-1}\Omega\{Q\})\mu_K\{Q\} \times \prod_{a\leq b} \mathrm{d}Q_{ab}, \tag{18}$$

where

$$\mu_K\{Q\} = \int_{\mathrm{R}^{K(u+1)}} \prod_{a\leq b} \delta(x_a \bullet x_b - KQ_{ab}) \times \prod_{a=0}^{u} [p(x_a)\mathrm{d}x_b] \tag{19}$$

and

$$\exp(\Omega\{Q\}) = \frac{1}{\sqrt{2\pi\sigma_0^2}} \int_R \overline{w_2}\mathrm{d}r \tag{20}$$

with

$$w_2 = \exp\left[-\frac{\beta}{2\sigma_0^2}\left(\frac{r}{\sqrt{\beta}} - v_0\{Q\}\right)\right] \times \prod_{a=1}^{u} \exp\left[-\frac{\beta}{2\sigma^2}\left(\frac{r}{\sqrt{\beta}} - v_a\{Q\}\right)\right].$$

In terms of Varadhan's large deviation theorem,[21−24] Ξ_u converges to the following expression in the $K \to \infty$ limit:

$$\lim_{K\to\infty} K^{-1}\log\Xi_u = \sup_{\{Q\}}(\beta^{-1}\Omega\{Q\} - I\{Q\}), \tag{21}$$

where $I\{Q\}$ is the rate function of $\mu_K\{Q\}$, which is based on Cramér's theorem of the theory of large deviations,[23,24] and given as

$$I\{Q\} \equiv \sup_{\{\widetilde{Q}\}} \left(\sum_{a\leq b} \widetilde{Q}_{ab}Q_{a_b} - \log M\left\{\widetilde{Q}\right\}\right) \tag{22}$$

with

$$M\left\{\widetilde{Q}\right\} \equiv \int_{\mathrm{R}^{u+1}} \exp\left(\sum_{a\leq b} \widetilde{Q}_{ab}x_a x_b\right) \prod_{a=0}^{u} Dx_a. \tag{23}$$

However, the optimization of expression (22) is intractable computationally, so we deal with this problem under the assumption of the replica symmetry (RS), i.e.

(i) $Q_{0a} = m$ for $a \neq 0$, and $Q_{ab} = q$ for $a \neq b$, and $a, b \neq 0$;

(ii) $\tilde{Q}_{0a} = E$ and $\tilde{Q}_{ab} = F$ for $a \neq b$, and $a, b \neq 0$;

(iii) $Q_{00} = p_0$, $Q_{aa} = p$, $\tilde{Q}_{00} = G_0/2$ and $\tilde{Q}_{aa} = G/2$, $a = 1, \ldots, n$.

Then the supremum of expression (22) is dependent on the parameter set $\{p_0, G_0, m, q, p, E, F, G\}$, which is given by the following saddle point equations:

$$\begin{aligned} m &= \frac{E}{1+F-G}, \qquad p = \frac{E^2+2F+1-G}{(1+F-G)^2}, \\ q &= \frac{E^2+F}{(1+F-G)^2}, \quad E = \frac{\beta^{-1}B}{1+B(p-q)}, \\ F &= \frac{\beta^{-1}B^2(B_0^{-1}+1-2m+q)}{[1+B(p-q)]^2}, \\ G &= F-E, \quad p_0 = 1, \quad \text{and} \quad G_0 = 0. \end{aligned} \tag{24}$$

The corresponding RS free energy is

$$\begin{aligned} F^{\mathrm{RS}}(r,\breve{S}) = &-\frac{1}{2}\left[\log(1-F+G) - \frac{E^2+F}{1+F-G}\right] \\ &-Em + \frac{1}{2}Fq - \frac{1}{2}Gp \\ &-\frac{1}{2\beta} \times \log[1+B(p-q)] \\ &-\frac{1}{2\beta} \times \frac{B(B_0^{-1}+1-2m+q)}{1+B(p-q)}, \end{aligned} \tag{25}$$

and the channel capacity can be expressed as

$$C(\breve{S}) = -\left(F^{\mathrm{RS}}(r,\breve{S})\Big|_{\sigma=\sigma_0} + \frac{1}{2\beta}\right). \tag{26}$$

4.2. Multiuser channel capacity with imperfect channel estimation

From expressions (21) and (26), it is easy to find that the free energy, i.e. the channel capacity is generally determined by $I\{Q\}$ and $\Omega\{Q\}$. In order to study the effect of channel estimation error on the channel capacity, the first step to do is evaluate how the channel estimation error affects the two terms $I\{Q\}$ and $\Omega\{Q\}$. An important fact is that the existence of channel estimation error does not change the term $I\{Q\}$ because of the orthogonality of spread sequences. Hence, we need only focus on the calculation of the term $\Omega\{Q\}$.

For the MMSE channel estimation, $\Gamma_k[l]$ is uncorrelated with $\tilde{S}_k$, thus resulting in $E\left\{S_k\widetilde{S_k}\right\} = 1-\sigma_S^2$. Then the convariances Q_{0a} and Q_{ab} are given, respectively, by

$$\begin{aligned} Q_{0a} \triangleq \overline{v_0 v_a} &= \overline{\left(\sum_{k=1}^{K} \widetilde{s_k}x_{0k}\right)\left(\sum_{k=1}^{K} \widetilde{s_k}x_{ak}\right)} \\ &= \frac{1-\sigma_S^2}{K}\sum_{k=1}^{K} x_{0k}x_{ak}, \quad \forall a > 0, \end{aligned}$$

and

$$Q_{ab} \triangleq \overline{v_a v_b} = \overline{\left(\sum_{k=1}^{K} \widetilde{s}_k x_{ak}\right)\left(\sum_{k=1}^{K} \widetilde{s}_k x_{bk}\right)} = \frac{1-\sigma_S^2}{K}\sum_{k=1}^{K} x_{ak}x_{bk}, \quad \forall a,b>0. \tag{27}$$

Then, variables v_0 and v_a can be constructed by three independent Gaussian random variables with a mean value of 0 and a variance of 1 as follows:

$$v_0 = u\left(p_0 - \frac{(1-\sigma_S^2)m^2}{q}\right)^{\frac{1}{2}} - t\frac{m\sqrt{(1-\sigma_S^2)}}{\sqrt{q}},$$

and

$$v_a = \sqrt{(1-\sigma_S^2)}(z_a(p-q)^{\frac{1}{2}} - t\sqrt{q}), \quad a = 1,\ldots,n. \tag{28}$$

Thus expression (20) becomes

$$\begin{aligned}&\exp(\Omega\{Q\})\\ &= -\frac{1}{2}\log\left[1+B(p-q)(1-\sigma_S^2)\right.\\ &\quad +nB\left[B_0^{-1}+p_0+(q-2m)(1-\sigma_S^2)\right]\\ &\quad \left.+\frac{1-n}{2}\log\left[1+B(p-q)(1-\sigma_S^2)\right]\right].\end{aligned} \tag{29}$$

Hence, the free energy and the capacity can be formulated by the following two expressions respectively:

$$\begin{aligned}&F^{\mathrm{RS}}(r,\tilde{S})\\ &= -\frac{1}{2}\left[\log(1-F+G) - \frac{E^2+F}{1+F-G}\right]\\ &\quad -Em + \frac{1}{2}Fq - \frac{1}{2}Gp\\ &\quad -\frac{1}{2\beta}\times\left\{\log\left[1+(1-\sigma_S^2)B(p-q)\right]\right\}\\ &\quad -\frac{1}{2\beta}\times\left\{\frac{B(B_0^{-1}+1+(1-\sigma_S^2)(q-2m))}{1+B(p-q)(1-\sigma_S^2)}\right\}\end{aligned} \tag{30}$$

and

$$C(\tilde{S}) = -\left(F^{\mathrm{RS}}(r,\tilde{S})\Big|_{\sigma=\sigma_0} + \frac{1}{2\beta}\right) \tag{31}$$

with the corresponding E, F and G given as

$$\begin{aligned}E &= \frac{\beta^{-1}B(1-\sigma_S^2)}{1+B(p-q)(1-\sigma_S^2)},\\ F &= \frac{(1-\sigma_S^2)\beta^{-1}B^2(B_0^{-1}+1-(q-2m)(1-\sigma_S^2))}{[1+B(p-q)(1-\sigma_S^2)]^2},\\ G &= F-E.\end{aligned} \tag{32}$$

5. Numerical simulation results

Numerical simulation results are provided in this section to illustrate the analysis in the former sections. The simulation results are mainly based on expressions (24)–(26) and (30)–(32). For comparison, in Fig.1, we show channel capacity versus the system load (β) and signal-to-noise ratio (SNR) when the channel state information is perfectly known at the receiver, namely, the convariance of the channel estimation error $\sigma_S^2 = 0$. Clearly, the channel capacity increases with signal-to-noise ratio but decreases with system load.

When channel estimation error is considered, the channel capacity is influenced by channel estimation error beside the parameters β and SNR. But we focus on the study of the effect of channel estimation error on channel capacity, and we have specified $\beta = 1$ in Fig.2 and SNR is set to be 2dB in Fig.3. As can be observed from the two figures, channel estimation error reduces the theoretic capacity of information generally. The result is expected, since as σ_S^2 increases, the channel estimation error increases.

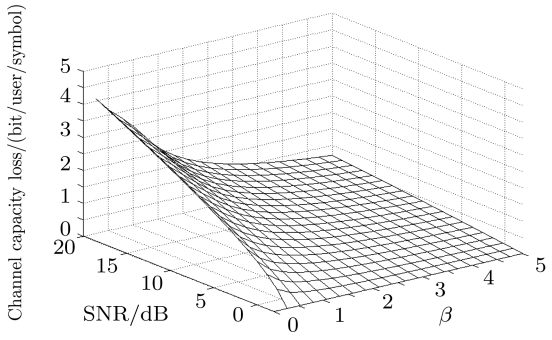


Fig.1. Channel capacity versus system load and signal-to-noise ratio.

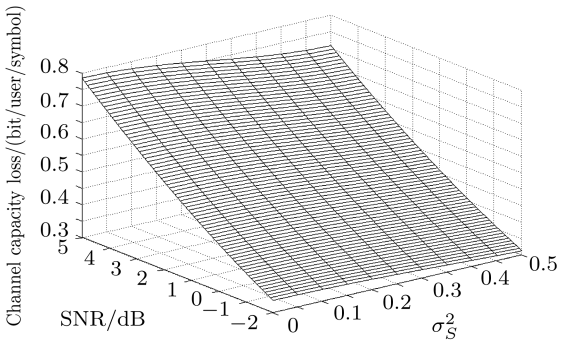


Fig.2. Channel capacity versus channel estimation error and signal-to-noise ratio.

To explicitly demonstrate the variation of channel capacity due to imperfect channel state information, we define capacity loss ΔC as the difference between the channel capacity with perfect channel state information and the capacity with channel estimation error. Figure 4 shows the relation between capacity loss and the convariance of the channel estimation error ($\beta = 1$). The legends in Fig.4 are the values of signal-to-noise ratios.

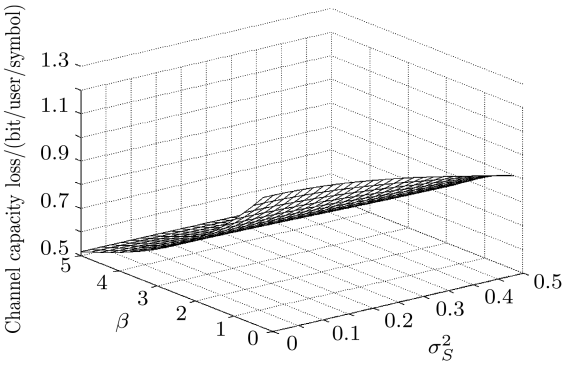


Fig.3. Channel capacity versus channel estimation error and system load.

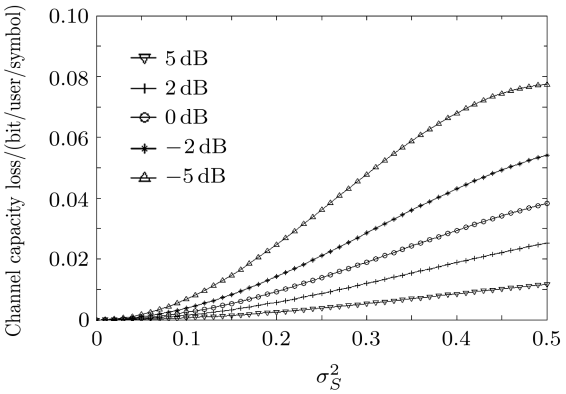


Fig.4. Channel capacity loss due to channel estimation error.

6. Conclusion and outlook

We have presented a general framework based on the replica method to discuss the effect of imperfect channel estimation error on multiuser channel capacity. Our analyses show that the channel capacity depends on the accuracy of the channel estimation, the system load and signal-to-noise ratio. Particularly, in practice, imperfect channel state information reduces the channel capacity of multiuser CDMA channel when the system load and the signal-to-noise ratio are specified.

In this paper, we obtain analytic results which match the performance of a practical CDMA system well, but those results are based on some assumptions. And those assumptions include. (1) the self-averaging assumption that can be applied to determining the free energy, (2) the replica method that is valid to large CDMA systems, (3) replica symmetry that holds. However, the validity of these assumptions have not been proved in the area of information theory. In other words, the sufficient conditions under which the replica method can be applied are still unknown to us.[11,12] These questions are difficult, challenging the mathematical physics, and wait for physical scientists to be solved in future.

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