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Integral sliding mode control for a class of nonlinear neutral systems with time-varying delays*

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This paper focuses on sliding mode control problems for a class of nonlinear neutral systems with time-varying delays. An integral sliding surface is firstly constructed. Then it finds a useful criteria to guarantee the global stability for the nonlinear neutral systems with time-varying delays in the specified switching surface, whose condition is formulated as linear matrix inequality. The synthesized sliding mode controller guarantees the reachability of the specified sliding surface. Finally, a numerical simulation validates the effectiveness and feasibility of the proposed technique.

Keywords: neutral systems, nonlinear, linear matrix inequality, integral sliding mode control
PACC: 0547

1. Introduction

Over the past two decades, the study on sliding mode control (SMC) has increased considerably owing to its insensitiveness to parametric uncertainties and disturbances.[1] SMC is an effective control approach for nonlinear systems.[2−6] In addition, SMC schemes have been developed to eliminate chaotic behaviours, see Refs.[4–10]. Recently, SMC has been exploited to a variety of applications in the robust control of uncertain systems with or without time-delays.[11−16] However, the integral SMC for nonlinear systems has received relatively little attention until recently.[17] In particular, the problem of integral SMC for nonlinear systems with neutral type and time-varying delays is still open.

It has been recognized that time-delays, which are inherent features of many physical processes, are often the cause for instability and poor performance of systems.[17] Hence, differential delay equations are among the most useful models in applications, and have attracted the attention of a considerable number of researchers. However, to the best of our knowledge, there has been less attention paid to questions surrounding integral SMC for nonlinear systems with neutral type and time-varying delays. The existence of time-varying delays and nonlinear term will make the problem more complex and challenging.

In this paper, we consider the integral SMC problem of nonlinear neutral systems with time-varying delays. It may involve a nonlinear unknown function. In the proposed methods, an integral-type sliding surface is constructed. By means of linear matrix inequality (LMI), a sufficient condition is given such that the neutral system in the specified switching surface is globally stable. The synthesized sliding mode controller guarantees the reachability of the specified sliding surface.

Notation: R^n and $R^{m\times n}$ denote, respectively, the n-dimensional Euclidean space, and the set of all $m\times n$ real matrices. A^{T} and A^{-1} denote the transpose and the inverse of any (square) matrix A respectively. $A > 0$ (or $A < 0$)denotes a positive-(negative-) definite matrix. I is used to denote the $n \times n$ identity matrix. $\|\cdot\|$ and $\|\cdot\|_1$ denote, respectively, the Euclidean norm and 1-norm of a vector or its induced matrix norm. If $a \in R^n$, we have $\|a\| \leq \|a\|_1$. The symbol '$\star$' within a matrix represents the symmetric term of the matrix.

2. Problem formulation

Consider the time-delay neutral systems which are described by the following form:

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$$\begin{aligned}\dot{x}(t) &= \boldsymbol{A}x(t) + \boldsymbol{A}_{\mathrm{d}}x(t-\tau(t)) \\ &\quad +\boldsymbol{C}\dot{x}(t-\tau(t)) + \boldsymbol{B}[u(t)+f(x(t),t)], \\ x(t) &= \phi(t),\ t\in[-d,0], \end{aligned} \tag{1}$$

where $x(t)\in R^n$ is the state, $u(t)\in R^m$ is the control input. $\boldsymbol{A}\in R^{n\times n}$, $\boldsymbol{A}_{\mathrm{d}}\in R^{n\times n}$, $\boldsymbol{C}\in R^{n\times n}$, $\boldsymbol{B}\in R^{n\times m}$ are known real constant matrices, $\tau(t)$ is the time-varying delay satisfying

$$0<\tau(t)\le d<\infty,\quad \dot{\tau}\le h<1, \tag{2}$$

where d and h are known real constant scalars, $\phi(t)$ is a continuous vector-valued initial function. $f(x(t),t)\in R^n$ is an unknown nonlinear function satisfying

$$\|f(x(t),t)\|\le\gamma\|x(t)\|, \tag{3}$$

where $\gamma>0$ is a known constant. Without loss of generality, it is assumed that the pair $(\boldsymbol{A},\boldsymbol{B})$ is controllable and the input matrix $\boldsymbol{B}$ has full column rank.

Now, let us formulate the problem to be dealt with in this paper.

SMC Design: For the neutral system in Eq.(1), a sliding mode controller is to be synthesized so that

(i) the sliding motion is asymptotically stable;

(ii) the state trajectory in Eq.(1) is globally driven onto the specified sliding surface, and maintained there for all subsequent time.

3. Main result

3.1. Integral-type sliding surface

In this work, we choose the integral switching function as follows:

$$s(t)=\boldsymbol{G}x(t)-\int_0^t\boldsymbol{G}(\boldsymbol{A}+\boldsymbol{B}\boldsymbol{K})x(s)\mathrm{d}s, \tag{4}$$

where $\boldsymbol{G}\in R^{m\times n}$ and $\boldsymbol{K}\in R^{m\times n}$ are constant matrices. The matrix K is chosen such that the matrix $\boldsymbol{A}+\boldsymbol{B}\boldsymbol{K}$ is Hurwitz, and the matrix $\boldsymbol{G}$ is to be designed so that $\boldsymbol{G}\boldsymbol{B}$ is nonsingular. It can be easily shown that the nonsingularity of $\boldsymbol{G}\boldsymbol{B}$ can be ensured by choosing $\boldsymbol{G}=\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}$ with $\boldsymbol{P}>0$, since $\boldsymbol{B}$ is assumed to be full column rank. It is noted that the switching function $s(t)$ in Eq.(4) is well defined for the solution $x(t)$ of the system (1). Moreover, it is seen that $s(t)$ is a semi-martingale, since the solution $x(t)$ is a semi-martingale. Actually, it can be obtained from Ref.[18] that the solution $x(t)$ of the system (1) is as follows:

$$\begin{aligned}x(t) &= \phi(0)+\int_0^t\Big[\boldsymbol{A}x(s)+\boldsymbol{A}_{\mathrm{d}}x(s-\tau(s)) \\ &\quad +\boldsymbol{C}\dot{x}(s-\tau(s)) \\ &\quad +\boldsymbol{B}\Big(u(s)+f(x(s),s)\Big)\Big]\mathrm{d}s. \end{aligned} \tag{5}$$

So, it can be obtained from Eqs.(4) and (5) such that

$$\begin{aligned}s(t) &= G\phi(0)+\int_0^t\Big[(-\boldsymbol{G}\boldsymbol{B}\boldsymbol{K})x(s) \\ &\quad +\boldsymbol{G}\boldsymbol{A}_{\mathrm{d}}x(s-\tau(s))+\boldsymbol{G}\boldsymbol{C}\dot{x}(s-\tau(s)) \\ &\quad +\boldsymbol{G}\boldsymbol{B}\Big(u(s)+f(x(s),s)\Big)\Big]\mathrm{d}s. \end{aligned} \tag{6}$$

Here, it is noted that $s(t)$ varies finitely, which means that it is rational to take the time derivation of $s(t)$. When the state trajectories of the system enter the sliding mode, we have $s(t)=0$ and $\dot{s}(t)=0$. Hence, it follows from Eq.(6) and $\dot{s}(t)=0$ that the equivalent control law in the sliding mode is given as

$$\begin{aligned}u(t) &= \boldsymbol{K}x(t)-(\boldsymbol{G}\boldsymbol{B})^{-1}\boldsymbol{G}\boldsymbol{A}_{\mathrm{d}}x(t-\tau(t)) \\ &\quad -(\boldsymbol{G}\boldsymbol{B})^{-1}\boldsymbol{G}\boldsymbol{C}\dot{x}(t-\tau(t))-f(x(t),t). \end{aligned} \tag{7}$$

Thus, the sliding mode dynamics can be obtained by substituting Eq.(7) into Eq.(1):

$$\begin{aligned}\dot{x}(t) &= \Big[\boldsymbol{A}+\boldsymbol{B}\boldsymbol{K}\Big]x(t) \\ &\quad +\Big[\boldsymbol{A}_{\mathrm{d}}-\boldsymbol{B}(\boldsymbol{G}\boldsymbol{B})^{-1}\boldsymbol{G}\boldsymbol{A}_{\mathrm{d}}\Big]x(t-\tau(t)) \\ &\quad +\Big[\boldsymbol{C}-(\boldsymbol{G}\boldsymbol{B})^{-1}\boldsymbol{G}\boldsymbol{C}\Big]\dot{x}(t-\tau(t)). \end{aligned} \tag{8}$$

For convenience, let $\widetilde{\boldsymbol{A}}=\boldsymbol{A}+\boldsymbol{B}\boldsymbol{K}$, $\widetilde{\boldsymbol{B}}=\boldsymbol{A}_{\mathrm{d}}-\boldsymbol{B}(\boldsymbol{G}\boldsymbol{B})^{-1}\boldsymbol{G}\boldsymbol{A}_{\mathrm{d}}$, $\widetilde{\boldsymbol{C}}=\boldsymbol{C}-\boldsymbol{B}(\boldsymbol{G}\boldsymbol{B})^{-1}\boldsymbol{G}\boldsymbol{C}$. Then Eq.(8) can reduce to

$$\dot{x}(t)=\widetilde{\boldsymbol{A}}x(t)+\widetilde{\boldsymbol{B}}x(t-\tau(t))+\widetilde{\boldsymbol{C}}\dot{x}(t-\tau(t)). \tag{9}$$

Some lemmas used in this paper are listed as follows.

Lemma 1 (Schur complement). Given constant matrices $\boldsymbol{\Omega}_1,\boldsymbol{\Omega}_2,\boldsymbol{\Omega}_3$, where $0<\boldsymbol{\Omega}_1=\boldsymbol{\Omega}_1^{\mathrm{T}}$ and $0<\boldsymbol{\Omega}_2=\boldsymbol{\Omega}_2^{\mathrm{T}}$ then $\boldsymbol{\Omega}_1+\boldsymbol{\Omega}_3^{\mathrm{T}}\boldsymbol{\Omega}_2^{-1}\boldsymbol{\Omega}_3$ if and only if

$$\begin{pmatrix}\boldsymbol{\Omega}_1 & \boldsymbol{\Omega}_3^{\mathrm{T}}\\ \boldsymbol{\Omega}_3 & -\boldsymbol{\Omega}_2\end{pmatrix}<0,\ \text{or}\ \begin{pmatrix}-\boldsymbol{\Omega}_2 & \boldsymbol{\Omega}_3\\ \boldsymbol{\Omega}_3^{\mathrm{T}} & \boldsymbol{\Omega}_1\end{pmatrix}<0.$$

Lemma 2 For any real matrices $\boldsymbol{\Sigma}_1,\boldsymbol{\Sigma}_2,\boldsymbol{\Sigma}_3$ with appropriate dimensions and $\boldsymbol{\Sigma}_3^{\mathrm{T}}\boldsymbol{\Sigma}_3\le I$, it follows that

$$\boldsymbol{\Sigma}_1^{\mathrm{T}}\boldsymbol{\Sigma}_2+\boldsymbol{\Sigma}_2^{\mathrm{T}}\boldsymbol{\Sigma}_1\le\varepsilon\boldsymbol{\Sigma}_1^{\mathrm{T}}\boldsymbol{\Sigma}_3\boldsymbol{\Sigma}_1+\varepsilon^{-1}\boldsymbol{\Sigma}_2^{\mathrm{T}}\boldsymbol{\Sigma}_3^{-1}\boldsymbol{\Sigma}_1,\ \forall\varepsilon>0.$$

Now, we proceed to the first task which analyses the stability of the sliding motion described by Eq.(9), and derive a sufficient condition by means of the LMI method.

3.2. Stability of sliding motion

Theorem 1 Consider the stochastic system Eq.(1) with assumptions Eqs.(2)–(3), and the switching surface (4). If there exist symmetric matrices $\boldsymbol{P} > 0$ and $\boldsymbol{Q} > 0$, and scalars satisfying

$$\begin{bmatrix} (1,1) & -2(\boldsymbol{A}+\boldsymbol{BK})^{\mathrm{T}}\boldsymbol{PC}+\boldsymbol{PA}_{\mathrm{d}} & 0 & 0 & \boldsymbol{PB} \\ \star & (2,2) & 2\boldsymbol{A}_{\mathrm{d}}^{\mathrm{T}}\boldsymbol{PB} & 2\boldsymbol{C}^{\mathrm{T}}\boldsymbol{PB} & 0 \\ \star & \star & -\boldsymbol{B}^{\mathrm{T}}\boldsymbol{PB} & 0 & 0 \\ \star & \star & \star & -\boldsymbol{B}^{\mathrm{T}}\boldsymbol{PB} & 0 \\ \star & \star & \star & \star & -\boldsymbol{B}^{\mathrm{T}}\boldsymbol{PB} \end{bmatrix} < 0, \tag{10}$$

where

$(1,1) = (\boldsymbol{A}+\boldsymbol{BK})^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}(\boldsymbol{A}+\boldsymbol{BK})+\boldsymbol{Q}+(\boldsymbol{A}+\boldsymbol{BK})^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{A}+\boldsymbol{BK})$,

$(2,2) = 2\boldsymbol{A}_{\mathrm{d}}^{\mathrm{T}}\boldsymbol{PA}_{\mathrm{d}}+2\boldsymbol{C}^{\mathrm{T}}\boldsymbol{PC}-(1-h)\boldsymbol{Q}$, and sliding mode matrix $\boldsymbol{G} = \boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}$, then the sliding mode dynamics Eq.(9) is stable.

Proof Let $N(t) = x(t)-\widetilde{\boldsymbol{C}}x(t-\tau(t))$. Now, consider the following Lyapunov functional candidate for system Eq.(9):

$$V(t) = N^{\mathrm{T}}(t)PN(t) + \int_{t-\tau(t)}^{t} x^{\mathrm{T}}(\theta)Qx(\theta)\mathrm{d}\theta. \tag{11}$$

By differentiating $V(t)$ along with the solutions of Eq.(9), we get

$$\begin{aligned}
\dot{V}(t) =\ & 2\dot{N}^{\mathrm{T}}(t)PN(t) + x^{\mathrm{T}}(t)Qx(t) \\
& -(1-\dot{\tau}(t))x^{\mathrm{T}}(t-\tau(t))Qx(t-\tau(t)) \\
=\ & 2\Big[\widetilde{\boldsymbol{A}}x(t)+\widetilde{\boldsymbol{B}}x(t-\tau(t))\Big]^{\mathrm{T}}\boldsymbol{P} \\
& \times\Big[x(t)-\widetilde{\boldsymbol{C}}x(t-\tau(t))\Big] + x^{\mathrm{T}}(t)\boldsymbol{Q}x(t) \\
& -(1-\dot{\tau}(t))x^{\mathrm{T}}(t-\tau(t))\boldsymbol{Q}x(t-\tau(t)) \\
=\ & 2x^{\mathrm{T}}(t)\widetilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}x(t) - 2x^{\mathrm{T}}(t)\widetilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}\widetilde{\boldsymbol{C}}x(t-\tau(t)) \\
& +2x^{\mathrm{T}}(t-\tau(t))\widetilde{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{P}x(t) \\
& -2x^{\mathrm{T}}(t-\tau(t))\widetilde{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{P}\widetilde{\boldsymbol{C}}x(t-\tau(t)) \\
& +x^{\mathrm{T}}(t)\boldsymbol{Q}x(t) \\
& -(1-\dot{\tau}(t))x^{\mathrm{T}}(t-\tau(t))\boldsymbol{Q}x(t-\tau(t)) \\
=\ & 2x^{\mathrm{T}}(t)\widetilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}x(t) - 2x^{\mathrm{T}}(t)\widetilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{PC}x(t-\tau(t)) \\
& +2x^{\mathrm{T}}(t)\widetilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{PB}(\boldsymbol{GB})^{-1}\boldsymbol{GC}x(t-\tau(t)) \\
& +2x^{\mathrm{T}}(t)\boldsymbol{PA}_{\mathrm{d}}x(t-\tau(t)) \\
& -2x^{\mathrm{T}}(t)\boldsymbol{PB}(\boldsymbol{GB})^{-1}\boldsymbol{GA}_{\mathrm{d}}x(t-\tau(t)) \\
& -2x^{\mathrm{T}}(t-\tau(t))\Big[\boldsymbol{A}_{\mathrm{d}}-\boldsymbol{B}(\boldsymbol{GB})^{-1}\boldsymbol{GA}_{\mathrm{d}}\Big]^{\mathrm{T}}P \\
& \times\Big[\boldsymbol{C}-\boldsymbol{B}(\boldsymbol{GB})^{-1}\boldsymbol{GC}\Big]x(t-\tau(t)) \\
& +x^{\mathrm{T}}(t)\boldsymbol{Q}x(t) \\
& -(1-\dot{\tau}(t))x^{\mathrm{T}}(t-\tau(t))\boldsymbol{Q}x(t-\tau(t)).
\end{aligned} \tag{12}$$

Note that $\boldsymbol{G} = \boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}$, and it follows from Lemma 2

$$\begin{aligned}
& -2x^{\mathrm{T}}(t)\boldsymbol{PB}(\boldsymbol{GB})^{-1}\boldsymbol{GA}_{\mathrm{d}}x(t-\tau(t)) \\
\le\ & x^{\mathrm{T}}(t)\boldsymbol{PB}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{PB})^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}x(t) \\
& +x^{\mathrm{T}}(t-\tau(t))\boldsymbol{A}_{\mathrm{d}}^{\mathrm{T}}\boldsymbol{PA}_{\mathrm{d}}x(t-\tau(t)),
\end{aligned} \tag{13}$$

$$\begin{aligned}
& 2x^{\mathrm{T}}(t)\widetilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{PB}(\boldsymbol{GB})^{-1}\boldsymbol{GC}x(t-\tau(t)) \\
\le\ & x^{\mathrm{T}}(t-\tau(t))C^{\mathrm{T}}\boldsymbol{PB}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{PB})^{-1} \\
& \times\boldsymbol{B}^{\mathrm{T}}\boldsymbol{PC}x(t-\tau(t)) + x^{\mathrm{T}}(t)\widetilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}\widetilde{\boldsymbol{A}}x(t),
\end{aligned} \tag{14}$$

$$\begin{aligned}
& 2x^{\mathrm{T}}(t-\tau(t))\Big[\boldsymbol{B}(\boldsymbol{GB})^{-1}\boldsymbol{GA}_{\mathrm{d}}\Big]^{\mathrm{T}}\boldsymbol{PC}x(t-\tau(t)) \\
\le\ & x^{\mathrm{T}}(t-\tau(t))\boldsymbol{A}_{\mathrm{d}}^{\mathrm{T}}\boldsymbol{PB}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{PB})^{-1} \\
& \times\boldsymbol{B}^{\mathrm{T}}\boldsymbol{PA}_{\mathrm{d}}x(t-\tau(t)) \\
& +x^{\mathrm{T}}(t-\tau(t))\boldsymbol{C}^{\mathrm{T}}\boldsymbol{PC}x(t-\tau(t)),
\end{aligned} \tag{15}$$

$$\begin{aligned}
& 2x^{\mathrm{T}}(t-\tau(t))\boldsymbol{A}_{\mathrm{d}}^{\mathrm{T}}\boldsymbol{P}\Big[\boldsymbol{B}(\boldsymbol{GB})^{-1}\boldsymbol{GC}\Big]x(t-\tau(t)) \\
\le\ & x^{\mathrm{T}}(t-\tau(t))C^{\mathrm{T}}\boldsymbol{PB}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{PB})^{-1} \\
& \times\boldsymbol{B}^{\mathrm{T}}\boldsymbol{PC}x(t-\tau(t)) \\
& +x^{\mathrm{T}}(t-\tau(t))\boldsymbol{A}_{\mathrm{d}}^{\mathrm{T}}\boldsymbol{PA}_{\mathrm{d}}x(t-\tau(t)),
\end{aligned} \tag{16}$$

$$\begin{aligned}
& -2x^{\mathrm{T}}(t-\tau(t))\boldsymbol{A}_{\mathrm{d}}^{\mathrm{T}}\boldsymbol{PB}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{PB})^{-1} \\
& \times\boldsymbol{B}^{\mathrm{T}}\boldsymbol{PC}x(t-\tau(t)) \\
\le\ & x^{\mathrm{T}}(t-\tau(t))\boldsymbol{A}_{\mathrm{d}}^{\mathrm{T}}\boldsymbol{PB}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{PB})^{-1} \\
& \times\boldsymbol{B}^{\mathrm{T}}\boldsymbol{PA}_{\mathrm{d}}x(t-\tau(t)) \\
& +x^{\mathrm{T}}(t-\tau(t))\boldsymbol{C}^{\mathrm{T}}\boldsymbol{PC}x(t-\tau(t)).
\end{aligned} \tag{17}$$

Substituting Eqs.(13)–(17) into Eq.(12) we get

$$\begin{aligned}
\dot{V}(t) \le\ & 2x^{\mathrm{T}}(t)\widetilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}x(t) - 2x^{\mathrm{T}}(t)\widetilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{PC}x(t-\tau(t)) \\
& +2x^{\mathrm{T}}(t)\boldsymbol{PA}_{\mathrm{d}}x(t-\tau(t)) \\
& +x^{\mathrm{T}}(t)\boldsymbol{PB}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{PB})^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}x(t)
\end{aligned}$$

$$
\begin{aligned}
&+x^{\mathrm{T}}(t-\tau(t))\boldsymbol{A}_{\mathrm{d}}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{A}_{\mathrm{d}}x(t-\tau(t))\\
&+x^{\mathrm{T}}(t-\tau(t))\boldsymbol{C}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{B}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{B})^{-1}\\
&\times\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}Cx(t-\tau(t))+x^{\mathrm{T}}(t)\widetilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}\widetilde{\boldsymbol{A}}x(t)\\
&+2x^{\mathrm{T}}(t-\tau(t))\boldsymbol{A}_{\mathrm{d}}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{B}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{B})^{-1}\\
&\times\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{A}_{\mathrm{d}}x(t-\tau(t))\\
&+x^{\mathrm{T}}(t-\tau(t))\boldsymbol{C}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{B}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{B})^{-1}\\
&\times\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{C}x(t-\tau(t))\\
&+x^{\mathrm{T}}(t-\tau(t))\boldsymbol{A}_{\mathrm{d}}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{A}_{\mathrm{d}}x(t-\tau(t))\\
&+2x^{\mathrm{T}}(t-\tau(t))\boldsymbol{C}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{C}x(t-\tau(t))\\
&+x^{\mathrm{T}}(t)\boldsymbol{Q}x(t)\\
&-(1-h)x^{\mathrm{T}}(t-\tau(t))\boldsymbol{Q}x(t-\tau(t))\\
\leq\ &\eta^{\mathrm{T}}(t)\boldsymbol{\Omega}\eta(t), \qquad (18)
\end{aligned}
$$

where

$$
\eta(t)=[x^{\mathrm{T}}(t)\ \ x^{\mathrm{T}}(t-\tau(t))\ \ \dot{x}^{\mathrm{T}}(t-\tau(t))]^{\mathrm{T}},
$$

$$
\boldsymbol{\Omega}=\begin{bmatrix}(1,1) & -2\widetilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{C}+\boldsymbol{P}\boldsymbol{A}_{\mathrm{d}}\\ \star & (2,2)\end{bmatrix}
$$

and

$$
\begin{aligned}
&(1,1)=\widetilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}\widetilde{\boldsymbol{A}}+\boldsymbol{M}+\boldsymbol{Q}+\widetilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}\widetilde{\boldsymbol{A}},\\
&(2,2)=2\boldsymbol{A}_{\mathrm{d}}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{A}_{\mathrm{d}}+2\boldsymbol{C}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{C}\\
&\qquad+2C^{\mathrm{T}}\boldsymbol{M}\boldsymbol{C}+2\boldsymbol{A}_{\mathrm{d}}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{A}_{\mathrm{d}}-(1-h)\boldsymbol{Q},\\
&\boldsymbol{M}=\boldsymbol{P}\boldsymbol{B}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{B})^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}.
\end{aligned}
$$

Hence, if $\boldsymbol{\Omega}<0$, for $\eta(t)\neq 0$, we have

$$
\dot{V}(t)<0. \qquad (19)
$$

Now, by a standard Lyapunov-type theorem in functional differential equations, it can be obtained from Eq.(19) that if there exist matrices $\boldsymbol{P}>0$, $\boldsymbol{Q}>0$ satisfying $\boldsymbol{\Omega}<0$, the sliding mode dynamics Eq.(9) is asymptotically stable. By Schur's complement, $\boldsymbol{\Omega}<0$ is equivalent to the LMI Eq.(10).

3.3. Reachability analysis

In the above section, the sufficient condition was derived for the stability of the system Eq.(9) in the switching surface $s(t)=0$. Now, we proceed to the second task, i.e. to synthesize an SMC law such that the reachability of the specified switching surface is ensured.

Theorem 2 Consider the time-delay neutral system Eq.(1) with assumptions Eqs.(2)–(3). Suppose that the switching function is given as Eq.(4) with $\boldsymbol{G}=\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}$ where $\boldsymbol{P}$ is the solution of LMI Eq.(10). Then, it can be shown that the reachability with probability 1 of the switching surface $s(t)=0$ is guaranteed by the following SMC law:

$$
\begin{aligned}
u(t)=\ &\boldsymbol{K}x(t)-(\boldsymbol{G}\boldsymbol{B})^{-1}\boldsymbol{G}\boldsymbol{A}_{\mathrm{d}}x(t-\tau(t))\\
&-(\boldsymbol{G}\boldsymbol{B})^{-1}\boldsymbol{G}\boldsymbol{C}\dot{x}(t-\tau(t))\\
&-\rho(t)\mathrm{sgn}(s(t)), \qquad (20)
\end{aligned}
$$

where the switching gain $\rho(t)$ is given as

$$
\rho(t)=\lambda+\gamma\|x(t)\|, \qquad (21)
$$

and $\lambda>0$ is a small constant.

Proof It follows from Eqs.(6) and (20),

$$
\dot{s}(t)=\boldsymbol{G}\boldsymbol{B}(-\rho(t)\mathrm{sgn}(s(t))+f(x(t),t)). \qquad (22)
$$

To analyse the reachability, we choose the Lyapunov function candidate as $V(t)=\dfrac{1}{2}\,s^{\mathrm{T}}(t)(\boldsymbol{G}\boldsymbol{B})^{-1}s(t)$. It is noted from Section (I) that $s(t)$ varies finitely. Hence, we have $\dot{V}(t)=s^{\mathrm{T}}(t)(\boldsymbol{G}\boldsymbol{B})^{-1}\dot{s}(t)$. Utilizing expression (22) we got

$$
\begin{aligned}
\dot{V}(t)=\ &s^{\mathrm{T}}(-\rho(t)\mathrm{sgn}(s(t))+f(x(t),t))\\
&-\rho(t)\|s(t)\|_1+\gamma\|s(t)\|\|x(t)\|. \qquad (23)
\end{aligned}
$$

Noting the fact that $\|s(t)\|_1\geq\|s(t)\|$, from Eqs.(21) and (23) we find that

$$
\dot{V}(t)\leq-\lambda\|s(t)\|<0\ \ for\ \|s(t)\|\neq 0. \qquad (24)
$$

This implies that the trajectories of the time-delay neutral system Eq.(1) will be globally driven onto the specified switching surface $s(t)=0$ with probability 1.

We see that the bound of $f(x(t),t)$ is required to synthesize the SMC law Eq.(20). In general, it is difficult to obtain the exact knowledge of the bound in practical application. Hence, an adaptive SMC law is further presented for the case when the bound γ is unavailable.

Theorem 3 Consider the system in Theorem 2, and suppose that the exact value of the bound γ is unavailable. If the adaptive SMC law is given by

$$
\begin{aligned}
u(t)=\ &\boldsymbol{K}x(t)-(\boldsymbol{G}\boldsymbol{B})^{-1}\boldsymbol{G}\boldsymbol{A}_{\mathrm{d}}x(t-\tau(t))\\
&-(\boldsymbol{G}\boldsymbol{B})^{-1}\boldsymbol{G}\boldsymbol{C}\dot{x}(t-\tau(t))\\
&-\hat{\rho}(t)\mathrm{sgn}(s(t)) \qquad (25)
\end{aligned}
$$

where

$$
\hat{\rho}(t)=\lambda+\hat{\gamma}\|x(t)\| \qquad (26)
$$

and parametric updating law is

$$\dot{\hat{\gamma}} = \xi \|s(t)\| \|x(t)\|, \tag{27}$$

here λ, ξ are positive constants, then the state trajectories of the time-delay neutral system will be guaranteed to tend to the sliding surface Eq.(6) and remain there with probability 1.

The proof of Theorem 3 can be obtained similar to the arguments in Theorem 2. Due to limited space, the details are omitted.

Remark It is seen from Theorem 3 that when the bound of γ is unknown, reachability may not happen in finite time.

4. Simulation example

We consider the system (1) with $n = m = 2$. The associated data are

$$\boldsymbol{A} = \begin{bmatrix} -2.6 & 1.5 \\ -1.2 & 0.4 \end{bmatrix}, \quad \boldsymbol{A}_{\mathrm{d}} = \begin{bmatrix} 0.2 & 0.3 \\ -0.1 & 0.1 \end{bmatrix},$$

$$\boldsymbol{B} = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} -0.2 & 0 \\ 0.1 & 1 \end{bmatrix},$$

the nonlinear functions are $f(x(t), t) = [0.3x_1(t) \quad 0.5x_2(t)]^{\mathrm{T}}$, and the time-varying delay $\tau(t) = 0.4\sin(t) + 0.6$ with $d = 1$, $h = 0.4$. Let $\phi(t) = [0 \quad 0]^{\mathrm{T}}$, $t \in [-1, 0]$. Here, we assume that the bound γ of $f(x(t), t)$ is unknown. The problem at hand is to design a sliding mode controller such that the sliding motion in the specified switching surface is stable, and the state trajectories can be driven onto the switching surface. To this end, we select matrix $\boldsymbol{K}$ as follows:

$$\boldsymbol{K} = \begin{bmatrix} -1.5 & 0 \\ -2 & -3.5 \end{bmatrix}.$$

By solving LMI (10), we obtain the following:

$$\boldsymbol{P} = \begin{bmatrix} 0.2534 & -0.3495 \\ -0.3495 & 0.5503 \end{bmatrix}, \quad \boldsymbol{Q} = \begin{bmatrix} 0.1892 & 0.0566 \\ 0.0566 & 0.0413 \end{bmatrix}.$$

The simulation result on $x(t)$ is given in Fig.1. We see that the states are globally asymptotically stable (in probability). It follows from Theorem 3 that the desired SMC law can be chosen as

$$\begin{aligned} u(t) = & \begin{bmatrix} -1.5 & 0 \\ -2 & -3.5 \end{bmatrix} x(t) \\ & - \begin{bmatrix} 0.35 & 0.15 \\ -0.05 & 0.05 \end{bmatrix} x(t - \tau(t)) \\ & - \begin{bmatrix} -8.15 & -3.9 \\ 1.55 & 2.1 \end{bmatrix} \dot{x}(t - \tau(t)) \\ & - \hat{\rho}(t)\mathrm{sgn}(s(t)), \end{aligned} \tag{28}$$

where

$$\hat{\rho}(t) = 0.5 + \hat{\gamma}\|x(t)\| \tag{29}$$

and parametric updating law is

$$\dot{\hat{\gamma}} = 0.1\|s(t)\|\|x(t)\|. \tag{30}$$

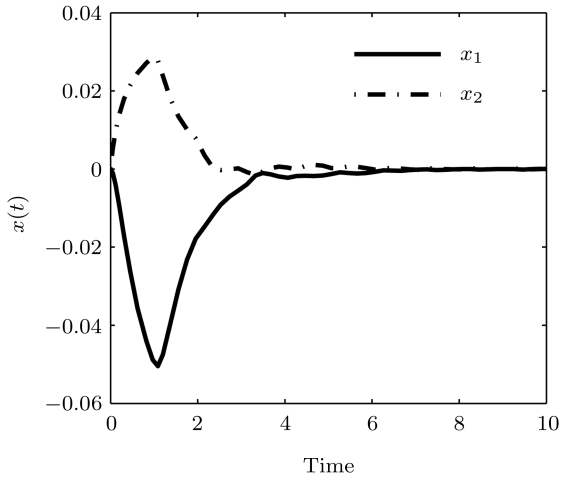


Fig.1. Trajectories of state $x(t)$.

5. Conclusions

The SMC design for nonlinear neutral systems with time-varying delays has been studied. A sufficient condition for the asymptotic stability of sliding motion has been proposed in terms of LMI. A control law for SMC has been synthesized such that the state trajectories of the closed-loop systems are globally driven onto the specified switching surface. Both the integral sliding surface and the SMC law have been obtained by means of the feasibility of LMI. An illustrative example is given to demonstrate the effectiveness of the obtained results.

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