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N-particles intermediate coordinate-momentum representation and its application*

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This paper constructs the new common eigenvectors of n intermediate coordinate-momentum operators which are complete and orthonormal. The intermediate coordinate-momentum representation of a multi-particles system is proposed and applied to a general$n$-mode quantum harmonic oscillators system with coordinate-momentum coupling.

Keywords: n-particles, intermediate coordinate-momentum representation, quantum harmonic oscillator

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1. Introduction

The representation theory and its applications had been studied widely in Refs.[1–12]. But, the intermediate coordinate-momentum representation for a multi-particles system has not been published yet. In this paper, the new intermediate coordinate-momentum representation of a multi-particles system is proposed and applied to a general n-mode quantum harmonic oscillators system with coordinate-momentum coupling. First, we construct $2n$ intermediate coordinate-momentum operators $\hat{X}_1,\ \hat{X}_2,\ \hat{X}_3,\ \ldots,\ \hat{X}_n$ and $\hat{P}_1,\ \hat{P}_2,\ \hat{P}_3,\ldots,\hat{P}_n$ for a n-particles system as follows

$$\hat{X}_i = \lambda_i \hat{x}_i + \nu_i \hat{p}_i,\ \hat{P}_i = \tau_i \hat{p}_i + \sigma_i \hat{x}_i,\quad i = 1,2,3,\ldots,n, \tag{1}$$

where $\lambda_i,\ \nu_i,\ \tau_i$ and σ_i are real numbers. If

$$\lambda_i \tau_i - \sigma_i \nu_i = 1, \tag{2}$$

it is easy to see that $\hat{P}_1,\ \hat{P}_2,\ \hat{P}_3,\ldots,\hat{P}_n$ are linearly independent, and $\hat{X}_1,\ \hat{X}_2,\ \hat{X}_3,\ldots,\hat{X}_n$ are linearly independent, which satisfy the following relations

$$[\hat{X}_i, \hat{P}_j] = \mathrm{i}\delta_{ij},\quad i,j = 1,2,3,\ldots,n. \tag{3}$$

We are challenged to search for the common eigenvectors $|\eta_1,\ \eta_2,\ \eta_3,\ldots,\eta_n\rangle \equiv |\boldsymbol{\eta}\rangle_{\lambda,\nu}$ of the intermediate coordinate-momentum operators $\{\hat{X}_1,\ \hat{X}_2,\ \hat{X}_3,\ldots,\hat{X}_n\}$. By virtue of the technique of integration within an ordered product (IWOP) of operators[11], we show that $|\boldsymbol{\eta}\rangle_{\lambda,\nu}$ makes a complete representation and can be used for solving some new n-particles dynamic problems.

2. Common eigenvectors of the intermediate coordinate-momentum operators $\{\hat{X}_1,\ \hat{X}_2,\ \hat{X}_3,\ \ldots,\ \hat{X}_n\}$

We begin with introducing the n-mode Fock space spanned by

$|l_1,\ l_2,\ l_3,\ldots,\ l_n\rangle = (\hat{a}_1^{\dagger l_1}\ \hat{a}_2^{\dagger l_2}\ \hat{a}_3^{\dagger l_3}\ \ldots\ \hat{a}_n^{\dagger l_n}/\sqrt{l_1!l_2!l_3!\ldots l_n!})\,|000\ldots0\rangle$, where $[\hat{a}_i,\ \hat{a}_j^{\dagger}] = \delta_{ij}$, $|000\ldots0\rangle \equiv |\mathbf{0}\rangle$ is the ground state, $\hat{a}_i$ and $\hat{a}_i^{\dagger}$ are related to $\hat{x}_i$ and $\hat{p}_i$ by

$$\hat{x}_i = \frac{\hat{a}_i + \hat{a}_i^{\dagger}}{\sqrt{2}},\quad \hat{p}_i = \frac{\hat{a}_i - \hat{a}_i^{\dagger}}{\mathrm{i}\sqrt{2}}. \tag{4}$$

We shall prove that the common eigenvectors $|\boldsymbol{\eta}\rangle_{\lambda,\nu}$ of $\{\hat{X}_1,\ \hat{X}_2,\ \hat{X}_3,\ldots,\hat{X}_n\}$ are given in n-mode Fock space by

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$$|\boldsymbol{\eta}\rangle_{\lambda,\nu}=\left[\pi\prod_{i=1}^{n}(\lambda_i^2+\nu_i^2)\right]^{-n/4}\exp\left\{-\frac{1}{2}\sum_{i=1}^{n}\frac{\eta_i^2}{\lambda_i^2+\nu_i^2}+\sqrt{2}\sum_{i=1}^{n}\frac{\eta_i}{\lambda_i-\mathrm{i}\nu_i}\hat{\boldsymbol{a}}_i^{\dagger}-\frac{1}{2}\sum_{i=1}^{n}\frac{\lambda_i+\mathrm{i}\nu_i}{\lambda_i-\mathrm{i}\nu_i}\hat{a}_i^{\dagger 2}-\mathrm{i}\frac{1}{2}\sum_{i=1}^{n}\frac{\lambda_i\sigma_i+\tau_i\nu_i}{\lambda_i^2+\nu_i^2}\eta_i^2\right\}|\mathbf{0}\rangle\,. \tag{5}$$

In fact, acting $\hat{a}_i$ on $|\boldsymbol{\eta}\rangle_{\lambda,\nu}$, we have

$$\hat{\boldsymbol{a}}_i\,|\boldsymbol{\eta}\rangle_{\lambda,\nu}=\left[\frac{\sqrt{2}\eta_i-(\lambda_i+\mathrm{i}\nu_i)\hat{\boldsymbol{a}}_i^{\dagger}}{\lambda_i-\mathrm{i}\nu_i}\right]|\boldsymbol{\eta}\rangle_{\lambda,\nu}\,,\quad i=1,2,3,\ldots,n. \tag{6}$$

The equation (6) leads to

$$\hat{\boldsymbol{X}}_i\,|\boldsymbol{\eta}\rangle_{\lambda,\nu}=\eta_i\,|\boldsymbol{\eta}\rangle_{\lambda,\nu}\,,\ i=1,2,3,\ldots n. \tag{7}$$

Thus we know that $|\boldsymbol{\eta}\rangle_{\lambda,\nu}$ is the common eigenvector of $\{\hat{\boldsymbol{X}}_1,\ \hat{\boldsymbol{X}}_2,\ \hat{\boldsymbol{X}}_3,\ldots,\hat{\boldsymbol{X}}_n\}$.

3. The completeness relation of $|\boldsymbol{\eta}\rangle_{\lambda,\nu}$

We now examine if $|\boldsymbol{\eta}\rangle_{\lambda,\nu}$ satisfies a completeness relation. By virtue of IWOP technique and the normal ordering form of the n-mode vacuum projector

$$|\mathbf{0}\rangle\langle\mathbf{0}|=:\exp\left\{-\sum\nolimits_{i=1}^{n}\hat{a}_i^{\dagger}\hat{a}_i\right\}:, \tag{8}$$

we can easily perform the following integration:

$$\begin{aligned}&\int\cdots\int_{-\infty}^{\infty}\left(\prod_{i=1}^{n}\mathbf{d}\eta_i\right)|\boldsymbol{\eta}\rangle_{\lambda,\nu\,\lambda,\nu}\langle\boldsymbol{\eta}|\\&=\left[\pi\prod_{i=1}^{n}(\lambda_i^2+\nu_i^2)\right]^{-n/2}\int\cdots\int_{-\infty}^{\infty}\left(\prod_{i=1}^{n}\mathbf{d}\eta_i\right):\exp\left\{-\sum_{i=1}^{n}\frac{\eta_i^2}{\lambda_i^2+\nu_i^2}+\sqrt{2}\sum_{i=1}^{n}\left(\frac{\eta_i}{\lambda_i-\mathrm{i}\nu_i}\hat{\boldsymbol{a}}_i^{\dagger}\right.\right.\\&\left.\left.+\frac{\eta_i}{\lambda_i+\mathrm{i}\nu_i}\hat{\boldsymbol{a}}_i\right)-\frac{1}{2}\sum_{i=1}^{n}\left(\frac{\lambda_i+\mathrm{i}\nu_i}{\lambda_i-\mathrm{i}\nu_i}\hat{\boldsymbol{a}}_i^{\dagger 2}+\frac{\lambda_i-\mathrm{i}\nu_i}{\lambda_i+\mathrm{i}\nu_i}\hat{\boldsymbol{a}}_i^{2}\right)-\sum_{i=1}^{n}\hat{\boldsymbol{a}}_i^{\dagger}\hat{\boldsymbol{a}}_i\right\}:=1.\end{aligned} \tag{9}$$

Using Eq.(7), we can also calculate

$$_{\lambda,\nu}\langle\boldsymbol{\eta}'|\,\hat{\boldsymbol{X}}_i\,|\boldsymbol{\eta}\rangle_{\lambda,\nu}=(\eta_i')_{\lambda,\nu}\langle\boldsymbol{\eta}'|\boldsymbol{\eta}\rangle_{\lambda,\nu}=(\eta_i)_{\lambda,\nu}\langle\boldsymbol{\eta}'|\boldsymbol{\eta}\rangle_{\lambda,\nu}\,,\quad(i=1,2,\ldots,n) \tag{10}$$

which tell us that $|\boldsymbol{\eta}\rangle_{\lambda,\nu}$ are orthonormal

$$_{\lambda,\nu}\langle\boldsymbol{\eta}'|\boldsymbol{\eta}\rangle_{\lambda,\nu}=\prod_{i=1}^{n}\delta(\eta_i'-\eta_i). \tag{11}$$

From Eqs.(9) and (11), we see that $|\boldsymbol{\eta}\rangle_{\lambda,\nu}$ are qualified for making up an intermediate coordinate-momentum representation.

Furthermore, we can obtain

$$_{\lambda,\nu}\langle\boldsymbol{\eta}|\,\hat{\boldsymbol{P}}_i=-\mathrm{i}\frac{\partial}{\partial\eta_i}{}_{\lambda,\nu}\langle\boldsymbol{\eta}|\,,\ (i=1,2,3,\ldots,n). \tag{12}$$

4. Simultaneously squeezing unitary transformation for $\hat{X}_i, i=1,2,3,\ldots,n$

We construct the following integral form unitary operator

$$U=\int\cdots\int_{-\infty}^{\infty}\left(\prod_{i=1}^{n}\mathrm{d}\eta_i\right)\sqrt{\prod_{i=1}^{n}\kappa_i}\,|\kappa_1\eta_1,\kappa_2\eta_2,\kappa_3\eta_3,\ldots,\kappa_n\eta_n\rangle_{\lambda,\nu}\langle\boldsymbol{\eta}|, \tag{13}$$

where $\kappa_1,\ \kappa_2,\ldots$ and κ_n are n independent positive numbers. The unitarity of U can be seen from Eq.(11) by calculating

$$\begin{aligned}\boldsymbol{U}\boldsymbol{U}^{\dagger}=&\left(\prod_{i=1}^{n}\kappa_i\right)\int\cdots\int_{-\infty}^{\infty}\left(\prod_{i=1}^{n}\mathrm{d}\eta_i\mathrm{d}\eta_i'\right)|\kappa_1\eta_1,\kappa_2\eta_2,\ldots,\kappa_n\eta_n\rangle\cdot{}_{\lambda,\nu}\langle\eta|\,|\eta'\rangle_{\lambda,\nu}\langle\kappa_1\eta_1',\kappa_2\eta_2',\ldots,\kappa_n\eta_n'|\\ =&\left(\prod_{i=1}^{n}\kappa_i\right)\int\cdots\int_{-\infty}^{\infty}\left(\prod_{i=1}^{n}\mathrm{d}\eta_i\mathrm{d}\eta_i'\right)|\kappa_1\eta_1,\kappa_2\eta_2,\ldots,\kappa_n\eta_n\rangle\langle\kappa_1\eta_1',\kappa_2\eta_2',\ldots,\kappa_n\eta_n'|\prod_{i=1}^{n}\delta(\eta_i'-\eta_i)\\ =&\left(\prod_{i=1}^{n}\kappa_i\right)\int\cdots\int_{-\infty}^{\infty}\left(\prod_{i=1}^{n}\mathrm{d}\eta_i\right)|\kappa_1\eta_1,\kappa_2\eta_2,\ldots,\kappa_n\eta_n\rangle\langle\kappa_1\eta_1,\kappa_2\eta_2,\ldots,\kappa_n\eta_n|=1.\end{aligned} \tag{14}$$

Using Eq.(7) and the IWOP technique, we can obtain

$$\begin{aligned}\boldsymbol{U}\hat{\boldsymbol{X}}_1\boldsymbol{U}^{\dagger}=&\left(\prod_{i=1}^{n}\kappa_i\right)\int\cdots\int_{-\infty}^{\infty}\left(\prod_{i=1}^{n}\mathrm{d}\eta_i\mathrm{d}\eta_i'\right)|\kappa_1\eta_1,\kappa_2\eta_2,\ldots,\kappa_n\eta_n\rangle\,{}_{\lambda,\nu}\langle\eta|\cdot\hat{\boldsymbol{X}}_1\,|\eta'\rangle_{\lambda,\nu}\langle\kappa_1\eta_1',\kappa_2\eta_2',\ldots,\kappa_n\eta_n'|\\ =&\left(\prod_{i=1}^{n}\kappa_i\right)\int\cdots\int_{-\infty}^{\infty}\left(\prod_{i=1}^{n}\mathrm{d}\eta_i\mathrm{d}\eta_i'\right)|\kappa_1\eta_1,\kappa_2\eta_2,\ldots,\kappa_n\eta_n\rangle\,{}_{\lambda,\nu}\langle\eta|\cdot\eta_1'\,|\eta'\rangle_{\lambda,\nu}\langle\kappa_1\eta_1',\kappa_2\eta_2',\ldots,\kappa_n\eta_n'|\\ =&\left(\prod_{i=1}^{n}\kappa_i\right)\int\cdots\int_{-\infty}^{\infty}\left(\prod_{i=1}^{n}\mathrm{d}\eta_i\mathrm{d}\eta_i'\right)|\kappa_1\eta_1,\kappa_2\eta_2,\ldots,\kappa_n\eta_n\rangle\cdot\eta_1'\left(\prod_{i=1}^{n}\delta(\eta_i'-\eta_i)\right)\langle\kappa_1\eta_1',\kappa_2\eta_2',\ldots,\kappa_n\eta_n'|\\ =&\left(\prod_{i=1}^{n}\kappa_i\right)\int\cdots\int_{-\infty}^{\infty}\eta_1\left(\prod_{i=1}^{N}\mathrm{d}\eta_i\right)|\kappa_1\eta_1,\kappa_2\eta_2,\ldots,\kappa_n\eta_n\rangle\langle\kappa_1\eta_1,\kappa_2\eta_2,\ldots,\kappa_n\eta_n|\\ =&\kappa_1^{-1}\hat{\boldsymbol{X}}_1,\end{aligned} \tag{15}$$

where we have used the mathematical formula $\displaystyle\int_{-\infty}^{\infty}x\mathrm{e}^{-\alpha x^2+\beta x}\mathrm{d}x=\frac{\beta}{2\alpha}\sqrt{\frac{\pi}{\alpha}}\mathrm{e}^{\frac{\beta^2}{4\alpha}}$, $\mathrm{Re}(\alpha)>0$.

Similarly, we have

$$U\hat{X}_2U^{\dagger}=\kappa_2^{-1}\hat{X}_2,\quad U\hat{X}_3U^{\dagger}=\kappa_3^{-1}\hat{X}_3,\ldots,U\hat{X}_nU^{\dagger}=\kappa_n^{-1}\hat{X}_n. \tag{16}$$

From Eqs.(15) and (16), we can see that U is indeed an operator simultaneously squeezing $\hat{X}_1$, $\hat{X}_2$, $\hat{X}_3$,... and $\hat{X}_n$.

5. The eigenvalue and eigenfunction of n-mode coupling quantum harmonic oscillator Hamiltonian

Consider a general n-mode quantum harmonic oscillator system with coordinate-momentum coupling whose Hamiltonian reads

$$\hat{H}=\sum\nolimits_{i=1}^{n}\left(\frac{\hat{p}_i^2}{2m_i}+\frac{1}{2}m_i\omega_i^2\hat{x}_i^2\right)+\sum_{i=1}^{n}\gamma_i(\hat{x}_i\hat{p}_i+\hat{p}_i\hat{x}_i)=\boldsymbol{\chi}D\boldsymbol{\chi}^{\dagger}, \tag{17}$$

where $\boldsymbol{\chi}=\left(\hat{p}_1\ \hat{p}_2\ \ldots\ \hat{p}_n\ m_1\omega_1\hat{x}_1\ m_2\omega_2\hat{x}_2\ \ldots\ m_n\omega_n\hat{x}_n\right)$, γ_i are all real numbers,

$$D=\begin{pmatrix}
\frac{1}{2m_1} & 0 & \cdots & \cdots & 0 & \frac{\gamma_1}{m_1\omega_1} & 0 & 0 & \cdots & 0\\
0 & \frac{1}{2m_2} & 0 & \cdots & 0 & 0 & \frac{\gamma_2}{m_2\omega_2} & 0 & \cdots & 0\\
\vdots & 0 & \ddots & \vdots & \vdots & \vdots & 0 & \ddots & \vdots & \vdots\\
\vdots & \vdots & \vdots & \ddots & 0 & \vdots & \vdots & \cdots & \cdots & 0\\
0 & 0 & \cdots & 0 & \frac{1}{2m_n} & 0 & 0 & \cdots & 0 & \frac{\gamma_n}{m_n\omega_n}\\
\frac{\gamma_1}{m_1\omega_1} & 0 & \cdots & \cdots & 0 & \frac{1}{2m_1} & 0 & \cdots & \cdots & 0\\
0 & \frac{\gamma_2}{m_2\omega_2} & 0 & \cdots & \cdots & 0 & \frac{1}{2m_2} & 0 & \cdots & 0\\
\vdots & 0 & \ddots & \ddots & \vdots & \cdots & \ddots & \ddots & \cdots & \vdots\\
\vdots & \vdots & \vdots & \ddots & 0 & \cdots & \cdots & \cdots & \ddots & 0\\
0 & \cdots & \cdots & 0 & \frac{\gamma_n}{m_n\omega_n} & 0 & \cdots & \cdots & 0 & \frac{1}{2m_n}
\end{pmatrix}_{2n\times 2n}.$$

D is a $2n\times 2n$ symmetrical matrix of quadratic form. We construct the following concrete unitary matrix $\tilde{A}$ for $\hat{H}$ by virtue of mathematical theory

$$\tilde{A}=\begin{pmatrix}
\frac{1}{\sqrt{2}} & 0 & \cdots & \cdots & 0 & \frac{1}{\sqrt{2}} & 0 & \cdots & \cdots & 0\\
0 & \frac{1}{\sqrt{2}} & 0 & \cdots & \vdots & 0 & \frac{1}{\sqrt{2}} & 0 & \cdots & \vdots\\
\vdots & 0 & \ddots & \ddots & \vdots & \vdots & 0 & \ddots & \ddots & \vdots\\
\vdots & \vdots & \ddots & \ddots & 0 & \vdots & \vdots & \ddots & \ddots & 0\\
0 & \vdots & & 0 & \frac{1}{\sqrt{2}} & 0 & \vdots & & \ddots & \frac{1}{\sqrt{2}}\\
\frac{1}{\sqrt{2}} & 0 & & & 0 & \frac{-1}{\sqrt{2}} & 0 & & & 0\\
0 & \frac{1}{\sqrt{2}} & 0 & \cdots & \vdots & 0 & \frac{-1}{\sqrt{2}} & \ddots & \ddots & \vdots\\
\vdots & 0 & \ddots & \ddots & \vdots & \vdots & 0 & \ddots & \ddots & \vdots\\
\vdots & \vdots & \ddots & \ddots & 0 & \vdots & & \ddots & \ddots & 0\\
0 & 0 & \cdots & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & \cdots & 0 & \frac{-1}{\sqrt{2}}
\end{pmatrix}_{2n\times 2n}.$$

Let $\boldsymbol{\chi}^\dagger=\tilde{A}\boldsymbol{\chi}'^\dagger$, where $\boldsymbol{\chi}'=\begin{pmatrix}\hat{P}_1 & \hat{P}_2 & \ldots & \hat{P}_n & m_1\omega_1\hat{X}_1 & m_2\omega_2\hat{X}_2 & \ldots & m_n\omega_n\hat{X}_n\end{pmatrix}$, then

$$\begin{aligned}\hat{H}=&\boldsymbol{\chi}\boldsymbol{D}\boldsymbol{\chi}^\dagger=\boldsymbol{\chi}'\boldsymbol{A}\boldsymbol{D}\tilde{\boldsymbol{A}}\boldsymbol{\chi}'^\dagger\\ =&\sum_{i=1}^{n}\left(\frac{\hat{P}_i^2}{2\mu_i}+\frac{1}{2}\mu_i\Omega_i^2\hat{X}_i^2\right),\end{aligned}\tag{18}$$

where

$$\begin{aligned}\hat{X}_i=&\frac{1}{\sqrt{2}}\hat{x}_i-\frac{1}{\sqrt{2}m_i\omega_i}\hat{p}_i,\\ \hat{P}_i=&\frac{1}{\sqrt{2}}\hat{p}_i+\frac{m_i\omega_i}{\sqrt{2}}\hat{x}_i,\end{aligned}$$

$$\begin{aligned}\mu_i=&m_i\frac{\omega_i}{\omega_i+2\gamma_i},\\ \Omega_i=&\sqrt{\omega_i^2-4\gamma_i^2},\ i=1,2,3,4,\ldots,n.\end{aligned}$$

That is to say, we choose $\lambda_i=\frac{1}{\sqrt{2}}$, $\nu_i=-\frac{1}{\sqrt{2}m_i\omega_i}$, $\tau_i=\frac{1}{\sqrt{2}}$, $\sigma_i=\frac{m_i\omega_i}{\sqrt{2}}$, which satisfy Eq.(2). Sandwiching Eq.(18) between ${}_{\lambda,\nu}\langle\boldsymbol{\eta}|$ and eigenstate $|E_j\rangle$ of $\hat{\boldsymbol{H}}$,

we obtain

$$\sum_{i=1}^{n}\left(-\frac{1}{2\mu_i}\frac{\partial^2}{\partial\eta_1^2}\varphi_j+\frac{1}{2}\mu_i\Omega_i^2\eta_i^2\varphi_j\right)=E_j\varphi_j, \quad (19)$$

where $\varphi_j \equiv {}_{\lambda,\nu}\langle\boldsymbol{\eta}|\, E_j\rangle$ is the eigenfunction. Thus the eigenvalues and eigenfunctions of the Schrödinger equation Eq.(19) in ${}_{\lambda,\nu}\langle\boldsymbol{\eta}|$ representation are respectively

$$\begin{aligned}\boldsymbol{E}_{j_1j_2\ldots j_n}=&\sum_{i=1}^{n}\left(\boldsymbol{j}_i+\frac{1}{2}\right)\hbar\Omega_i,\\ \varphi_{j_1j_2\ldots j_n}=&\prod_{i=1}^{n}\varphi_{j_i}(\eta_i),\ \varphi_{j_i}(\eta_i)\\ =&\boldsymbol{N}_{j_i}\exp(-\alpha_i\eta_i^2/2)\boldsymbol{H}_{j_i}(\alpha_i\eta_i),\\ j_i=&0,1,2,3,\ldots,\end{aligned}$$

where $\alpha_i=\sqrt{\mu_i\Omega_i/\hbar}$. To obtain the eigenfunctions in $\langle x_1,\ x_2,\ldots,x_n|$ representation, we now calculate the overlap

$$\begin{aligned}\langle x_1,x_2,\ldots,x_n|\boldsymbol{\eta}\rangle_{\lambda,\nu}=&\int\ldots\int\frac{\prod_{i=1}^{n}(\mathrm{d}^2z_i)}{\pi^n}\langle x_1,x_2,\ldots,x_n\mid z_1,z_2,\ldots,z_n\rangle\langle z_1,z_2,\ldots,z_n\mid\boldsymbol{\eta}\rangle_{\lambda,\nu}\\ =&\frac{\sqrt{\prod_{i=1}^{n}(m_i\omega_i)}}{(\sqrt{2}\pi\hbar)^{n/2}}\exp\left\{\mathrm{i}\sum_{i=1}^{n}\left(\frac{m_i\omega_i}{2\hbar}x_i^2\right)-\mathrm{i}\sum_{\mathrm{i}=1}^{n}\left(\frac{\sqrt{2}m_i\omega_i}{\hbar}x_i\eta_i\right)+\mathrm{i}\sum_{i=1}^{n}\left(\frac{m_i\omega_i}{2\hbar}\eta_i^2\right)\right\},\end{aligned} \quad (20)$$

where we have used the over-completeness of n-mode coherent states and recovered Planck constant $\hbar$ and omitted the complex number in the normalized coefficient whose modulus is 1. Then we obtain

$$\begin{aligned}\varphi_j(\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_n)=&\langle\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_n|\boldsymbol{E}_{j_1j_2\ldots j_n}\rangle\\ =&\int\ldots\int_{-\infty}^{\infty}\left(\prod_{i=1}^{n}\mathbf{d}\eta_i\right)\langle\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_n|\eta\rangle_{\lambda,\nu}\,{}_{\lambda,\nu}\langle\eta|\boldsymbol{E}_{j_1j_2\ldots j_n}\rangle\\ =&\prod_{i=1}^{n}\left\{\sqrt{\frac{\boldsymbol{m}_i\omega_i}{\sqrt{2}\pi\hbar}}\boldsymbol{N}_{j_i}\exp\left(\mathrm{i}\frac{\boldsymbol{m}_i\omega_i}{2\hbar}\boldsymbol{x}_i^2\right)\boldsymbol{F}_{j_i}(\boldsymbol{x}_i)\right\},\end{aligned} \quad (21)$$

where

$$\begin{aligned}\boldsymbol{F}_{j_i}(\boldsymbol{x}_i)=&\int_{-\infty}^{\infty}\mathbf{d}\eta_i\exp\left\{\mathrm{i}\frac{\boldsymbol{m}_i\omega_i}{2\hbar}\eta_i^2-\mathrm{i}\frac{\sqrt{2}\boldsymbol{m}_i\omega_i}{\hbar}\boldsymbol{x}_i\eta_i-\frac{\alpha_i^2}{2}\eta_i^2\right\}\boldsymbol{H}_{j_i}(\alpha_i\eta_i)\\ =&\frac{\mathbf{d}^{j_i}}{\mathbf{d}t^{j_i}}\int_{-\infty}^{\infty}\mathbf{d}\eta_i\exp\left\{\mathrm{i}\frac{\boldsymbol{m}_i\omega_i}{2\hbar}\eta_i^2-\mathrm{i}\frac{\sqrt{2}\boldsymbol{m}_i\omega_i}{\hbar}x_i\eta_i-\frac{\alpha_i^2}{2}\eta_i^2+2\alpha_i\eta_i\boldsymbol{t}-\boldsymbol{t}^2\right\}\Bigg|_{t=0}\\ =&\sqrt{\frac{2\pi\hbar}{\alpha_i^2\hbar-\mathrm{i}\boldsymbol{m}_i\omega_i}}\frac{\mathbf{d}^{j_i}}{\mathbf{d}t^{j_i}}\exp\left\{\frac{(2\alpha_i\hbar\boldsymbol{t}-\mathrm{i}\sqrt{2}\boldsymbol{m}_i\omega_i\boldsymbol{x}_i)^2}{2(\alpha_i^2\hbar^2-\mathrm{i}\boldsymbol{m}_i\omega_i\hbar)}-\boldsymbol{t}^2\right\}\Bigg|_{t=0},\end{aligned} \quad (22)$$

and we have used the mathematical formula $H_{j_i}(\xi)=\dfrac{\mathrm{d}^{j_i}}{\mathrm{d}t^{j_i}}\exp\{2\xi t-t^2\}\Big|_{t=0}$.

The front three $\boldsymbol{F}_{j_i}(\boldsymbol{x}_i)$ have forms as follows:

$$\begin{aligned}F_0(x_i)=&\sqrt{\frac{2\pi\hbar}{\mu_i\Omega_i-\mathrm{i}m_i\omega_i}}\exp\left\{\frac{-2m_i^2\omega_i^2x_i^2}{\mu_i\Omega_i\hbar-\mathrm{i}m_i\omega_i\hbar}\right\},\\ F_1(x_i)=&\sqrt{\frac{2\pi\hbar}{\mu_i\Omega_i-\mathrm{i}m_i\omega_i}}\frac{-\mathrm{i}2\sqrt{2\mu_i\Omega_i/\hbar}m_i\omega_ix_i}{\mu_i\Omega_i-\mathrm{i}m_i\omega_i}\exp\left\{\frac{-2m_i^2\omega_i^2x_i^2}{\mu_i\Omega_i\hbar-\mathrm{i}m_i\omega_i\hbar}\right\},\\ F_2(x_i)=&\sqrt{\frac{2\pi\hbar}{\mu_i\Omega_i-\mathrm{i}m_i\omega_i}}\frac{2\hbar(\mu_i^2\Omega_i^2+m_i^2\omega_i^2)-8\mu_i\Omega_im_i^2\omega_i^2x_i^2}{\hbar(\mu_i\Omega_i-\mathrm{i}m_i\omega_i)^2}\exp\left\{\frac{-2m_i^2\omega_i^2x_i^2}{\mu_i\Omega_i\hbar-\mathrm{i}m_i\omega_i\hbar}\right\},\\ &\ldots.\end{aligned} \quad (23)$$

In particular, when $\gamma_i = 0$, then $\mu_i = m_i$, $\Omega_i = \omega_i$, $\alpha_i = \sqrt{m_i\omega_i/\hbar}$, we obtain

$$
\begin{aligned}
|\varphi_0(\boldsymbol{x}_i)|^2 =& \sqrt{\frac{\boldsymbol{m}_i\omega_i}{\pi\hbar}}\exp\left(-\frac{\boldsymbol{m}_i\omega_i}{\hbar}\boldsymbol{x}_i^2\right),\\
|\varphi_1(\boldsymbol{x}_i)|^2 =& \sqrt{\frac{\boldsymbol{m}_i\omega_i}{\pi\hbar}}\frac{2\boldsymbol{m}_i\omega_i}{\hbar}\boldsymbol{x}_i^2\exp\left(-\frac{\boldsymbol{m}_i\omega_i}{\hbar}\boldsymbol{x}_i^2\right),\\
|\varphi_2(\boldsymbol{x}_i)|^2 =& \sqrt{\frac{\boldsymbol{m}_i\omega_i}{\pi\hbar}}\frac{1}{2}\left(\frac{2\boldsymbol{m}_i\omega_i}{\hbar}\boldsymbol{x}_i^2-1\right)^2\exp\left(-\frac{\boldsymbol{m}_i\omega_i}{\hbar}\boldsymbol{x}_i^2\right),\\
&\ldots,
\end{aligned}
\tag{24}
$$

which is just the situation of the quantum harmonic oscillator system without coupling.

In summary, we have found an intermediate coordinate-momentum representation $|\boldsymbol{\eta}\rangle_{\lambda,\nu}$ in n-mode Fock space, which possesses orthonormal and completeness relation, and provides a new approach for solving some dynamic problems of n-particles systems with coordinate-momentum interaction. The IWOP technique plays an essential role in our derivations.

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