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Effect of multi-velocity-difference in traffic flow*

Mo Ye-Liu(莫业柳)[a], He Hong-Di(何红弟)[b], Xue Yu(薛 郁)[a)c)],
Shi Wei(时 伟)[a)c)], and Lu Wei-Zhen(卢伟真)[b)]

[a)] *Institute of Physical Science and Engineering, Guangxi University, Nanning* 530004, *China*
[b)] *Department of Building and Construction, City University of Hong Kong, Hong Kong, China*
[c)] *Shanghai Institute of Applied Mathematics and Mechanics, Shanghai University, Shanghai* 200072, *China*

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Based on the optimal velocity models, an extended model is proposed, in which multi-velocity-difference ahead is taken into consideration. The damping effect of the multi-velocity-difference ahead has been investigated by means of analytical and numerical methods. Results indicate that the multi-velocity-difference leads to the enhancement of stability of traffic flow, suppression of the emergence of traffic jamming, and reduction of the energy consumption.

Keywords: traffic flow, optimal velocity model, stable condition, linear stability analysis
PACC: 0550, 0520, 6470

1. Introduction

Traffic problems attract extensive attention of scientists all the time, because of the appearance of observed non-equilibrium phase transitions and various nonlinear dynamical phenomena in the process of traffic congestion. Many traffic models such as cellular automaton, car-following, gas kinetic and hydrodynamic models are proposed to explain the formation of traffic jamming in different light.[1,2]

The classical optimal velocity (OV) model is a kind of car-following model proposed by Bando *et al* in 1995.[3] The dynamical equation of the OV model is given as follows:

$$\ddot{x}_j(t) = a\{V(\Delta x_j(t)) - \dot{x}_j(t)\}, \tag{1}$$

where $x_j(t)$ is position of car j at time t, $\Delta x_j(t) = x_{j+1}(t) - x_j(t)$ is the headway of car j at time t, a is the driver's sensitivity and equals the inverse of driver's reaction time τ. The OV model is a very simple car-following model and has succeeded in reproducing the dynamical formation of congestion. However, many investigators have analysed the characteristics of the OV model[4] and found that it cannot avoid crash at values of sensitivity smaller than about $a \leq 1\mathrm{s}^{-1}$ for it shows unrealistic values of the acceleration for various optimal functions.[5] Recently, Davis,[6] Lenz *et al*[7] and Wilson *et al*[5] have studied the reason for the OV model's drawback and proposed separately the improved model to remove the OV model's drawbacks, in which Wilson *et al* presented three multiple look-ahead models A, B and C with many-neighbour and non-locality. In what follows, for simplicity, we call the 3rd model as the model C expressed as

$$\ddot{x}_j(t) = a\left(V\left(\sum_{l=1}^{n}\beta_l \Delta x_{j+l-1}(t)\right) - \dot{x}_j(t)\right). \tag{2}$$

In addition, Ge *et al*[8] has also proposed a similar multiple look-ahead model in the form of difference equation by considering the effect of multi-headway ahead on optimal velocity. Their studies indicate that the property with many-neighbour and non-locality is beneficial to the enhancement of the stability of traffic flow.

Herman *et al*[9] observed that variation of multiple cars relative velocity ahead affected the current drivers in real traffic. In 1961, Gazis *et al*[10] proposed a follow-the-leader model by considering the relative velocity in a single lane (called GM model for short). Since the acceleration depends on the vehicle in front and obtains a larger value to exceed the acceleration in real traffic, GM model is not applicable to very low traffic density.[2,11] Treiber *et al*[11] pointed out that the effect of relative velocity is capable of enhancing the stability of traffic flow. They proposed the ID model (Intelligent Drivers model) to reproduce the real traffic phenomena. Later, Jiang *et al*[12] proposed the full-velocity difference model, which has a

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good starting process and is capable of simulating the phase transition from free to synchronization in the real traffic. Nakayama *et al*[13] studied the effect of the interplay between back and forth on traffic. Xue *et al*[14] also introduced the relative velocity to improve the stability of the OV model.

It is able to record position and velocity of many cars in front to determine their states in the intelligent traffic system. Thus, it is in favour of safety cooperative driving and increasing probability to suppress unrealistic large acceleration in the OV model via considering the damping effect of multi-velocity-difference in front. Based on the OV model, we propose a multi-velocity-difference model in Section 2. We would make sure whether or not the extended model displays marked improvement of stability of traffic flow. Linear stability analysis indicates that the effect of multiple velocity difference ahead enhances the stability of traffic in great measure. In Section 3, we conduct the simulation and analysis of the stability of traffic and the additional energy consumption of the whole flow. Finally, the conclusion is given in Section 4.

2. Model and stability analysis

Based on the multiple look-ahead model, the stimulus-response equation proposed by Gazis *et al*[10] and symmetrical consideration, the dynamical equation with multi-velocity-difference ahead reads

$$\ddot{x}_j(t) = a\left[V\left(\sum_{l=1}^{n}\beta_l\Delta x_{j+l-1}\right)-\dot{x}_j(t)\right] + a\sum_{l=1}^{n}\frac{v_{j+l-1}^{m}}{\Delta x_{j+l-1}^{\rho}}\Delta v_{j+l-1}(t), \tag{3}$$

where Δv_{j+l-1} and Δx_{j+l-1} are respectively velocity difference (relative velocity) and headway between jth car and $(j+l-1)$th car ahead at time t, β_l is a weight to reflect the different preoccupation about the variations of many-vehicle states ahead; m and ρ are phenomenological parameters to be fixed by comparison with empirical data; they are chosen in practice to be $m = 0.8$ and $\rho = 2.8$[15,16] or $m = 0.953$ and $\rho = 3.05$.[17,18] This extended model assumes that a driver can adjust the vehicle's acceleration $\ddot{x}_j(t)$ via considering many-neighbour-vehicle's headway and relative velocity ahead. The optimal velocity is given by

$$V(\Delta x_j(t)) = (V_{\max}/2)\left[\tanh\left(\sum_{l=1}^{n}\beta_l\Delta x_{j+l-1}(t)-h_{\rm c}\right)+\tanh(h_{\rm c})\right], \tag{4}$$

where $V_{\max}$ is the maximal velocity, $h_{\rm c}$ is the safety distance. Based on the properties of weight $\beta_l (l = 1, 2, \ldots, n)$ in Ref.[8], we choose a kind of general weight as follows:

$$n>1,\quad \beta_l=\frac{\eta-1}{\eta^l},\ \text{for } l\neq n;\qquad \beta_l=\frac{1}{\eta^{n-1}},\quad \text{for}\quad l=n, \tag{5}$$

where η is a parameter in the weight, with a value $\eta \geq 3$.[19]

We study the stability of the uniform traffic flow as a function of disturbance. The uniform traffic flow is defined as such a state that all cars move with the same headway h and the optimal velocity $V(h)$. The solution of the uniformly steady state in Eq.(3) is given by

$$x_{j,0}(t) = hj + V(h)t, \tag{6}$$

where $h = L/N$, N is the total number of vehicles, L is the road length, and h is the mean headway. Let $y_j(t)$ be a small perturbations in the uniform solution $x_{j,0}(t)$: $x_j(t) = x_{j,0}(t) + y_j(t)$. Then, the linear equation is obtained from Eq.(3),

$$\frac{{\rm d}^2y_j(t)}{{\rm d}t^2} = a\left[V'(h)\sum_{l=1}^{n}\beta_l\Delta y_{j+l-1} + \sum_{l=1}^{n}\frac{V^m(h)}{h^{\rho}}\frac{{\rm d}\Delta y_{j+l-1}}{{\rm d}t}\right], \tag{7}$$

where $V'(h)$ is the derivative of optimal velocity $V(\Delta x)$ at $\Delta x = h$ and $\Delta y_{j+l-1} = y_{j+l} - y_{j+l-1}$. By expanding $y_j(t)$ in Fourier-modes: $\Delta y_j(t) = {\rm C}\exp({\rm i}kj + zt)$, one obtains

$$z^2 + az - az\frac{V^m(h)}{h^{\rho}}\sum_{l=1}^{n}\left({\rm e}^{{\rm i}kl}-{\rm e}^{{\rm i}k(l-1)}\right) - aV'(h)\sum_{l=1}^{n}\beta_l\left({\rm e}^{{\rm i}kl}-{\rm e}^{{\rm i}k(l-1)}\right)=0. \tag{8}$$

Expanding $z = z_1({\rm i}k) + z_2({\rm i}k)^2 + \ldots$ and inserting it in Eq.(8) lead to the first-order and second-order terms of coefficients in the expression of z respectively, as follows:

$$z_1 = V'(h); \qquad z_2 = \frac{-V'^2(h) + an\dfrac{(V(h))^m}{h^\rho}V'(h) - aV'(h)\displaystyle\sum_{l=1}^{n}\beta_l\frac{1-2l}{2}}{a}. \tag{9}$$

Thus the neutral stability criterion is given by

$$\frac{V'(h)}{a} = \sum_{l=1}^{n}\beta_l\frac{2l-1}{2} + n\frac{(V(h))^m}{h^\rho}. \tag{10}$$

For small disturbances with long wavelengths, the uniform traffic flow is unstable under the following condition:

$$\frac{V'(h)}{a} < \sum_{l=1}^{n}\beta_l\frac{2l-1}{2} + n\frac{(V(h))^m}{h^\rho}. \tag{11}$$

Obviously, the stability criterion (Eq.(10)) is determined by the value of the weight function β_l. Based on Eq.(5), the parameter of weight functions is chosen as $\eta = 3$ to analyse the stability. Comparison of the neutral stability lines in the extended model with one of the Wilson's models(model C) is shown in Fig.1, for $n = 1, 2, 3$, and 5. In Fig.1, the apex of each curve denotes the critical point (h_c, a_c), the traffic flow in the region above each curve is stable and the traffic jam does not occur. Below each curve, the traffic flow loses its stability and the density wave emerges. From Fig.1, it can be found that taking into consideration of more cars ahead, the critical points and the neutral stability curves are lowered, which means a more stable traffic flow. Moreover, just considering only two cars ahead, the velocity difference of the two cars in extended model is more stable than that of arbitrary number of cars ahead in Wilson's model C. That is to say, the stability of the extended model has been strengthened, and traffic jam is suppressed effectively.

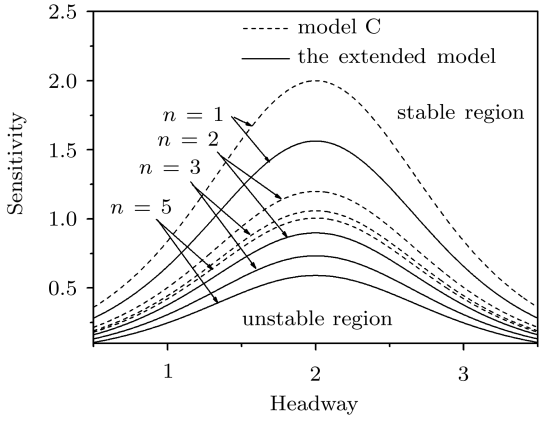


Fig.1. neutral stability curves in the headway-sensitivity space. The neutral stability curves in model C (dash lines) and the extended model (solid lines) are shown for $\eta = 3, n = 1, 2, 3$, and 5.

3. Numerical simulation

In order to demonstrate how the stability of traffic flow is enhanced in the extended model, we have carried out computer simulations for Wilson's model C and the extended traffic flow model. Both models investigated must have the same uniform flow solution for any density. The initial situation is as follows: 200 cars are distributed on the single lane with the cycle of length 400,the mean headway $h = 2$ and the mean velocity tanh(2). In addition, the safety distance is $h_c = 2.0$, $m = 0.8$ and $\rho = 2.8$ and the time-step is 0.01.

First, we want to understand how the velocity changes prominently under the influence of disturbance. The leading car has a larger headway than others. Figure 2 exhibit the snapshot of velocity configuration of 100 vehicles of the three models (including OV model, Wilson's model C, and the extended model) in the unstable condition $a = 0.5$ at time step $t = 500$. In accordance with the results of linear stability analysis, two cars ahead and their velocity differences in the extended model and three cars ahead in Wilson's model C are taken into account. It is found that the negative velocity appears in the OV model while the Wilson's model C and the extended model still show normal velocity in Fig.2. When the sensitivity a is less than 0.47, the simulation exhibits the appearance of negative velocity in Wilson's model C

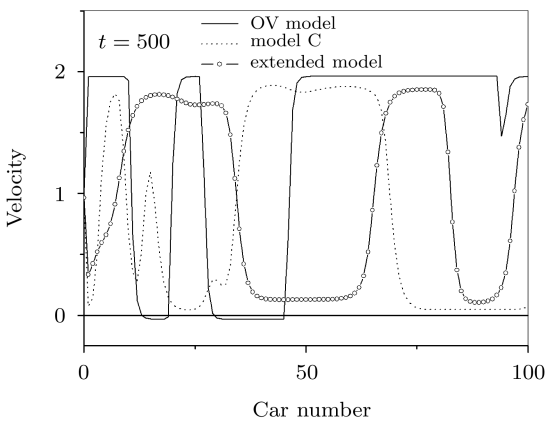


Fig.2. A snapshot of velocities of the 100 vehicles at $t = 500$ of the three models (OV model, Wilson's model C, and the extended model) under the unstable condition $a = 0.5$, $N = 200$, $L = 400$.

as illustrated in Fig.3. The snapshots of velocity of 100 vehicles for Wilson's model C and extended model with $a = 0.3$ at $t = 500$ and 5000 are shown in Fig.3. It indicates that the driver will go backwards within the large reaction time τ, which is inconsistent with driver's behaviour in the real traffic. From Figs.3(a) and 3(b), it is found that the negative velocity in the extended model does not appear in the process of evolution. It is very clear that the multiple velocity differences play an important role in the stability of the traffic flow. Moreover, Fig.4 shows the velocity curves of the 100$^{\text{th}}$ vehicle for the Wilson's model C ($n = 3$) and the extended model ($n = 2$) at the sensitivity $a = 1.0$. Results demonstrate that the extended model is very easy to arrive at a stable state due to damping effect of the multi-velocity-difference.

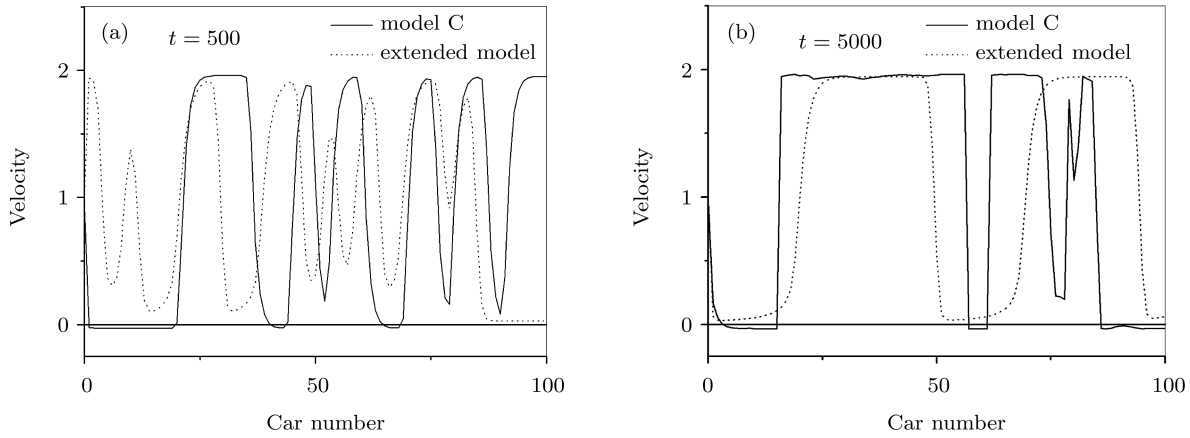


Fig.3. (a) A snapshot of velocities of 100 vehicles at $t = 500$ for Wilson's model C and the extended model with $a = 0.3$. $N = 200$, $L = 400$. (b) A snapshot of velocities of 100 vehicles at t=5000 for Wilson's model C and the extended model with $a = 0.3$. $N = 200$, $L = 400$.

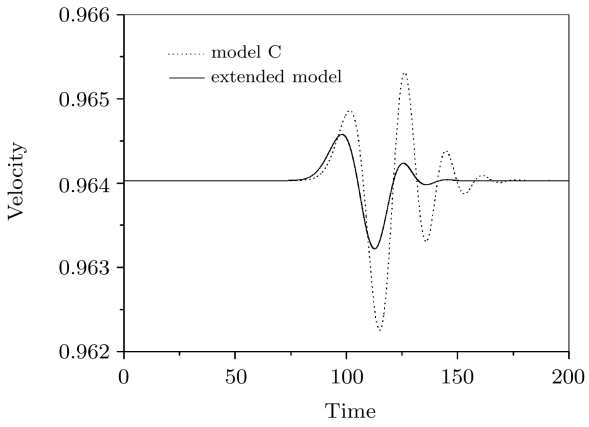


Fig.4. Comparison of speed profiles of the 100$^{\text{th}}$ vehicle for the model C ($n = 3$) and the extended model ($n = 2$), with $a = 1.0$, $N = 200$, $L = 400$.

On the other hand, the energy consumption in traffic flow can display the stable properties of the model. Nakayama *et al*[14] presented the following equation to estimate the additional energy consumption for the whole traffic flow,

$$E = \sum_{\text{cars}} \sum_{\text{waves}} \frac{1}{2} \left(v_{\max}^2 - v_{\min}^2 \right), \tag{12}$$

where $\sum_{\text{waves}}$ denotes the summation of all periods of oscillation until the disturbance disappears.[13,20] Shi *et al*[19] analysed the additional energy consumption of some typical car-following models and linked the minimizing of the energy consumption with the stability of the model.[20]

Figure 5 exhibits the additional energy consumption of Wilson's model C and the extended model for $n = 2$ and 3, and the parameter of weight parameter $\eta = 3$. One can see that the consumption of total

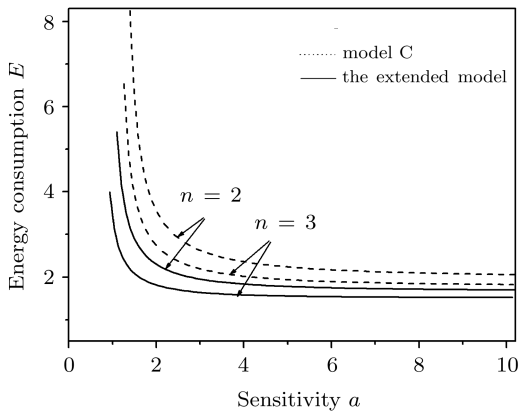


Fig.5. Energy consumption as a function of sensitivity. The dashed lines represents the energy consumption for model C, and the solid line indicates that for the extended model, for $\eta = 3$, $n = 2, 3$.

energy decreases with increasing sensitivity. It indicates that a larger value of sensitivity means a shorter reaction time of driver, making velocity change fast and the additional consumption of energy break down. On the contrary, when traffic flow does not satisfy the stable condition, the perturbation effect on the traffic flow will be amplified and oscillated ceaselessly to induce a stop-and-go traffic. We notice also that the energy consumption curves will be dropped with increasing value of the parameter n in each model. The energy consumption in the extended model with $n = 2$ is lower than that in Wilson's model C with $n = 3$. Thus, it also indicates that the damping effect of the multi-velocity-difference can decrease the energy consumption because frequent acceleration of cars in the real traffic just gives rise to the increase of energy consumption.

4. Summary

We have presented a multi-velocity-difference optimal velocity model. The stability of the extended model displays marked improvement. Surely, it acquires a strong ability in considering various complicated factors that affects traffic flow in different measure. However, drivers in the intelligent transport system can conveniently receive information of headway and multiple velocity changes ahead. The damping effect of the multi-velocity-difference ahead can improve traffic, suppress the emergence of a traffic jam, and reduce the energy consumption.

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