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Direct implementation of a scalable non-local multi-qubit controlled phase gate via optical fibres and adiabatic passage*

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This paper presents a direct implementation scheme of the non-local multi-qubit controlled phase gate by using optical fibres and adiabatic passage. The smaller operation number for implementing the multi-qubit controlled phase gate and needlessness for addressing individually save physical resource and lower the difficulties of experiment. Meanwhile, the scheme is immune from some decoherence effects such as the atomic spontaneous emission and fibre loss. In principle, it is scalable.

Keywords: adiabatic passage, multi-qubit controlled phase gate
PACC: 0367, 4250

1. Introduction

Deterministic multi-qubit phase gate is required for multi-party quantum communication and quantum computation, such as Shor's algorithm for prime factoring,[1] Grover's search algorithm,[2] and distributed quantum computing.[3−5] Although multi-qubit phase gate can be realized by decomposing it into basic quantum gate,[6,7] many physical resources will be wasted and the decoherence time of the system must be long due to the fact that the procedure of decomposing multiqubit gates into elementary gates becomes complicated as the number of qubits increases. So much attention is concentrated on direct implementation of multi-qubit quantum gate.[8−12] In most of previous schemes, all qubits involved in multi-qubit quantum gate are confined in the same region. However, multi-qubit quantum gate with qubits distributed in different regions[13−16] is useful for distributed quantum computation and for quantum communication since it can operate on qubits far from each other.

The stimulated Raman adiabatic passage, which has some advantages that decoherence due to spontaneous emission from excited states can be compressed and it is robust against some experimental parameter errors,[17] has become an amazing and sophisticated technique in coherent population transferring and many other quantum information process, such as preparing entangled state,[18,19] transferring quantum information,[20,21] realizing qubit rotation,[22] quantum teleportation,[23,24] entanglement distillation,[25] and so on. Based on the technology, Goto *et al* proposed a scheme to locally implement a multi-qubit controlled unitary gate.[9] Recently, several proposals have been suggested to engineer entanglement or quantum computation between separate subsystems via optical fibres.[4,14] In this paper, we combine the advantages of adiabatic passage and the scalability brought by using optical fibres to propose a scheme for directly realizing multi-atom non-local controlled phase gate. Similar idea has been used to prepare high-dimensional or multi-qubit entanglement state and implement quantum logic gate.[26−28] Our scheme is constructed by n spatially separated cavities connected by optical fibres, where each cavity traps one atom. Our scheme has the following features: 1) direct implementation of multi-qubit controlled phase gate saves physical resource and reduces the difficulties of experiment; 2) addressing individually is unnecessary, this keeps the scheme easier to realize from the experimental point of view; 3) because the whole process is through dark state, it is free from some decoherence

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effects, such as atomic spontaneous emission and fibre loss; 4) cavities are connected by optical fibres, so it is scalable.

2. Realization of three-qubit controlled phase gate

As shown in Fig.1, three five-level atoms are trapped in three cavities which are connected by fibres, respectively. The lower states $|0\rangle$, $|1\rangle$, and $|d\rangle$ are three stable ground states, while the upper states $|e\rangle$ and $|f\rangle$ are excited states. The qubit is encoded in the states $|0\rangle$ and $|1\rangle$. Assuming that three-atom system is initially in the state $|\Psi\rangle = \prod_{i=1}^{3}(\alpha_i |0\rangle_i + \beta_i |1\rangle_i)$ with $|\alpha_i|^2 + |\beta_i|^2 = 1$, we find that three-qubit controlled phase gate will be realized by the following three steps.

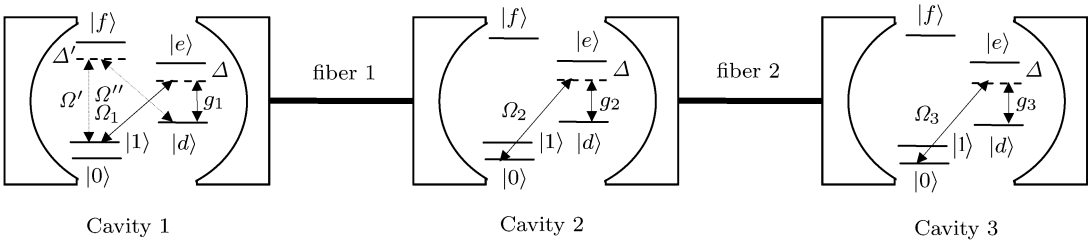


Fig.1. Schematic setup for directly implementing the three-qubit controlled phase gate, where three five-level atoms are trapped in three spatially separated cavities, respectively, and the cavities are connected by two optical fibres.

Step 1 The state $|1\rangle_1$ is firstly transferred to $-|d\rangle_1$ by adiabatic passage. The transitions $|1\rangle \to |f\rangle$ and $|d\rangle \to |f\rangle$ of atom 1 trapped in cavity 1 are driven by two detuned laser fields with Rabi frequencies $\Omega'(t)$ and $\Omega''(t)$. Both detunings of the fields from the excited state $|f\rangle$ are Δ'. The Hamiltonian of the system of atom 1 and cavity 1 in the interaction picture and in the rotating-wave approximation is (setting $\hbar = 1$)

$$\begin{aligned} H'(t) =& \Omega'(t)\mathrm{e}^{-\mathrm{i}\Delta' t}\,|f\rangle\langle 1| \\ &+ \Omega''(t)\mathrm{e}^{-\mathrm{i}\Delta' t}\,|f\rangle\langle d| + \mathrm{H.c.}, \end{aligned} \tag{1}$$

where H.c. denotes the Hermitian conjugate, and the terms describing the coupling between atom 1 and cavity 1 mode have been removed since they have no effect during this process.

This Hamiltonian has the following dark state with the eigenvalue corresponding to zero,

$$|D(t)\rangle \propto \Omega''(t)\,|1\rangle - \Omega'(t)\,|d\rangle\,, \tag{2}$$

which is decoupled from the upper state $|f\rangle$. In the adiabatic condition, if we design the pulse shapes satisfying

$$\lim_{t\to-\infty}\frac{\Omega'(t)}{\Omega''(t)} = 0, \tag{3}$$

$$\lim_{t\to\infty}\frac{\Omega''(t)}{\Omega'(t)} = 0, \tag{4}$$

we can adiabatically transfer the state $|1\rangle$ of atom 1 to $-|d\rangle$, i.e.

$$\begin{aligned} |100\rangle_a &\to -|d00\rangle_a\,, \\ |110\rangle_a &\to -|d10\rangle_a\,, \\ |101\rangle_a &\to -|d01\rangle_a\,, \\ |111\rangle_a &\to -|d11\rangle_a\,, \end{aligned} \tag{5}$$

while the states $|000\rangle_a$, $|010\rangle_a$, $|001\rangle_a$, $|011\rangle_a$ remain unchanged, where $|m_1m_2m_3\rangle_a$ $(m_{1,2,3} = 0, 1, d)$ is the state of three-atom system.

Step 2 The states $-|d00\rangle_a$, $-|d10\rangle_a$, and $-|d01\rangle_a$ are adiabatically appended by the phase factor $\mathrm{e}^{-\mathrm{i}\phi}$. The transition $|1\rangle \to |e\rangle$ of atom 1 is driven by the detuned classical field with Rabi frequency $\Omega_1(t)$ while the transitions $|0\rangle \to |e\rangle$ of atoms 2 and 3 are driven by two detuned classical fields with Rabi frequencies $\Omega_2(t)$ and $\Omega_3(t)$ (see Fig.1), respectively. The single cavity mode couples to the transition $|d\rangle_k \to |e\rangle_k$ with coupling coefficients g_k $(k = 1, 2, 3)$. All detunings of the fields and cavity modes from the excited state $|e\rangle_k$ are Δ. In the interaction picture and in the rotating wave approximation, the total Hamiltonian of the system including three atoms,

two fibres, and three cavities is

$$\begin{aligned} H(t) =& \Omega_1(t)\mathrm{e}^{-\mathrm{i}\Delta t}\left|e\right\rangle_1\left\langle 1\right| + \sum_{k=2,3}\Omega_k(t)\mathrm{e}^{-\mathrm{i}\Delta t}\left|e\right\rangle_k\left\langle 0\right| \\ &+ \sum_{k=1}^{3} g_k a_k \mathrm{e}^{-\mathrm{i}\Delta t}\left|e\right\rangle_k\left\langle d\right| \\ &+ \sum_{k=1,2}\upsilon_k b_k(a_k^\dagger + a_{k+1}^\dagger) + \mathrm{H.c.}, \end{aligned} \tag{6}$$

where a_k and $a_k^\dagger$ are the annihilation operator and the creation operator for photons in the mode of cavity k, respectively, b_k denotes the annihilation operator for photons in the fibre mode k, υ_k represents the coupling strength between the fibre mode k and the interrelated cavity-mode, and the short fibre limit has been taken so that only resonant mode b_k of the fibre interacts with the interrelated cavity-mode.[4,14,20]

Define the excitation number operator

$$\begin{aligned} N_e =& \left|1\right\rangle_1\left\langle 1\right| + \sum_{k=2,3}\left|0\right\rangle_k\left\langle 0\right| + \sum_{k=1,2} b_k^\dagger b_k \\ &+ \sum_{k=1,2,3}(\left|e\right\rangle_k\left\langle e\right| + a_k^\dagger a_k), \end{aligned} \tag{7}$$

which commutes with $H(t)$ so that the excitation number is conserved during the evolution. The Hamiltonian $H(t)$ has the following six dark states:

$$\begin{aligned} \left|D_{000}\right\rangle \propto& (g_2g_3\left|000\right\rangle_a\left|000\right\rangle_c - g_2\Omega_3\left|00d\right\rangle_a\left|100\right\rangle_c + \Omega_2g_3\left|0d0\right\rangle_a\left|100\right\rangle_c + \Omega_2\Omega_3\left|0dd\right\rangle_a\left|110\right\rangle_c \\ &+ g_2\Omega_3\left|00d\right\rangle_a\left|010\right\rangle_c - \Omega_2g_3\left|0d0\right\rangle_a\left|010\right\rangle_c - \Omega_2\Omega_3\left|0dd\right\rangle_a\left|101\right\rangle_c - \frac{1}{\sqrt{2}}\Omega_2\Omega_3\left|0dd\right\rangle_a\left|200\right\rangle_c \\ &- g_2\Omega_3\left|00d\right\rangle_a\left|001\right\rangle_c + \Omega_2g_3\left|0d0\right\rangle_a\left|001\right\rangle_c + \Omega_2\Omega_3\left|0dd\right\rangle_a\left|011\right\rangle_c - \frac{1}{\sqrt{2}}\Omega_2\Omega_3\left|0dd\right\rangle_a\left|020\right\rangle_c \\ &- \frac{1}{\sqrt{2}}\Omega_2\Omega_3\left|0dd\right\rangle_a\left|002\right\rangle_c), \\ \left|D_{010}\right\rangle \propto& (g_3\left|010\right\rangle_a\left|000\right\rangle_c - \Omega_3\left|01d\right\rangle_a\left|100\right\rangle_c + \Omega_3\left|01d\right\rangle_a\left|010\right\rangle_c - \Omega_3\left|01d\right\rangle_a\left|001\right\rangle_c), \\ \left|D_{001}\right\rangle \propto& (g_2\left|001\right\rangle_a\left|000\right\rangle_c + \Omega_2\left|0d1\right\rangle_a\left|100\right\rangle_c - \Omega_2\left|0d1\right\rangle_a\left|010\right\rangle_c + \Omega_2\left|0d1\right\rangle_a\left|001\right\rangle_c), \\ \left|D_{d10}\right\rangle \propto& (-g_3\Omega_1\left|d10\right\rangle_a\left|000\right\rangle_c + \Omega_1\Omega_3\left|d1d\right\rangle_a\left|100\right\rangle_c - \Omega_1\Omega_3\left|d1d\right\rangle_a\left|010\right\rangle_c + \Omega_1\Omega_3\left|d1d\right\rangle_a\left|001\right\rangle_c \\ &- g_1\Omega_3\left|11d\right\rangle_a\left|000\right\rangle_c), \\ \left|D_{d01}\right\rangle \propto& (-g_2\Omega_1\left|d01\right\rangle_a\left|000\right\rangle_c - \Omega_1\Omega_2\left|dd1\right\rangle_a\left|100\right\rangle_c + \Omega_1\Omega_2\left|dd1\right\rangle_a\left|010\right\rangle_c - \Omega_1\Omega_2\left|dd1\right\rangle_a\left|001\right\rangle_c \\ &+ \Omega_2g_1\left|1d1\right\rangle_a\left|000\right\rangle_c), \\ \left|D_{d00}\right\rangle \propto& (-g_2g_3\Omega_1\left|d00\right\rangle_a\left|000\right\rangle_c + \Omega_1g_2\Omega_3\left|d0d\right\rangle_a\left|100\right\rangle_c - \Omega_1\Omega_2g_3\left|dd0\right\rangle_a\left|100\right\rangle_c - \Omega_1\Omega_2\Omega_3\left|ddd\right\rangle_a\left|110\right\rangle_c \\ &- g_1\Omega_2\Omega_3\left|1dd\right\rangle_a\left|100\right\rangle_c - \Omega_1g_2\Omega_3\left|d0d\right\rangle_a\left|010\right\rangle_c + \Omega_1\Omega_2g_3\left|dd0\right\rangle_a\left|010\right\rangle_c + \Omega_1\Omega_2\Omega_3\left|ddd\right\rangle_a\left|101\right\rangle_c \\ &+ g_1\Omega_2\Omega_3\left|1dd\right\rangle_a\left|010\right\rangle_c + \Omega_1g_2\Omega_3\left|d0d\right\rangle_a\left|001\right\rangle_c - \Omega_1\Omega_2g_3\left|dd0\right\rangle_a\left|001\right\rangle_c - \Omega_1\Omega_2\Omega_3\left|ddd\right\rangle_a\left|011\right\rangle_c \\ &- g_1\Omega_2\Omega_3\left|1dd\right\rangle_a\left|001\right\rangle_c - g_1g_2\Omega_3\left|10d\right\rangle_a\left|000\right\rangle_c + g_1\Omega_2g_3\left|1d0\right\rangle_a\left|000\right\rangle_c + \frac{1}{\sqrt{2}}\Omega_1\Omega_2\Omega_3\left|ddd\right\rangle_a\left|200\right\rangle_c \\ &+ \frac{1}{\sqrt{2}}\Omega_1\Omega_2\Omega_3\left|ddd\right\rangle_a\left|020\right\rangle_c + \frac{1}{\sqrt{2}}\Omega_1\Omega_2\Omega_3\left|ddd\right\rangle_a\left|002\right\rangle_c), \end{aligned} \tag{8}$$

where we have discarded the states of two fibre modes since they are always in the vacuum state, $\left|n_1n_2n_3\right\rangle_c$ ($n_{1,2,3} = 0, 1, 2$) denotes the field state with n_1 photons in the mode of cavity 1, n_2 in the mode of cavity 2, n_3 in the mode of cavity 3. Under the conditions $g_1 = g_2 = g_3 = g$ and $g \gg \Omega_k$ ($k = 1, 2, 3$), these dark states become

$$\left|D_{000}\right\rangle \propto \left|000\right\rangle_a, \tag{9}$$

$$\left|D_{010}\right\rangle \propto \left|010\right\rangle_a, \tag{10}$$

$$\left|D_{001}\right\rangle \propto \left|001\right\rangle_a, \tag{11}$$

$$|D_{d10}\rangle \propto -\Omega_1 |d10\rangle_a - \Omega_3 |11d\rangle_a , \quad (12)$$

$$|D_{d01}\rangle \propto -\Omega_1 |d01\rangle_a + \Omega_2 |1d1\rangle_a , \quad (13)$$

$$|D_{d00}\rangle \propto -\Omega_1 |d00\rangle_a + \Omega_2 |1d0\rangle_a - \Omega_3 |10d\rangle_a , \quad (14)$$

where we have also discarded the states of three cavity modes since they are in the vacuum state.

Then, in the adiabatic condition, if we design the pulse shapes gratifying

$$\lim_{t\to-\infty} \frac{\Omega_2}{\Omega_1} = 0, \quad \lim_{t\to-\infty} \frac{\Omega_3}{\Omega_1} = 0, \quad (15)$$

$$\lim_{t\to\infty} \frac{\Omega_1}{\Omega_2} = 0, \quad \lim_{t\to\infty} \frac{\Omega_1}{\Omega_3} = 0, \quad (16)$$

we have

$$\begin{aligned} |D_{d10}\rangle &\xrightarrow{t\to-\infty} \propto -|d10\rangle_a \xrightarrow{t\to\infty} \propto -|11d\rangle_a , \\ |D_{d01}\rangle &\xrightarrow{t\to-\infty} \propto -|d01\rangle_a \xrightarrow{t\to\infty} \propto |1d1\rangle_a , \\ |D_{d00}\rangle &\xrightarrow{t\to-\infty} \propto -|d00\rangle_a \xrightarrow{t\to\infty} \propto \Omega_2 |1d0\rangle_a \\ &\quad - \Omega_3 |10d\rangle_a . \end{aligned} \quad (17)$$

Here and the following, the states (9)–(11) remain unchanged, $|011\rangle_a$ and $|d11\rangle_a$ are not evolved since $H(t)|011\rangle_a |000\rangle_c |00\rangle_f = 0$ and $H(t)|d11\rangle_a |000\rangle_c |00\rangle_f = 0$.

If we design again adiabatically the pulse shapes satisfying

$$\lim_{t\to-\infty} \frac{\Omega_1}{\Omega_2} = 0, \quad \lim_{t\to-\infty} \frac{\Omega_1}{\Omega_3} = 0, \quad (18)$$

$$\lim_{t\to\infty} \frac{\Omega_2}{\Omega_1} = 0, \quad \lim_{t\to\infty} \frac{\Omega_3}{\Omega_1} = 0, \quad (19)$$

we can obtain

$$\begin{aligned} -|11d\rangle_a &\to -e^{-i\phi}|d10\rangle_a , \\ |1d1\rangle_a &\to -e^{-i\phi}|d01\rangle_a , \\ -\Omega_3 |10d\rangle_a + \Omega_2 |1d0\rangle_a &\to -e^{-i\phi}|d00\rangle_a , \end{aligned} \quad (20)$$

where the phase shift $-\phi$ is implemented by the phase shift of $\Omega_1(t)$.[9]

After steps 1 and 2, the state of three-atom system is changed into

$$\begin{aligned} |000\rangle_a &\to |000\rangle_a , \\ |010\rangle_a &\to |010\rangle_a , \\ |001\rangle_a &\to |001\rangle_a , \\ |011\rangle_a &\to |011\rangle_a , \\ |100\rangle_a &\to -e^{-i\phi}|d00\rangle_a , \\ |110\rangle_a &\to -e^{-i\phi}|d10\rangle_a , \\ |101\rangle_a &\to -e^{-i\phi}|d01\rangle_a , \\ |111\rangle_a &\to -|d11\rangle_a . \end{aligned} \quad (21)$$

Step 3 Just the reverse of the first step, we design the pulse shapes satisfying

$$\lim_{t\to-\infty} \frac{\Omega''(t)}{\Omega'(t)} = 0, \quad (22)$$

$$\lim_{t\to\infty} \frac{\Omega'(t)}{\Omega''(t)} = 0, \quad (23)$$

so that the state $-|d\rangle$ of atom 1 is transferred to $e^{i\phi}|1\rangle$, i.e.

$$\begin{aligned} -e^{-i\phi}|d00\rangle_a &\to |100\rangle_a , \\ -e^{-i\phi}|d10\rangle_a &\to |110\rangle_a , \\ -e^{-i\phi}|d01\rangle_a &\to |101\rangle_a , \\ -|d11\rangle_a &\to e^{i\phi}|111\rangle_a , \end{aligned} \quad (24)$$

where the phase shift ϕ is realized by the phase shift of $\Omega''(t)$, other four states $|000\rangle_a$, $|010\rangle_a$, $|001\rangle_a$, and $|011\rangle_a$ remain unchanged.

After the above three steps, we have

$$\begin{aligned} |000\rangle_a &\to |000\rangle_a , \\ |010\rangle_a &\to |010\rangle_a , \\ |001\rangle_a &\to |001\rangle_a , \\ |011\rangle_a &\to |011\rangle_a , \\ |100\rangle_a &\to |100\rangle_a , \\ |110\rangle_a &\to |110\rangle_a , \\ |101\rangle_a &\to |101\rangle_a , \\ |111\rangle_a &\to e^{i\phi}|111\rangle_a . \end{aligned} \quad (25)$$

This corresponds to a three-qubit controlled phase gate for the three atoms respectively trapped in three cavities.

3. Realization of n-qubit controlled phase gate

Next, we extend the scheme to directly implement a n-qubit controlled phase gate. As shown in Fig.2, $n-1$ optical fibres connect n cavities with each cavity trapping a five-level atom. Similar to Sec. 2, after three steps, we can implement an n-qubit controlled phase gate, where the first and last steps are the same as the process of implementing three-qubit controlled phase gate but step 2 is replaced by the following step 2′.

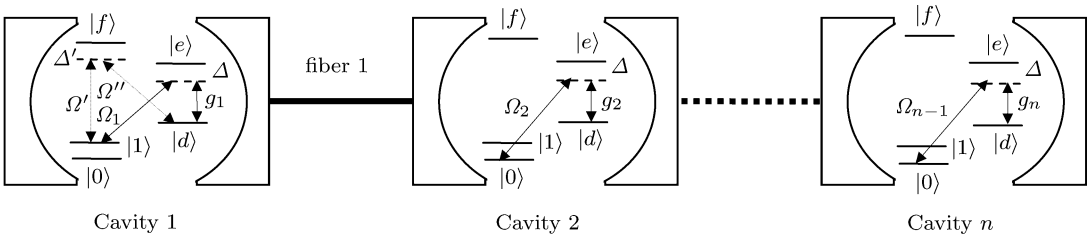


Fig.2. Schematic setup for directly realizing the n-qubit controlled phase gate, where n five-level atoms are trapped in n spatially separated cavities, respectively, and the cavities are linked through optical fibres.

Step 2′ The states $-|dm_2m_3\ldots m_n\rangle_a$ $(m_{2,3,\ldots,n}=0,1)$ are adiabatically appended by the phase factor $\mathrm{e}^{-\mathrm{i}\phi}$ except for the state $-|d11\ldots\rangle_a$. The transition $|1\rangle\rightarrow|e\rangle$ of atom 1 is driven by the detuned classical field with Rabi frequency $\Omega_1(t)$ while the transitions $|0\rangle\rightarrow|e\rangle$ of the others n-1 atoms are driven by detuned classical fields with Rabi frequencies $\Omega_k(t)$ $(k=2,\ldots,n)$, respectively. The single cavity mode couples to the transition $|d\rangle_k\rightarrow|e\rangle_k$ with coupling coefficients g_k $(k=1,\ldots,n)$. All detunings of the fields and cavity modes from the excited state $|e\rangle_k$ are Δ. In the interaction picture and in the rotating wave approximation, the total Hamiltonian of the system including n atoms, $n-1$ fibres, and n cavities is

$$\begin{aligned}H''(t)=&\Omega_1(t)\mathrm{e}^{-\mathrm{i}\Delta t}|e\rangle_1\langle 1|+\sum_{k=2}^{n}\Omega_k(t)\mathrm{e}^{-\mathrm{i}\Delta t}|e\rangle_k\langle 0|\\&+\sum_{k=1}^{n}g_ka_k\mathrm{e}^{-\mathrm{i}\Delta t}|e\rangle_k\langle d|\\&+\sum_{k=1}^{n-1}v_kb_k(a_k^{\dagger}+a_{k+1}^{\dagger})+\mathrm{H.c.}.\end{aligned}\tag{26}$$

Define the excitation number operator

$$N_e=|1\rangle_1\langle 1|+\sum_{k=2}^{n}|0\rangle_k\langle 0|+\sum_{k=1}^{n-1}b_k^{\dagger}b_k+\sum_{k=1}^{n}(|e\rangle_k\langle e|+a_k^{\dagger}a_k),\tag{27}$$

which commutes with $H''(t)$ so that the excitation number is conserved during the evolution. The Hamiltonian has the following dark states:

$$\begin{aligned}|D_{00\ldots0}\rangle\propto&g_2g_3\ldots g_n|0\ldots0\rangle_a|0\ldots0\rangle_c+\Omega_2g_3\ldots g_n(|0d0\ldots0\rangle_a|10\ldots0\rangle_c\\&-|0d0\ldots0\rangle_a|010\ldots0\rangle_c+\ldots)+\ldots\\&+(-1)^ng_2\ldots g_{n-1}\Omega_n(|0\ldots0d\rangle_a|10\ldots0\rangle_c-|0\ldots0d\rangle_a|010\ldots0\rangle_c+\ldots)\\&+\ldots+\Omega_2\ldots\Omega_n[(-1)^{(n-1)(n+2)}|0d\ldots dd\rangle_a|1\ldots10\rangle_c+\ldots\\&+(-1)^{(n-1)(n+6)/2}\frac{1}{\sqrt{n-1}}|0d\ldots dd\rangle_a|n-1,0\ldots0\rangle_c\\&+\ldots+(-1)^{(n-1)(3n+4)/2}\frac{1}{\sqrt{n-1}}|0d\ldots dd\rangle_a|0\ldots0,n-1\rangle_c],\\|D_{01\ldots10}\rangle\propto&g_n|01\ldots10\rangle_a|0\ldots0\rangle_c+(-1)^n\Omega_n(|01\ldots1d\rangle_a|10\ldots0\rangle_c-|01\ldots1d\rangle_a|010\ldots0\rangle_c+\ldots),\\&\ldots,\\|D_{d1\ldots10}\rangle\propto&-\Omega_1g_n|d1\ldots10\rangle_a|0\ldots0\rangle_c+(-1)^{n+1}\Omega_1\Omega_n(|d1\ldots1d\rangle_a|10\ldots0\rangle_c\\&-|d1\ldots1d\rangle_a|010\ldots0\rangle_c+\ldots)+(-1)^ng_1\Omega_n|11\ldots d\rangle_a|0\ldots0\rangle_c,\\&\ldots,\end{aligned}$$

$$
\begin{aligned}
|D_{d0\ldots0}\rangle \propto & -\Omega_1 g_2 \ldots g_n \,|d0\ldots0\rangle_a \,|0\ldots0\rangle_c + g_1 \Omega_2 g_3 \ldots g_n \,|1d0\ldots0\rangle_a \,|0\ldots0\rangle_c + \ldots \\
& + (-1)^n g_1 g_2 \ldots g_{n-1} \Omega_n \,|10\ldots0d\rangle_a \,|0\ldots0\rangle_c \\
& - \Omega_1 \Omega_2 g_3 \ldots g_{n-1} g_n (|dd0\ldots0\rangle_a \,|10\ldots0\rangle_c - |dd0\ldots0\rangle_a \,|01\ldots0\rangle_c + \ldots) + \ldots \\
& + (-1)^{n+1} \Omega_1 g_2 \ldots g_{n-1} \Omega_n (|d0\ldots0d\rangle_a \,|10\ldots0\rangle_c - |d0\ldots0d\rangle_a \,|010\ldots0\rangle_c + \ldots) \\
& - g_1 \Omega_2 \Omega_3 g_4 \ldots g_{n-1} g_n (|1dd0\ldots0\rangle_a \,|10\ldots0\rangle_c - |1dd0\ldots0\rangle_a \,|010\ldots0\rangle_c + \ldots) \\
& + \ldots - \Omega_1 \ldots \Omega_n [(-1)^{(n-1)(n+3)} (|d\ldots d\rangle_a \,|01\ldots1\rangle_c - |d\ldots d\rangle_a \,|101\ldots1\rangle_c + \ldots) \\
& + \ldots + (-1)^{(n-1)(n+6)/2} \frac{1}{\sqrt{n-1}} \,|d\ldots d\rangle_a \,|n-1,0\ldots0\rangle_c + \ldots \\
& + (-1)^{(n-1)(3n+4)/2} \frac{1}{\sqrt{n-1}} \,|d\ldots d\rangle_a \,|0\ldots0,n-1\rangle_c], \qquad (28)
\end{aligned}
$$

where we have omitted the states of $n-1$ fibres since they are always in the vacuum state. Under the conditions $g_1 = g_2 = \ldots = g_n = g$ and $g \gg \Omega_k$ $(k = 1,2,\ldots,n)$, these dark states become

$$
\begin{aligned}
|D_{00\ldots0}\rangle &\propto |00\ldots0\rangle_a \,, \\
|D_{01\ldots10}\rangle &\propto |01\ldots10\rangle_a \,, \\
&\ldots, \\
|D_{d1\ldots10}\rangle &\propto -\Omega_1 \,|d1\ldots10\rangle_a + (-1)^n \Omega_n \,|1\ldots1d\rangle_a \,, \\
&\ldots, \\
|D_{d0\ldots0}\rangle &\propto -\Omega_1 \,|d0\ldots0\rangle_a + \Omega_2 \,|1d0\ldots0\rangle_a - \Omega_3 \,|10d0\ldots0\rangle_a + \ldots + (-1)^n \Omega_n \,|10\ldots0d\rangle_a \,, \qquad (29)
\end{aligned}
$$

where we have discarded the state of $n-1$ fibres and n cavities since they are in the vacuum state.

Then, in the adiabatic condition, if we design the pulse shapes satisfying

$$
\lim_{t\to-\infty} \frac{\Omega_k}{\Omega_1} = 0, \qquad \lim_{t\to\infty} \frac{\Omega_1}{\Omega_k} = 0, \qquad k = 2,\ldots,n, \qquad (30)
$$

we have

$$
\begin{aligned}
|D_{d1\ldots10}\rangle \xrightarrow{t\to-\infty} \propto & -|d1\ldots10\rangle_a \xrightarrow{t\to\infty} \propto (-1)^n \,|1\ldots1d\rangle_a \,, \\
& \ldots, \\
|D_{d0\ldots0}\rangle \xrightarrow{t\to-\infty} \propto & -|d0\ldots0\rangle_a \xrightarrow{t\to\infty} \propto \Omega_2 \,|1d0\ldots0\rangle_a - \Omega_3 \,|10d0\ldots0\rangle_a + \ldots \\
& + (-1)^n \Omega_n \,|10\ldots0d\rangle_a \,. \qquad (31)
\end{aligned}
$$

Here and the following, the states $|0m_2m_3\ldots m_n\rangle_a$ $(m_{2,3,\ldots,n} = 0,1)$ remain unchanged except for the state $|01\ldots1\rangle_a$. $|01\ldots1\rangle_a$ and $|d1\ldots1\rangle_a$ are not evolved since $H''(t)\,|01\ldots1\rangle_a\,|0\ldots0\rangle_c\,|0\ldots0\rangle_f = 0$ and $H''(t)\,|d1\ldots1\rangle_a\,|0\ldots0\rangle_c\,|0\ldots0\rangle_f = 0$.

If we design again adiabatically the pulse shapes satisfying

$$
\lim_{t\to-\infty} \frac{\Omega_1}{\Omega_k} = 0, \quad \lim_{t\to\infty} \frac{\Omega_k}{\Omega_1} = 0, k = 2,\ldots,n, \qquad (32)
$$

we can gain

$$
\begin{aligned}
& -|1\ldots1d\rangle_a \to -\mathrm{e}^{-\mathrm{i}\phi} \,|d1\ldots10\rangle_a \,, \\
& \ldots, \\
& \Omega_2 \,|1d0\ldots0\rangle_a - \Omega_3 \,|10d0\ldots0\rangle_a + \ldots \\
& +(-1)^n \Omega_n \,|10\ldots0d\rangle_a \to -\mathrm{e}^{-\mathrm{i}\phi} \,|d0\ldots0\rangle_a \,, \qquad (33)
\end{aligned}
$$

where the phase shift ϕ is implemented by the phase shift of $\Omega_1(t)$.

After all the three steps, similar to Sec. 2, only the state $|11\ldots1\rangle$ obtains a phase $\mathrm{e}^{\mathrm{i}\phi}$, the other states are unchanged. Thus we have realized the n-qubit controlled phase gate for n atoms respectively trapped in n cavities.

4. Discussion and conclusion

Figure 3 shows the time dependence of the quantum basis state's coefficient for the three-qubit controlled-phase (with phase shift equals to π) gate, where the initial state of three-atom is $\frac{1}{2\sqrt{2}}(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle)$. Rabi frequencies of Gaussian shape and of full width at half maximum T_P=100 ns are chosen, the couplings are parametrized by $\Omega'_{\max}/2\pi = \Omega''_{\max}/2\pi = \Omega_{1\,\max}/2\pi = \Omega_{2\,\max}/2\pi$=5 MHz, $\Omega_{3\,\max}/2\pi$=3 MHz, and $g/2\pi$=34 MHz, which satisfies the condition $g \gg \Omega_k$ (k = 1,2,...,n) and can be presently obtained experimentally with recent technologies.[29,30] As expected, the state $|111\rangle$ finally achieves the phase π while other seven states $|000\rangle_a$, $|010\rangle_a$, $|001\rangle_a$, $|011\rangle_a$, $|100\rangle_a$, $|110\rangle_a$, and $|101\rangle_a$ are unchanged. Obviously, this scheme can be almost perfectly implemented under the adiabatic conditions $\Omega_{k,\max} \cdot T_P$, $g_k \cdot T_P \gg 1$, $\Omega_{k,\max}$, $g_k \gg \kappa$,$1/\tau$, where κ and $1/\tau$ are the cavity decay rate and the spectral linewidth of the excited atomic states, respectively. Remarkably, the decomposition of the multiqubit gates into the elementary gates requires a number of gate steps,[31,32] for example, n-qubit controlled unitary gates can be decomposed into $(2^n - 3)$ two-qubit controlled gates.[31] However, the realization of multi-qubit controlled phase gate only needs three steps in our scheme. The smaller step number for implementing the multi-qubit controlled phase gate saves physical resource and reduces the difficulties of experiment, and the advantages become even more significant with an increasing number of the qubits.

In summary, we propose a scalable scheme to directly realize the n-qubit phase gate via optical fibres and adiabatic passage. The implementation of n-qubit controlled phase gate in our scheme is much more efficient than decomposing into the elementary gates. Addressing individually is not necessary, which further lowers the complexity of experiment. In addition, this scheme is robust against decoherence effects due to spontaneous emission from the excited state and the fibre loss.

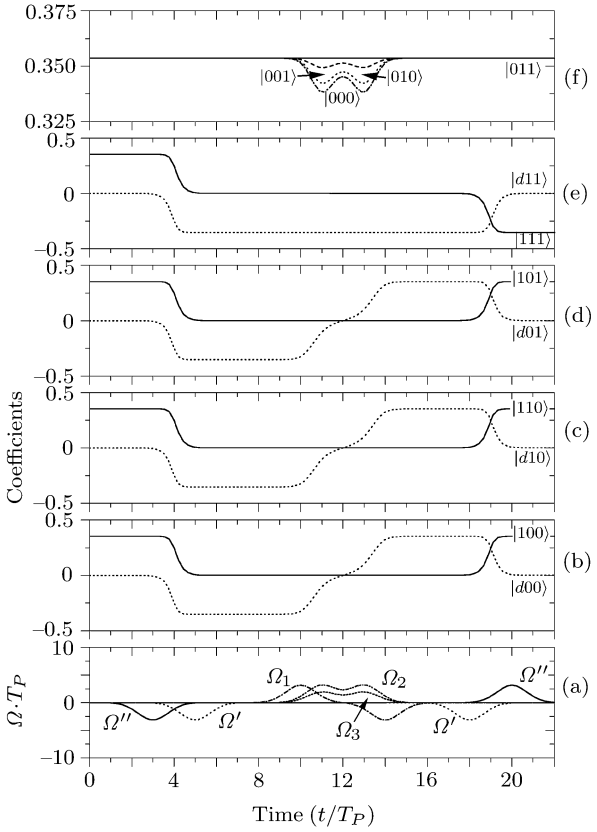


Fig.3. Coefficients' evolution of three-qubit controlled-phase gate. (a) The Rabi frequencies: solid line $\Omega''(t)$; dotted line $\Omega'(t)$; dashed dotted line Ω_1; dashed dotted line Ω_2; short dashed line Ω_3. (b)–(f) are the time evolution of the coefficients associated with all the states.

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