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Noether symmetries of discrete mechanico-electrical systems*

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This paper focuses on studying Noether symmetries and conservation laws of the discrete mechanico-electrical systems with the nonconservative and the dissipative forces. Based on the invariance of discrete Hamilton action of the systems under the infinitesimal transformation with respect to the generalized coordinates, the generalized electrical quantities and time, it presents the discrete analogue of variational principle, the discrete analogue of Lagrange–Maxwell equations, the discrete analogue of Noether theorems for Lagrange–Maxwell and Lagrange mechanico-electrical systems. Also, the discrete Noether operator identity and the discrete Noether-type conservation laws are obtained for these systems. An actual example is given to illustrate these results.

Keywords: Noether symmetry, conservation law, discrete mechanico-electrical system, dissipative function

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1. Introduction

Lie group theory has long been used to study differential equations. It has developed into a powerful tool to solve differential equations, to classify them and to establish properties of their solution space. These aspects of Lie group theory have been described in many books and lecture notes.[1−17]

Applications of Lie group theory to discrete equations, such as difference equations, differential-difference equations, are much more recent.[18−37] Several different approaches are pursued. One approach is considering the transformation of the dependent variables only, while the independent variables are fixed, and consequently so is the lattice.[18,19] The restriction is very strong and excludes most of the symmetries of the physical problems. The second approach is studying a difference, or differential-difference equation, together with a fixed lattice, on which the discrete variables take their values.[28,29] In this approach it is necessary to go beyond point symmetries and to consider Lie group actions at different point of the lattice simultaneously. This method leads to the deformations of the admitted Lie algebra with certain losses in useful properties, it is applicable mainly to linear, linearizable or integrable equations. The third approach deals with symmetries of a differential equation and introduces a difference equation together with a symmetry-adapted mesh in such a way that all the symmetries of the original differential equation are preserved.[22−27] Here we use the third approach.

In this paper, we study the Noether symmetry of the discrete mechanico-electrical dynamical systems with the nonconservative and the dissipative forces. Our point of view is to pose the question: how does one discretize a Lagrange–Maxwell equation while preserving all of its Lie point symmetries for the mechanico-electrical systems. Here we start from the Lagrange–Maxwell equation and find that its Lie algebra are a priori given. We then look for a difference Lagrange–Maxwell equations and a time mesh that have same symmetry group and the same symmetry algebra. The use of symmetry group methods thus leads to a natural and powerful technique for driving an uniform mesh discretizations of Lagrange–Maxwell equations.

We know that the structure of the symmetry group essentially affects the construction of ordinary equations and mesh. Group transformations can dis-

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tort the geometric structure of a mesh which influences the approximation and other properties of the difference Lagrange–Maxwell equations. In this paper, we construct the difference lattice based on the symmetries of Dorodnitsyn's method.[22−24]

The structure of this paper is as follows. In Section 2 we define the operators of discrete transformation and the operators of discrete differentiation to the right and left, and introduce on the uniform mesh of the discrete Lagrange function for the mechanico-electrical systems to give the discrete analogue of variational principle. In Section 3 we present the discrete analogue of Noether theorems and discrete Noether-type conservation laws for Lagrange–Maxwell and Lagrange mechanico-electrical systems. In Section 4 we also discuss an example of circuit of plank capacitor to obtain the discrete Lagrange–Maxwell equations, discrete gauge function and conservation laws associate with this system. Finally some conclusions are drawn in the Section 5.

2. Discrete analogue of variational principle

In a mechanico-electrical system, the mechanical part, constituted by N particles, is described by n generalized coordinates q^i $(i = 1, \ldots, n)$, and the electrodynamical part, constituted by m electric circuits, is described by the model of electrics. Consider a mechanico-electrical system composed of m circuits consisting of linear conductors and capacitors. For circuit j, we denote by i^j the current, by u_j the electric potentials, by e^j $(\dot{e}^j = i^j)$ the charge in the capacitor, by R_j the resistance, and by C_j the capacitance.

We consider the space z of sequence $(t, q^i, e^j, \dot{q}^i, \dot{e}^j)$, and in the space we define a map (differentiation), acting according to the rule: $D(t) = 1, D\left(q^i\right) = \dot{q}^i, D\left(e^j\right) = \dot{e}^j, D\left(\dot{q}^i\right) = \ddot{q}^i, \quad D\left(\dot{e}^j\right) = \ddot{e}^j, \ldots$. Identify D with the action of the first order linear differential operator

$$D = \frac{\partial}{\partial t} + \dot{q}^i \frac{\partial}{\partial q^i} + \dot{e}^j \frac{\partial}{\partial e^j} + \ddot{q}^i \frac{\partial}{\partial \dot{q}^i} + \ddot{e}^j \frac{\partial}{\partial \dot{e}^j} + \ldots . \quad (1)$$

We fix an arbitrary value of the parameter $h > 0$ and with the help of the tangent field (1) we form a pair of the operators which we shall call, respectively, the operators of discrete transformation to the right and left:

$$S_{+h} = e^{hD} = \sum_{s=0}^{\infty} \frac{h^s}{s!} D^s,$$

$$S_{-h} = e^{-hD} = \sum_{s=0}^{\infty} \frac{(-h)^s}{s!} D^s, \quad (2)$$

the operators S_{+h}, S_{-h} commute with one another and satisfy $S_{+h} \cdot S_{-h} = S_{-h} \cdot S_{+h} = 1$.

Using the simplest invariant mesh h (see Fig.1) in the t-direction, we introduce a pair of linear difference operators S_{-h} and S_{+h} defined by

$$\begin{aligned} S_{-h} q^i &= q^{i-}, \quad S_{+h} q^i = q^{i+}, \\ S_{-h} e^j &= e^{j-}, \quad S_{+h} e^j = e^{j+}, \end{aligned} \quad (3)$$

and S_{-h} and S_{+h} are corresponding shift operators (or the operators of discrete translation to the right and left) . Moreover, with the help of S_{-h} and S_{+h} we form a pair of operators of discrete (finite-difference) differentiation to the right and left:

$$\begin{aligned} D_{+h} &= \frac{S_{+h} - 1}{h} = \sum_{s=1}^{\infty} \frac{h^{(s-1)}}{s!} D^s, \\ D_{-h} &= \frac{1 - S_{-h}}{h} = \sum_{s=1}^{\infty} \frac{(-h)^{(s-1)}}{s!} D^s \end{aligned} \quad (4)$$

The operators S_{+h}, S_{-h}, D_{+h} and D_{-h} commute in any combination, while $D_{+h} = D_{-h} S_{+h}, \quad D_{-h} = D_{+h} S_{-h}$.

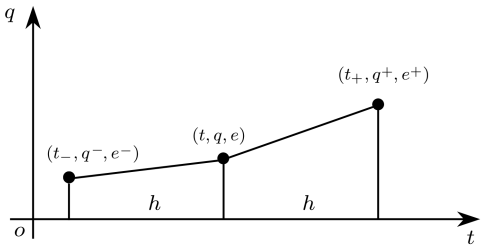


Fig.1. Simplest invariant mesh for a three-point difference equation.

We introduce the infinitesimal transformations with respect to time, generalized coordinates and electrical quantities as

$$\begin{aligned} t^* &= t + \varepsilon \xi_0 \left(t, q^i, e^j\right), \\ q^{i*} &= q^i + \varepsilon \xi_i \left(t, q^i, e^j\right), \\ e^{j*} &= e^j + \varepsilon \eta_j \left(t, q^i, e^j\right) \end{aligned} \quad (5)$$

where $\xi_0 \left(t, q^i, e^j\right)$, $\xi_i \left(t, q^i, e^j\right)$ and $\eta_j \left(t, q^i, e^j\right)$ constitute the Lie group of infinitesimal transformations. For obtaining the discrete Noether-type conserved quantities, with the invariant mesh h, we introduce the infinitesimal transformation operator (i.e. generalized the Noether-type operator identity):

$$X = \xi_0 \frac{\partial}{\partial t} + \xi_i \frac{\partial}{\partial q^i} + \eta_i \frac{\partial}{\partial e^j}$$

$$+\left(D_{+h}\left(\xi_i\right)-q_k^i D_{+h}\left(\xi_0\right)\right)\frac{\partial}{\partial q_k^j}$$
$$+\left(D_{+h}\left(\eta_j\right)-e_k^j D_{+h}\left(\xi_0\right)\right)\frac{\partial}{\partial e_k^j}$$
$$+hD_{+h}\left(\xi_0\right)\frac{\partial}{\partial h}. \tag{6}$$

We will initially use the simplest invariant mesh which is regular with the constant steps h.

On this mesh the discrete Lagrange function for a mechanico-electrical system is

$$L_{\mathrm{d}}=T\left(q^i,q_k^i\right)-V\left(q^i\right)+W_{\mathrm{m}}\left(q^i,e_k^i\right)-W_{\mathrm{e}}\left(q^i,e^i\right), \tag{7}$$

where T and V are, respectively, the discrete kinetic energy and the discrete potential energy, $W_{\mathrm{e}}=\left(e^j\right)^2/2C_j$ the electric field energy, $W_{\mathrm{m}}=L_{j\beta}i_j i_\beta/2\quad(j,\beta=1,\ldots,m)$ the magnetic field energy, and $L_{j\beta}(j\neq\beta)$ is the inductance on circuit j due to circuit β and L_{jj} is self inductance of circuit j. The discrete dissipative function for the system is $F_{\mathrm{d}}=F_e\left(e_k^j\right)+F_m\left(q^i,q_k^i\right)$. In Eq.(6) and the expression of F

$$q_k^i=D_{+h}\left(q^i\right)=\frac{q^{i+}-q^i}{t_+-t}=\frac{q^{i+}-q^i}{h},$$

$$e_k^j=D_{+h}\left(e^j\right)=\frac{e^{j+}-e^j}{t_+-t}=\frac{e^{j+}-e^j}{h} \tag{8}$$

are the corresponding right difference derivatives.

For a positive integer k, the discrete action of the mechanico-electrical system is the map defined by

$$M_{\mathrm{d}}=\sum L_{\mathrm{d}}\left(t,q^i,e^j,q_k^i,e_k^j\right)h. \tag{9}$$

Discrete Hamilton action M_{d} of the systems under the infinitesimal transformation (5) is

$$M_{\mathrm{d}}^*=\sum L_{\mathrm{d}}\left(t^*,q^{i*},e^{j*},q_k^{i*},e_k^{j*}\right)h. \tag{10}$$

Now the authors calculate variation of the discrete Hamilton action M_{d} as

$$\begin{aligned}\Delta M_{\mathrm{d}}&=M_{\mathrm{d}}^*-M_{\mathrm{d}}\\&\approx\sum\left[L_{\mathrm{d}}\left(t^*,q^{i*},e^{j*},q_k^{i*},e_k^{j*}\right)h\right.\\&\quad\left.-L_{\mathrm{d}}\left(t,q^i,e^j,q_k^i,e_k^j\right)h\right].\end{aligned} \tag{11}$$

We use generalized Noether-type operator identity (6) and direct calculations by applying a specific difference Leibnitz rule which is given in Refs.[11–13] to get

$$\begin{aligned}&\xi_0\frac{\partial L_{\mathrm{d}}}{\partial t}+\xi_i\frac{\partial L_{\mathrm{d}}}{\partial q^i}+\eta_j\frac{\partial L_{\mathrm{d}}}{\partial e^j}+\left(D_{+h}\left(\xi_i\right)-q_k^iD_{+h}\left(\xi_0\right)\right)\frac{\partial L_{\mathrm{d}}}{\partial q_k^i}+\left(D_{+h}\left(\eta_j\right)-e_k^jD_{+h}\left(\xi_0\right)\right)\frac{\partial L_{\mathrm{d}}}{\partial e_k^j}+L_{\mathrm{d}}D_{+h}\left(\xi_0\right)\\&=\xi_0\left\{\frac{\partial L_{\mathrm{d}}}{\partial t}+D_{-h}\left(q_k^i\frac{\partial L_{\mathrm{d}}}{\partial q_k^i}+e_k^j\frac{\partial L_{\mathrm{d}}}{\partial e_k^j}-L_{\mathrm{d}}\right)\right\}+\xi_i\left\{\frac{\partial L_{\mathrm{d}}}{\partial q^i}-D_{-h}\left(\frac{\partial L_{\mathrm{d}}}{\partial q_k^i}\right)\right\}+\eta_j\left\{\frac{\partial L_{\mathrm{d}}}{\partial e^j}-D_{-h}\left(\frac{\partial L_{\mathrm{d}}}{\partial e_k^j}\right)\right\}\\&\quad+D_{+h}\left\{\xi_0L_{\mathrm{d}}^-+\left(\xi_i-\xi_0S_{-h}\left(q_k^i\right)\right)\frac{\partial L_{\mathrm{d}}^-}{\partial q_k^i}\left(\eta_j-\xi_0S_{-h}(e_k^j)\right)\frac{\partial L_{\mathrm{d}}^-}{\partial e_k^j}\right\}.\end{aligned} \tag{12}$$

3. Discrete analogue of Noether symmetries transformation

Suppose $Q_{\mathrm{d}}^i-\dfrac{\partial F_{\mathrm{d}}}{\partial q_k^i}$ are the generalized non-potential forces of the discrete mechanico-electrical system, and $u_{\mathrm{d}}^j-\dfrac{\partial F_{\mathrm{d}}}{\partial e_k^j}$ the generalized electrokinetic potentials of kth return circuits for the discrete system. If the Lagrangian of the systems satisfies the condition that Hamilton action of the discrete mechanico-electrical system is generalized quasi-invariant with respect to the infinitesimal transformation (5), i.e.

$$\begin{aligned}&\sum L_{\mathrm{d}}\left(t,q^i,e^j,q_k^i,e_k^j\right)h\\&=\sum L_{\mathrm{d}}\left(t^*,q^{i*},e^{j*},q_k^{i*},e_k^{j*}\right)h\\&\quad+\sum\left\{\left(Q_{\mathrm{d}}^i-\frac{\partial F_{\mathrm{d}}}{\partial q_k^i}\right)\delta_{\mathrm{d}}q^i\Delta t\right.\\&\quad\left.+\left(u_{\mathrm{d}}^j-\frac{\partial F_{\mathrm{d}}}{\partial e_k^j}\right)\delta_{\mathrm{d}}e^j+D_{+h}\left(G\right)\right\}h,\end{aligned} \tag{13}$$

then we give the definition as follows:

Definition 3 If Hamilton action of the discrete mechanico-electrical systems is generalized quasi-invariant under the infinitesimal transformation (5), i.e. the infinitesimal transformations satisfy

$$\Delta M_{\mathrm{d}}=-\sum\left\{D_{+h}\left(G\right)+\left(Q_{\mathrm{d}}^i-\frac{\partial F_{\mathrm{d}}}{\partial q_k^i}\right)\delta_{\mathrm{d}}q^i\right.$$

$$+\left(u_{\mathrm{d}}^{j}-\frac{\partial F_{\mathrm{d}}}{\partial e_{k}^{j}}\right)\delta_{\mathrm{d}}e^{j}\Bigg\}h, \tag{14}$$

where the $G=G\left(t,q^{i},e^{j}\right)$ is the discrete gauge function, $\left(Q_{\mathrm{d}}^{i}-\dfrac{\partial F_{\mathrm{d}}}{\partial q_{k}^{i}}\right)\delta q^{i}$ is the discrete analogue of summation of the virtual work for non-potential generalize forces, and $\left(u_{\mathrm{d}}^{j}-\dfrac{\partial F_{\mathrm{d}}}{\partial e_{k}^{j}}\right)\delta e^{j}$ is the analogue of summation of the virtual work for generalize electromotive forces. We call the infinitesimal transformation (5) the discrete analogue of the generalized Noether quasi-symmetrical transformation. To apply the difference analogue of the Noether theorem for the system, we generalize the Noether-type operator identity to give

$$\begin{aligned}
&\xi_{0}\frac{\partial L_{\mathrm{d}}}{\partial t}+\xi_{i}\frac{\partial L_{\mathrm{d}}}{\partial q^{i}}+\eta_{j}\frac{\partial L_{\mathrm{d}}}{\partial e^{j}}+\left[D_{+h}\left(\xi_{i}\right)-q_{k}^{i}D_{+h}\left(\xi_{0}\right)\right]\frac{\partial L_{\mathrm{d}}}{\partial q_{k}^{i}}+\left[D_{+h}\left(\eta_{j}\right)-e_{k}^{j}D_{+h}\left(\xi_{0}\right)\right]\frac{\partial L_{\mathrm{d}}}{\partial e_{k}^{j}}\\
&+L_{\mathrm{d}}D_{+h}\left(\xi_{0}\right)+\left(Q_{\mathrm{d}}^{i}-\frac{\partial F_{\mathrm{d}}}{\partial q_{k}^{i}}\right)\left(\xi_{i}-q_{k}^{i}\xi_{0}\right)+\left(u_{\mathrm{d}}^{j}-\frac{\partial F_{\mathrm{d}}}{\partial e_{k}^{j}}\right)\left(\eta_{j}-e_{k}^{j}\xi_{0}\right)+D_{+h}\left(G\right)\\
=&\ \xi_{0}\left\{\frac{\partial L_{\mathrm{d}}}{\partial t}+D_{-h}\left(q^{i}\frac{\partial L_{\mathrm{d}}}{\partial q_{k}^{i}}+e^{j}\frac{\partial L_{\mathrm{d}}}{\partial e_{k}^{j}}-L_{\mathrm{d}}\right)\right\}+\xi_{i}\left\{\frac{\partial L_{\mathrm{d}}}{\partial q^{i}}-D_{-h}\left(\frac{\partial L_{\mathrm{d}}}{\partial q_{k}^{i}}\right)+\left(Q_{\mathrm{d}}^{i}-\frac{\partial F_{\mathrm{d}}}{\partial q_{k}^{i}}\right)\right\}\\
&+\eta_{j}\left\{\frac{\partial L_{\mathrm{d}}}{\partial e^{j}}-D_{-h}\left(\frac{\partial L_{\mathrm{d}}}{\partial e_{k}^{j}}\right)+\left(u_{\mathrm{d}}^{j}-\frac{\partial F_{\mathrm{d}}}{\partial e_{k}^{j}}\right)\right\}\\
&+D_{+h}\left\{\xi_{0}L_{\mathrm{d}}^{-}+\left(\xi_{i}-\xi_{0}S_{-h}\left(q_{k}^{i}\right)\right)\frac{\partial L_{\mathrm{d}}^{-}}{\partial q_{k}^{i}}(\eta_{j}-\xi_{0}S_{-h}(e_{k}^{j}))\frac{\partial L_{\mathrm{d}}^{-}}{\partial e_{k}^{j}}+G\right\},
\end{aligned} \tag{15}$$

where all derivatives are difference derivatives.

If the left-hand side of Eq.(15) equals 0, i.e.

$$\begin{aligned}
&\xi_{0}\frac{\partial L_{\mathrm{d}}}{\partial t}+\xi_{i}\frac{\partial L_{\mathrm{d}}}{\partial q^{i}}+\eta_{j}\frac{\partial L_{\mathrm{d}}}{\partial e^{j}}+\left[D_{+h}\left(\xi_{i}\right)-q_{k}^{i}D_{+h}\left(\xi_{0}\right)\right]\frac{\partial L_{\mathrm{d}}}{\partial q_{k}^{i}}+\left[D_{+h}\left(\eta_{j}\right)-e_{k}^{j}D_{+h}\left(\xi_{0}\right)\right]\frac{\partial L_{\mathrm{d}}}{\partial e_{k}^{j}}\\
&+\left(Q_{\mathrm{d}}^{i}-\frac{\partial F_{\mathrm{d}}}{\partial q_{k}^{i}}\right)\left(\xi_{i}-q_{k}^{i}\xi_{0}\right)+\left(u_{\mathrm{d}}^{j}-\frac{\partial F_{\mathrm{d}}}{\partial e_{k}^{j}}\right)\left(\eta_{j}-e_{k}^{j}\xi_{0}\right)+L_{\mathrm{d}}D_{+h}\left(\xi_{0}\right)+D_{+h}\left(G\right)=0,
\end{aligned} \tag{16}$$

the Eq.(16) is called the discrete analogue of Noether operator identity for this mechanico-electrical systems.

Here the right-hand side of Eq.(15) equals 0, and then we obtain some results as follows.

When there exists some other equation

$$\begin{aligned}
&\xi_{0}\left\{\frac{\partial L_{\mathrm{d}}}{\partial t}+D_{-h}\left(q^{i}\frac{\partial L_{\mathrm{d}}}{\partial q_{k}^{i}}+e^{j}\frac{\partial L_{\mathrm{d}}}{\partial e_{k}^{j}}-L_{\mathrm{d}}\right)\right\}+\xi_{i}\left\{\frac{\partial L_{\mathrm{d}}}{\partial q^{i}}-D_{-h}\left(\frac{\partial L_{\mathrm{d}}}{\partial q_{k}^{i}}\right)+\left(Q_{\mathrm{d}}^{i}-\frac{\partial F_{\mathrm{d}}}{\partial q_{k}^{i}}\right)\right\}\\
&+\eta_{j}\left\{\frac{\partial L_{\mathrm{d}}}{\partial e^{j}}-D_{-h}\left(\frac{\partial L_{\mathrm{d}}}{\partial e_{k}^{j}}\right)+\left(u_{\mathrm{d}}^{j}-\frac{\partial F_{\mathrm{d}}}{\partial e_{k}^{j}}\right)\right\}=0,
\end{aligned} \tag{17}$$

then we have the difference Lagrange–Maxwell equations

$$D_{-h}\left(\frac{\partial L_{\mathrm{d}}}{\partial q_{k}^{i}}\right)-\frac{\partial L_{\mathrm{d}}}{\partial q^{i}}+\frac{\partial F_{\mathrm{d}}}{\partial q_{k}^{i}}=Q_{\mathrm{d}}^{i}, \tag{18}$$

$$D_{-h}\left(\frac{\partial L_{\mathrm{d}}}{\partial e_{k}^{j}}\right)-\frac{\partial L_{\mathrm{d}}}{\partial e^{j}}+\frac{\partial F_{\mathrm{d}}}{\partial e_{k}^{j}}=u_{\mathrm{d}}^{j}, \tag{19}$$

and energy equation

$$\frac{\partial L_{\mathrm{d}}}{\partial t}+D_{-h}\left(q^{i}\frac{\partial L_{\mathrm{d}}}{\partial q_{k}^{i}}+e^{j}\frac{\partial L_{\mathrm{d}}}{\partial e_{k}^{j}}-L_{\mathrm{d}}\right)=0. \tag{20}$$

and this system possesses the conservation law

$$D_{+h}\left\{\xi_{0}L_{\mathrm{d}}^{-}+\left(\xi_{i}-\xi_{0}q_{k}^{-i}\right)\frac{\partial L_{\mathrm{d}}^{-}}{\partial q_{k}^{i}}\right.$$

$$+(\eta_j - \xi_0 e_k^j)\frac{\partial L_{\rm d}^-}{\partial e_k^j} + G\Big\} = 0, \tag{21}$$

namely

$$I_{\rm d} = \xi_0 L_{\rm d}^- + \left(\xi_i - \xi_0 q_k^{-i}\right)\frac{\partial L_{\rm d}^-}{\partial q_k^i}$$
$$+(\eta_j - \xi_0 e_k^j)\frac{\partial L_{\rm d}^-}{\partial e_k^j} + G = \text{const}, \tag{22}$$

the difference equations (21) and (22) are called the difference conservation laws associated with such a difference mechanico-electrical system. The difference equations (21) and (22) form the invariant schemes on an uniform regular mesh h and thus coincide with the difference conservation laws.

4. Discrete analogue of Noether theorems of the mechanico-electrical systems

We have obtained that a discrete mechanico-electrical system, with the generalized nonconservation forces $Q_{\rm d}^i - \dfrac{\partial F_{\rm d}}{\partial q_k^i}$ and $u_{\rm d}^j - \dfrac{\partial F_{\rm d}}{\partial e_k^j}$, is governed by Eqs.(18) and (19), and then we can obtain the discrete conserved quantities associating to the system from the following discrete analogue of Noether theorems.

Theorem 1 If Lie group (5) of the infinitesimal transformations for the discrete Lagrange–Maxwell mechanico-electrical systems (18) and (19) on an uniform mesh h, where there is the discrete gauge function G, satisfies the discrete Noether identity (16), then the system possesses the discrete Noether conserved quantities (21) or (22).

If the discrete mechanico-electrical system satisfies conditions $Q_{\rm d}^i - \dfrac{\partial F_{\rm d}}{\partial q_k^i} = 0$, $u_k^j - \dfrac{\partial F_{\rm d}}{\partial e_k^j} = 0$, then from Eqs.(18) and (19) we can obtain

$$D_{-h}\left(\frac{\partial L_{\rm d}}{\partial q_k^i}\right) - \frac{\partial L_{\rm d}}{\partial q^i} = 0, \tag{23}$$

$$D_{-h}\left(\frac{\partial L_{\rm d}}{\partial e_k^j}\right) - \frac{\partial L_{\rm d}}{\partial e^j} = 0, \tag{24}$$

we call the system (23) and (24) the discrete Lagrange mechanico-electrical system.

If the left-hand side of Eq.(15) equals 0, i.e.

$$\xi_0\frac{\partial L_{\rm d}}{\partial t} + \xi_i\frac{\partial L_{\rm d}}{\partial q^i} + \eta_j\frac{\partial L_{\rm d}}{\partial e^j}$$
$$+\left[D_{+h}\left(\xi_i\right) - q_k^i D_{+h}\left(\xi_0\right)\right]\frac{\partial L_{\rm d}}{\partial q_k^i}$$
$$+\left[D_{+h}\left(\eta_j\right) - e_k^j D_{+h}\left(\xi_0\right)\right]\frac{\partial L_{\rm d}}{\partial e_k^j}$$
$$+L_{\rm d}D_{+h}\left(\xi_0\right) + D_{+h}\left(G\right) = 0, \tag{25}$$

which is called the discrete Noether identity for the Lagrange mechanico-electrical system (23) and (24), and these are the discrete Noether conserved quantity of the forms (21) or (22) with the discrete gauge function G associating with Eq.(25).

Theorem 2 If Lie group (5) of the infinitesimal transformations for the discrete Lagrange mechanico-electrical systems (23) and (24) on an uniform mesh h, where there is the discrete gauge function G, satisfies the discrete Noether identity (25), then the system possesses the discrete analogy of Noether conserved quantity of the forms (21) or (22)

5. An example

Capacitor C is expressed with the following Fig.2. It's an electrode plank (unit mass $m = 1$) which is hanged on a spring with stiffness k. The plank is vibrating along vertical direction under the action of gravity, elastic and electric field force.

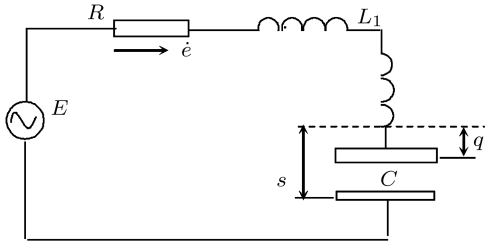


Fig.2. A electrocircuit of capacitor.

For this system, we choose the position of equilibrium (O point) of active plank of capacitor as the origin of the coordinate, and the active plank described by generalized coordinate q. The electrodynamical part is described by generalized electric quantity e. We denote by $T = \dfrac{1}{2}\dot{q}^2$ the kinetic energy of active plank of capacitor, by $W_{\rm m} = \dfrac{1}{2}\dot{e}^2$ the magnetic energy of capacitor, by $V = \dfrac{1}{2}kq^2$ the potential of spring, by $W_{\rm e} = \dfrac{1}{2}\dfrac{s-q}{A}e^2$ the electric energy of capacitor, by R the resistance, and by $E = E_0\sin\omega t$ the electromotive force. Then discrete Lagrange function $L_{\rm d}$ of the

system with an uniform mesh h is

$$L_{\mathrm{d}} = \frac{1}{2}\left(\frac{q^{+}-q}{h}\right)^{2} + \frac{1}{2}\left(\frac{e^{+}-e}{h}\right)^{2} - \frac{1}{2}kq^{2} - \frac{1}{2}\frac{s-q}{A}e^{2}, \tag{26}$$

where s and A are constants. The discrete dissipative function is

$$F_{\mathrm{d}} = F_{\mathrm{e}} = \frac{1}{2}R\left(\frac{e^{+}-e}{h}\right)^{2}, \tag{27}$$

and the non-conservative generalized force is

$$u_r = E_0 \sin\omega t. \tag{28}$$

In accordance with Noether theorem one can find the following operators of infinitesimal transformations [38]

$$X_1 = \frac{\partial}{\partial t}, \tag{29}$$

$$X_2 = \frac{\partial}{\partial t} + \dot{q}\frac{\partial}{\partial q} + \dot{e}\frac{\partial}{\partial e}, \tag{30}$$

$$X_3 = \dot{q}\frac{\partial}{\partial q} + \dot{e}\frac{\partial}{\partial e}, \tag{31}$$

$$X_4 = \frac{\partial}{\partial t} + \dot{q}\frac{\partial}{\partial q}, \tag{32}$$

$$X_5 = \frac{\partial}{\partial t} + \dot{e}\frac{\partial}{\partial e}, \tag{33}$$

$$X_6 = \dot{q}\frac{\partial}{\partial q}, \tag{34}$$

$$X_7 = \dot{e}\frac{\partial}{\partial e}. \tag{35}$$

Substituting Lie group associated with operator (29) into the discrete Noether identity (16), we obtain

$$G_1 = -E_0 e\sin\omega t + Re, \tag{36}$$

then using discrete Lagrangian and G_1 in Eq.(21) we get the discrete Noether conservation law

$$D_{+h}\left\{-\frac{1}{2}\left(\frac{q-q^{-}}{h}\right)^{2} - \frac{1}{2}\left(\frac{e-e^{-}}{h}\right)^{2} - \frac{1}{2}k\left(q^{-}\right)^{2} - \frac{1}{2}\frac{q^{-}}{A}\left(e^{-}\right)^{2} - E_0\sin\omega t + Re\right\} = 0, \tag{37}$$

namely the system possesses the discrete Noether conserved quantity

$$I_{\mathrm{d1}} = \frac{1}{2}\left(\frac{q-q^{-}}{h}\right)^{2} + \frac{1}{2}\left(\frac{e-e^{-}}{h}\right)^{2} + \frac{1}{2}k\left(q^{-}\right)^{2} + \frac{1}{2}\frac{q^{-}}{A}\left(e^{-}\right)^{2} - E_0 e^{-}\sin\omega t + Re^{-} = \mathrm{const}. \tag{38}$$

Substituting operator (30) into the discrete Noether identity (16) one has

$$G_2 = -\frac{1}{2}\left(\frac{q^{+}-q}{h}\right)^{2} - \frac{1}{2}\left(\frac{e^{+}-e}{h}\right)^{2} - \frac{1}{2}kq^{2} + \frac{1}{2A}e^{2} - \frac{s-q}{2A}e^{2} + E_0 e, \tag{39}$$

then using discrete Lagrangian and gauge function G_2 in Eq.(21) we have

$$D_h\left\{\frac{1}{2}\left(\frac{q-q^{-}}{h}\right)^{2} + \frac{1}{2}\left(\frac{e-e^{-}}{h}\right)^{2} - \frac{1}{2}k\left(q^{-}\right)^{2} - \frac{1}{2}\frac{q^{-}}{A}\left(e^{-}\right)^{2} + G\right\} = 0, \tag{40}$$

namely

$$I_{\mathrm{d2}} = \frac{1}{2A}\left(e^{-}\right)^{2} - \frac{s-q}{A}\left(e^{-}\right)^{2} + Re^{-} = \mathrm{const}. \tag{41}$$

We should show that we can find the numerical value solution of discrete gauge function G, and further substitute the result into Eq.(21) to obtain the discrete conserved quantity.

Substituting operator (31) into the discrete Noether identity (16) we obtain

$$G_3 = -\frac{1}{2}\frac{q^{+}-q}{h} - \frac{1}{2}\frac{e^{+}-e}{h} - \frac{1}{2}kq^{2} + \frac{1}{2A}e^{2} - \frac{s-q}{2A}e^{2} + E_0\sin\omega t, \tag{42}$$

then using discrete Lagrangian and gauge function G_3 in Eq.(21) we get

$$D_{+h}\left\{\frac{1}{2}\left(\frac{q-q^{-}}{h}\right)^{2} + \frac{1}{2}\left(\frac{e-e^{-}}{h}\right)^{2} - \frac{1}{2}kq^{2} + \frac{1}{2A}\left(e^{-}\right)^{2} - \frac{s-q}{2A}\left(e^{-}\right)^{2} + E_0 e\sin\omega t\right\} = 0, \tag{43}$$

namely

$$I_{\mathrm{d3}} = \frac{1}{2}\left(\frac{q-q^{-}}{h}\right)^{2} + \frac{1}{2}\left(\frac{e-e^{-}}{h}\right)^{2} - \frac{1}{2}kq^{2} + \frac{1}{2A}\left(e^{-}\right)^{2} - \frac{s-q}{2A}\left(e^{-}\right)^{2} + E_0 e^{-}\sin\omega t = \mathrm{const} \tag{44}$$

Similarly, for vector fields (32), (33), (34) and (35), we can obtain the discrete gauge functions and conserved quantities as follows:

$$G_4 = -\frac{1}{2}\left(\frac{q^+ - q}{h}\right)^2 - \frac{1}{2}kq^2 + \frac{1}{2A}e^2 - E_0 e\sin\omega t + Re,$$

$$I_{\mathrm{d}4} = \frac{1}{2}\left(\frac{e - e^-}{h}\right)^2 - \frac{s - q^-}{2A}\left(e^-\right)^2 + \frac{1}{2A}\left(e^-\right)^2 - E_0 e^-\sin\omega t + Re^- = \mathrm{const}; \quad (45)$$

$$G_5 = -\frac{1}{2}\frac{e^+ - e}{h} - \frac{s - q}{2A}e^2 + Re,$$

$$I_{\mathrm{d}5} = -\frac{1}{2}\frac{q - q^-}{h} - \frac{1}{2}k\left(q^-\right)^2 - \frac{s - q}{A}\left(e^-\right)^2 + Re^- = \mathrm{const}; \quad (46)$$

$$G_6 = -\frac{1}{2}\left(\frac{q^+ - q}{h}\right)^2 - \frac{1}{2}kq^2 + \frac{1}{2A}e^2,$$

$$I_{\mathrm{d}6} = \frac{1}{2}\frac{q - q^-}{h} - \frac{1}{2}k\left(q^-\right)^2 + \frac{1}{2A}\left(e^-\right)^2; \quad (47)$$

$$G_7 = -\frac{1}{2}\left(\frac{e^+ - e}{h}\right)^2 - \frac{s - q}{2A}e^2 + E_0 e\sin\omega t,$$

$$I_{\mathrm{d}7} = \frac{1}{2}\left(\frac{e - e^-}{h}\right)^2 - \frac{s - q^-}{2A}\left(e^-\right)^2 + E_0 e^-\sin\omega t. \quad (48)$$

6. Conclusion

In this work, we have studied Noether symmetries and conservation law of the discrete mechanico-electrical systems. The results here present significant approaches to seek conservation laws for these discrete systems.

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